Imaging of the Pion on a Fine Lattice

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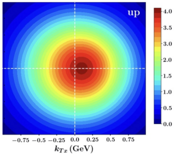
3D Imaging of Hadrons

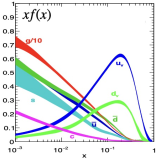
Transvers momentum distributions (TMDs)



Cammarota, et al. (JAM), PRD **102** (2020).

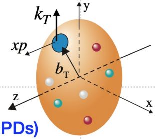
$\int d^2\vec{k}_T$

PDFs



NNPDF, EPJ C77 (2017)

$\int d^2\vec{b}_T$



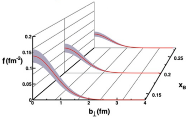
$\int d^2\vec{b}_T$

Wigner distributions/Generalized TMDs

$W(x, \vec{k}_T, \vec{b}_T)$

$\int d^2\vec{k}_T$

Generalized parton distributions (GPDs)

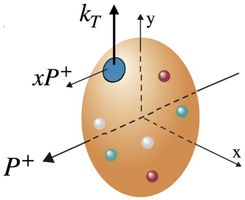


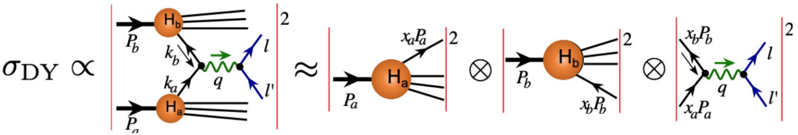
W. Armstrong et al., arXiv: 1708.00888.

From Y. Zhao

TMDPDFs

- Distribution densities to find a parton with specific momentum;
- Related to the cross section of TMD processes;
- Our knowledge is still limited.

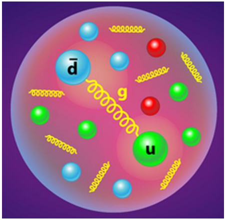




Boussarie, et al. 2304.03302

Global fitting: limited data points to fit multi-dimensional functions.

Pion TMDPDFs



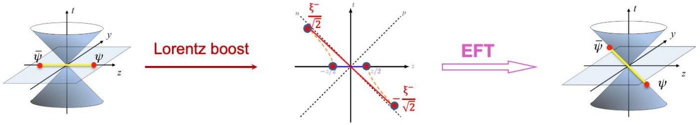
By APS/Alan Stonebraker

- Physically: pseudo-Goldstone boson; chiral symmetry breaking
- Numerically: best signal on lattice; easy to get large Lorentz factor; benchmark for other hadrons
- Experimentally: Pion-induced Drell-Yan process of FNAL and COMPASS(CERN)

Large Momentum Effective Theory (LaMET)

- Light-cone distribution: separate in time direction, universal.
- Quasi-distribution: equal time, P^z dependent.

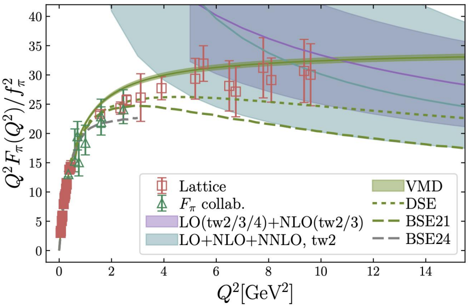
$$\frac{\overbrace{\boxed{\tilde{f}_i(x,b_\perp,\mu,P^z)}}^{\text{Quasi-TMDPDF}}}{\underbrace{\boxed{\sqrt{S_r^q(b_\perp,\mu)}}}_{\text{Soft function}}} = \overbrace{\boxed{C_q(\mu,xP^z)}}^{\text{Hard kernel}} \exp\left[\frac{1}{2}\underbrace{\boxed{K_q(b_\perp,\mu)}}_{\text{Collins-Soper kernel}} \ln\frac{(2xP^z)^2}{\zeta}\right] \underbrace{\boxed{f_i(x,b_\perp,\mu,\zeta)}}_{\text{Light-cone TMDPDF}} + \overbrace{\boxed{\ldots}}^{\text{Power corrections}}$$



Lattice Setup

- Volume: $48^3 \times 64$ with $a = 0.06$ fm, generated by HotQCD;
- Sea quark: $N_f = 2 + 1$ Highly Improved Staggered Quark (HISQ) action with physical quark masses;
- Valence quark: Wilson-Clover action on 1-step hypercubic (HYP) smeared gauge configurations, valance pion mass $m_\pi = 300$ MeV;
- Hadron momentum: $P_z = \{1.29, 1.72, 2.15\}$ GeV;
- Multiple exact and sloppy Dirac operator inversions using All-Mode Averaging (AMA);
- Boosted Gaussian smearing for quarks to achieve better overlap with boosted hadrons (Radius ≈ 0.59 fm, $m_q \approx 0.7\, m_\pi$).

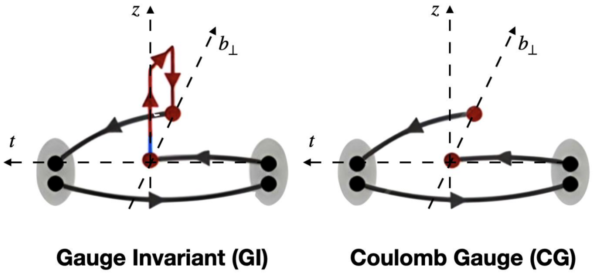
Momentum Smearing: Pion Electromagnetic Form Factors



H. T. Ding, et al. 2404.04412

- HotQCD: HISQ + Wilson-clover + HYP, $m_\pi = 140$ MeV, $a = 0.076$ fm, V = $64^3 \times 64$;
- Boosted Gaussian smearing helps to reach very large $Q^2 \sim 10\ \mathrm{GeV}^2$.

Quasi-TMDPDF under Coulomb Gauge without Wilson Line



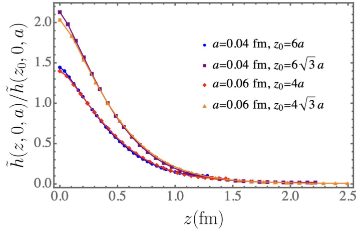
The quasi-TMD matrix elements of the pion under CG are defined as

$$\tilde{h}_\Gamma(b_\perp, z, \mu, P^z) = \langle \pi^+; P_z | \overline{\psi}(\frac{b_\perp}{2}, \frac{z}{2}) \Gamma \psi(-\frac{b_\perp}{2}, -\frac{z}{2}) |_{\nabla \cdot \vec{A}=0} | \pi^+; P_z \rangle$$

CG v.s. GI: Simple Renormalization

The renormalization of the quark correlator is simply multiplicative and z-independent, take the pion quasi-PDF as an example:

$$\bar{\psi}_B(z)\Gamma\psi_B(0) = Z_\psi(a)\left[\bar{\psi}(z)\Gamma\psi(0)\right]_R$$

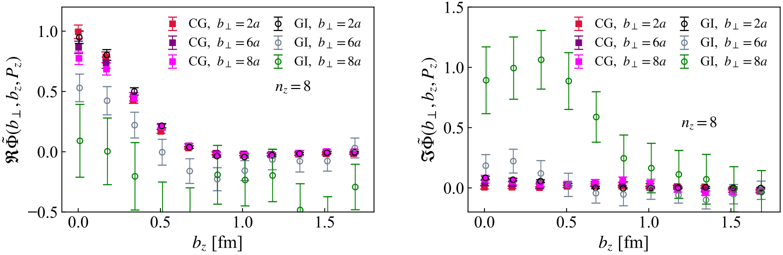


X. Gao, et al. 2306.14960

- HotQCD: HISQ + Wilson-clover + HYP, m_π = 300 MeV, $a = \{0.04, 0.06\}$ fm, V = $\{64^3 \times 64, 48^3 \times 64\}$;
- $\tilde{h}(z,a)/\tilde{h}(z_0,a)$ should be independent of a, i.e. no linear div.

CG v.s. GI: Better Signal

Pion quasi-TMDWF with CG and GI approaches.

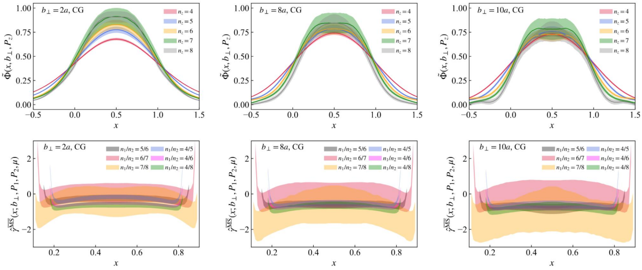


D. Bollweg, et al. 2403.00664

- DWF: Mobius DW + Iwasaki, physical m_π, a = 0.0836 fm, V = $64^3 \times 128 \times 12$;
- CG has much better signal than GI;
- CG has smaller tail at large z, does not need extrapolation.

CS Kernel Extracted from Quasi-TMDWF

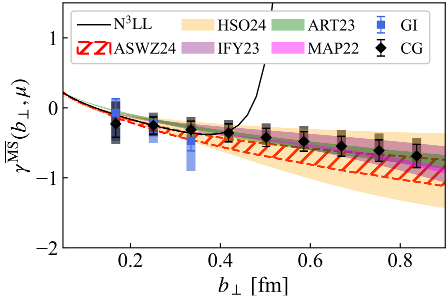
$$K(b_\perp,\mu) = \frac{1}{\ln\left(P_1^z/P_2^z\right)} \ln \frac{H^\pm\left(xP_2^z,\mu\right)\tilde{\Psi}^\pm\left(x,b_\perp,\mu,P_1^z\right)}{H^\pm\left(xP_1^z,\mu\right)\tilde{\Psi}^\pm\left(x,b_\perp,\mu,P_2^z\right)}$$



D. Bollweg, et al. 2403.00664

- Quasi-TMDWF has a good behavior even for large $b_\perp \sim 0.85$ fm;
- The x-independent CS kernel can be extracted from good plateaus in x.

CS Kernel Results



A. Avkhadiev, et al. 2402.06725 (ASWZ24)

- MILC: HISQ + Symanzik, physical m_π, $a = \{0.09, 0.12, 0.15\}$ fm, $V = \{64^3 \times 96, 48^3 \times 64, 32^3 \times 48\}$;
- Good consistency with the state-of-the-art GI lattice calculation;
- Good consistency with phenomenological results;
- Smaller uncertainties to reach the large separation region.

Intrinsic soft function

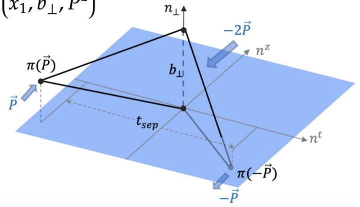
$$\frac{\tilde{f}_i(x, b_\perp, \mu, P^z)}{\boxed{\sqrt{S_r^q(b_\perp, \mu)}}} = C_q(\mu, xP^z) \exp\left[\frac{1}{2} K_q(b_\perp, \mu) \ln \frac{(2xP^z)^2}{\zeta}\right] f_i(x, b_\perp, \mu, \zeta)$$

Soft function

The soft function can be extracted from the quasi-TMDWFs (*D. Bollweg, et al. 2403.00664*) and the light-meson form factor F (*Q. A. Zhang, et al. (LPC) PRL 125 (2020)*).

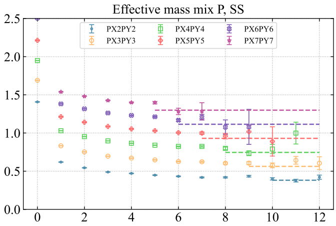
$$S_r(b_\perp, \mu) = \frac{F(b_\perp, P_1, P_2, \Gamma, \mu)}{\int dx_1 dx_2 H(x_1, x_2, \Gamma) \tilde{\Psi}^{\pm}_{\bar{C}G}(x_2, b_\perp, P^z) \tilde{\Psi}^{\pm}_{\bar{C}G}(x_1, b_\perp, P^z)}$$

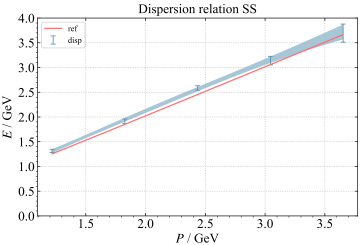
$$F(b_\perp, P_z) = \left\langle \pi(-\vec{P}) \middle| \bar{q}(b_\perp)\Gamma q(b_\perp)\bar{q}(0)\Gamma' q(0) \middle| \pi(\vec{P}) \right\rangle$$



2pt of Pion

Dispersion relation: $E^2 = m^2 + c_1 P^2 + c_2 a^2 P^4$

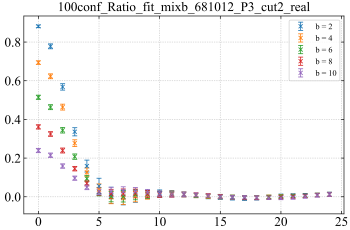


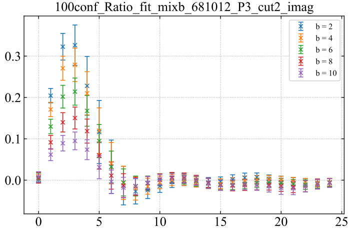


- Good signal-to-noise ratio even for large momentum;
- Fit results: $c_1 = 1.16(13)$ and $c_2 = -0.10(15)$.

Bare Quasi-TMDPDF

The preliminary results (100 configurations) of pion quasi-tmdpdf with momentum $P^z = 1.8$ GeV.





- Good signal-to-noise ratio even for large $b_\perp$;
- Small tails at long range, easy to extrapolate.

Estimated Cost

Task	Symbol	Time (sec.)
Extended source/sink creation	t_{ext}	17
MG light-quark inversion sloppy	t_{inv}^{l}	17
MG light-quark inversion exact	t_{inv}^{l}	26
Contraction for pion 2pt function	t_{2pt}^{π}	13
Contraction for 3pt function for quasi-TMD	$t_{\text{3pt}}^{\text{TMD}}$	0.25×25^2

Table 1: Times needed for various parts of the calculations on the $48^3 \times 64$, $a = 0.06$ fm lattice, with $m_\pi = 300$ MeV, using 1 node of Polaris at ALCF (4 A100 cards).

Resource Request

The time of the calculations on a single gauge configuration and one source is

$$t = N_{\text{mom}} \times \left(2t^{l}_{\text{inv}} + 2t^{\pi}_{\text{2pt}} + 4t_{\text{ext}} + (t^{l}_{\text{inv}} + t_{\text{ext}} + t^{\text{TMD}}_{\text{3pt}}) \times N_{\text{sep}}\right)$$

Computaional Resources:

- 105k GPU hours on FNAL

Storage Resources:

- 50 TB disk storage
- 100 TB long-term storage

Summary

- Pion TMDs are important for understanding the chiral symmetry breaking and QCD structure, it is also a good benchmark;
- Coulomb gauge method has significant advantages in the lattice calculation of TMDs, especially in large separation region ($b_\perp \gtrsim 1\text{fm}$);
- Our methods and technologies have been examed in many previous works, we are able to calculate the pion TMDs with controllable uncertainties.

Q & A

Q. Systematics from using meson form factors in literature?
A. Soft function can be checked by comparing with perturbative results; a 25% error is still meaningful.

Q. Compute the pion quasi-TMD with the standard gauge invariant definition?
A. Yes, they share the same quark propagators, will do partial computation.

Q. Preliminary results to demonstrate the computational effectiveness?
A. Pion PDF; Pion quasi-TMDWF & CS kernel; Preliminary 2pt and 3pt results of Pion TMDPDF.

Q. Estimation of how the noise increases with each unit of momentum?
A. Effective mass and dispersion relation plots.

Thank You For Attention!

Backup: Estimated Cost 2023

Task	Symbol	Time (sec.)
Extended source/sink creation	t_{ext}	10
MG light-quark inversion sloppy	t^{l}_{inv}	26
MG light-quark inversion exact	t^{l}_{inv}	50
Contraction for kaon 2pt function	t^{π}_{2pt}	10
Contraction for 3pt function for FF	$t^{\text{FF}}_{\text{3pt}}$	4

Table 2: Timings using 16 nodes of the BNL IC cluster (32 K80 cards)

Backup: Gribov Copies - Quasi Pion PDF

- Mainly affect the gauge propagator
- Lattice Gribov noise + measurement distortion

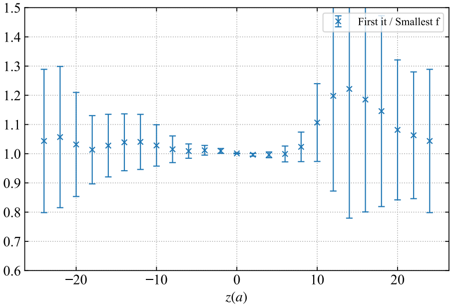


Figure 1: The ratio of quasi pion PDF from two sets of Gribov copies.

Backup: Gribov Copies - Quark Spatial Propagator

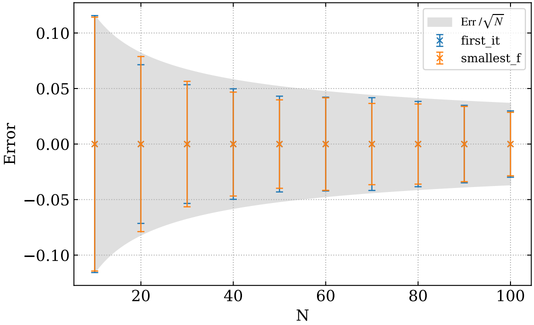


Figure 2: The error of effective mass at $z = 10$ a as a function of number of configurations N. The grey band shows the expected error that decreases as $1/\sqrt{N}$.

Backup: GI Quasi-TMDPDF