3D Imaging of the Pion on a Fine Lattice

Phys. Rev. D 112 (2025) 2504.04625

And on going projects …

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CFNS Seminar 2025/10/15

Center for Frontiers in Nuclear Science

Stony Brook University

June 14, 2024

We are pleased to provide this certificate of participation for:

Jinchen He
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who attended the

2024 CFNS Summer School
on the Physics of the Electron-Ion Collider,

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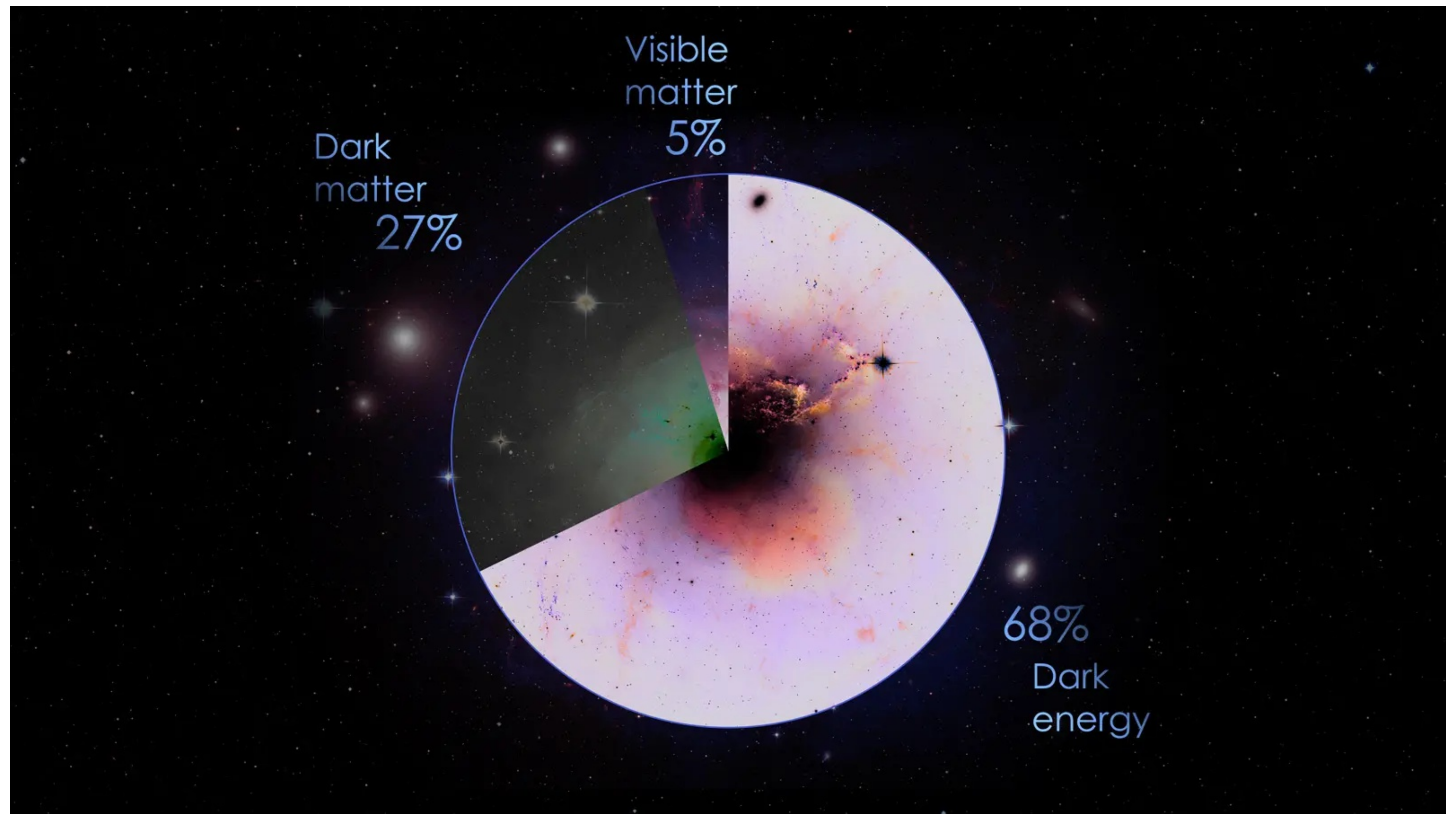
Contents

Introduction

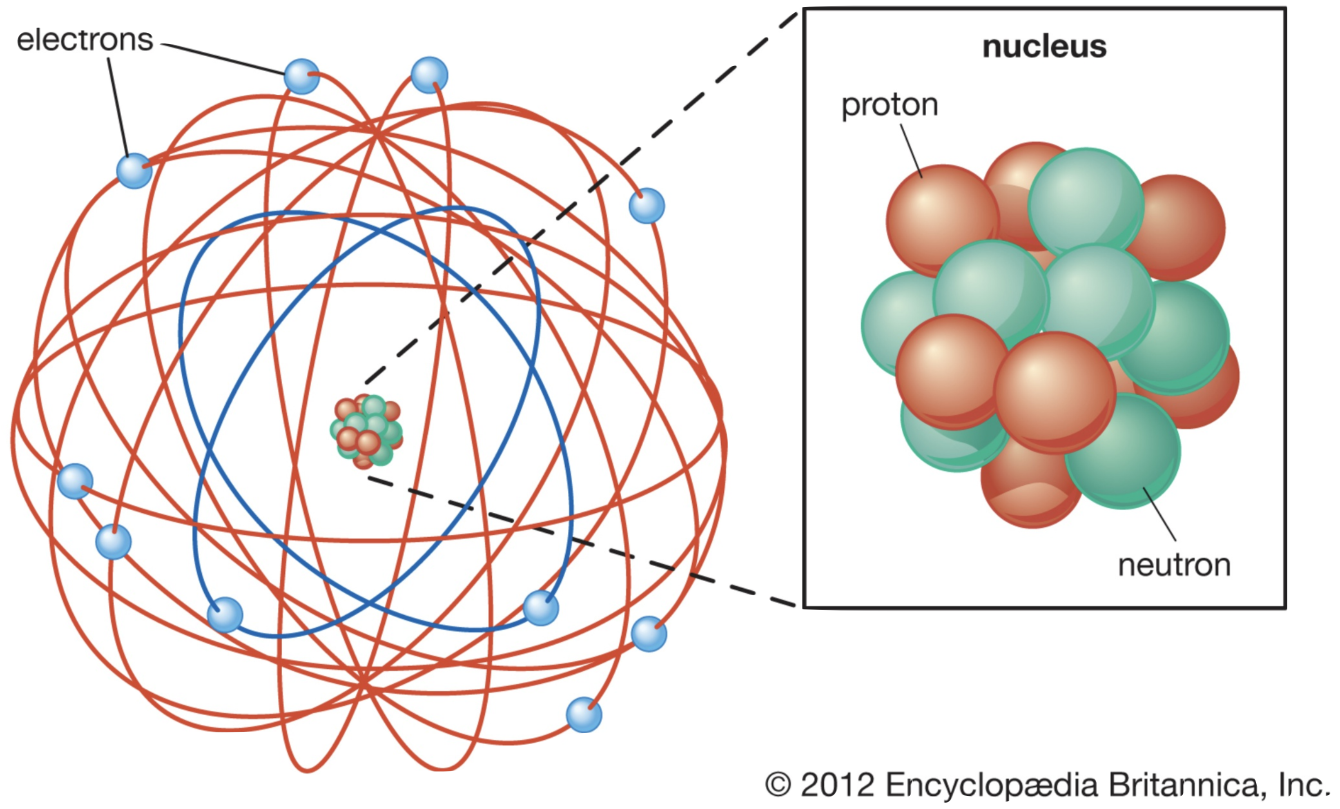
Visible Universe

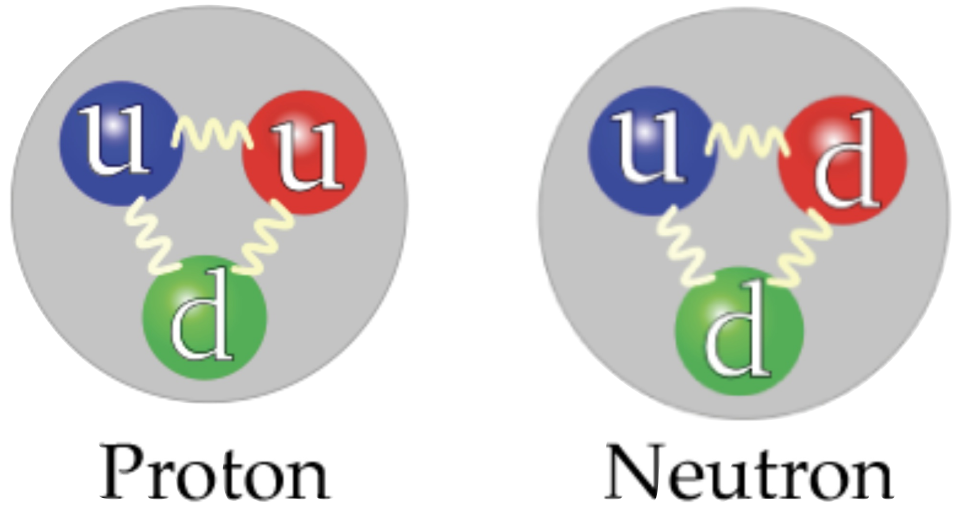
- Only 5% of the universe is visible. *Spergel, David N. "The dark side of cosmology: Dark matter and dark energy." Science 347.6226 (2015): 1100-1102.*

- The visible universe is made up of protons and neutrons, the inner structure of hadrons are sophisticated if we step closer.



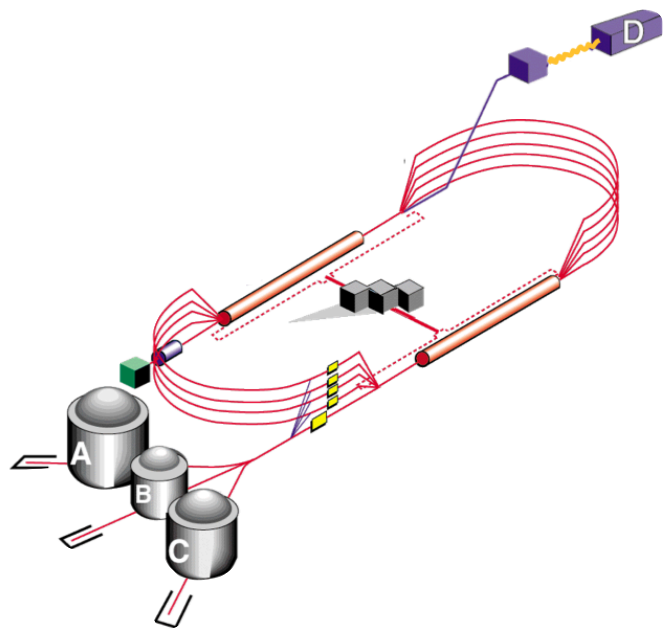
Cr. NASA's Goddard Space Flight Center





- Many experiments have been designed to probe the internal structure of hadrons. *P.S. The list of experiments here is not complete.*

CEBAF(JLab)



Cr. Dave Gaskell

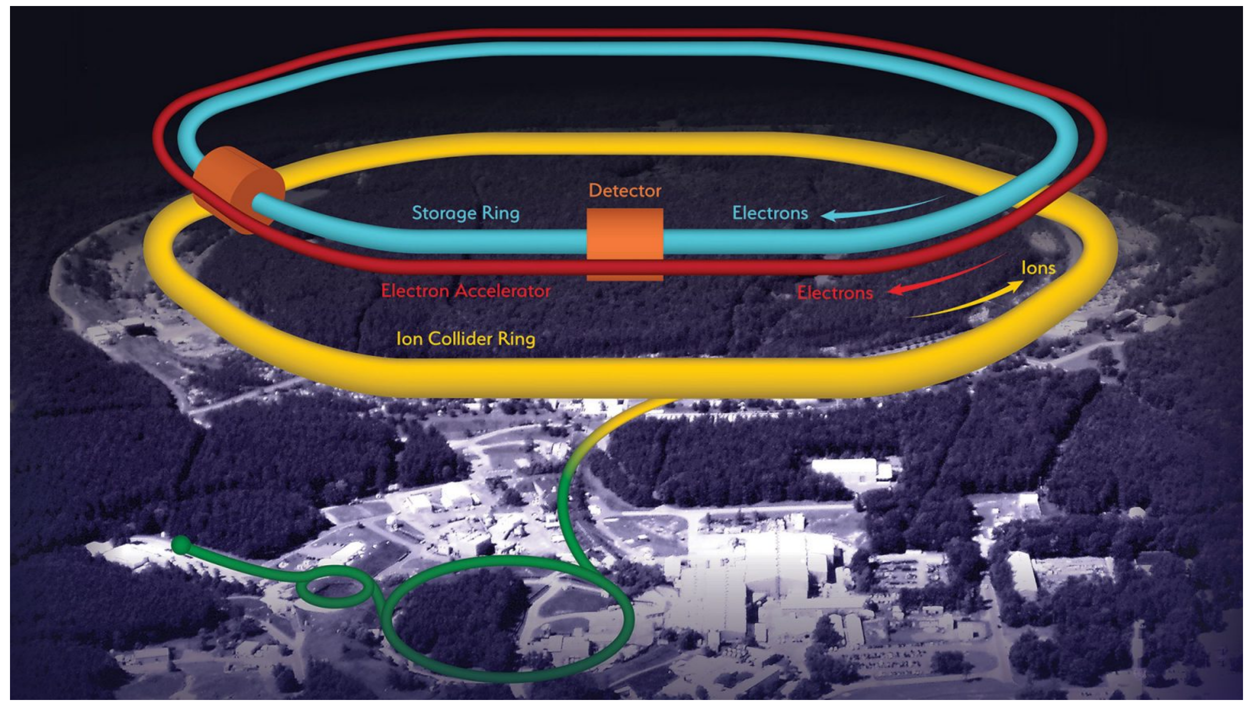
HERA

Cr. DESY

LHC

Cr. CERN

EIC



Cr. BNL

Parton Physics

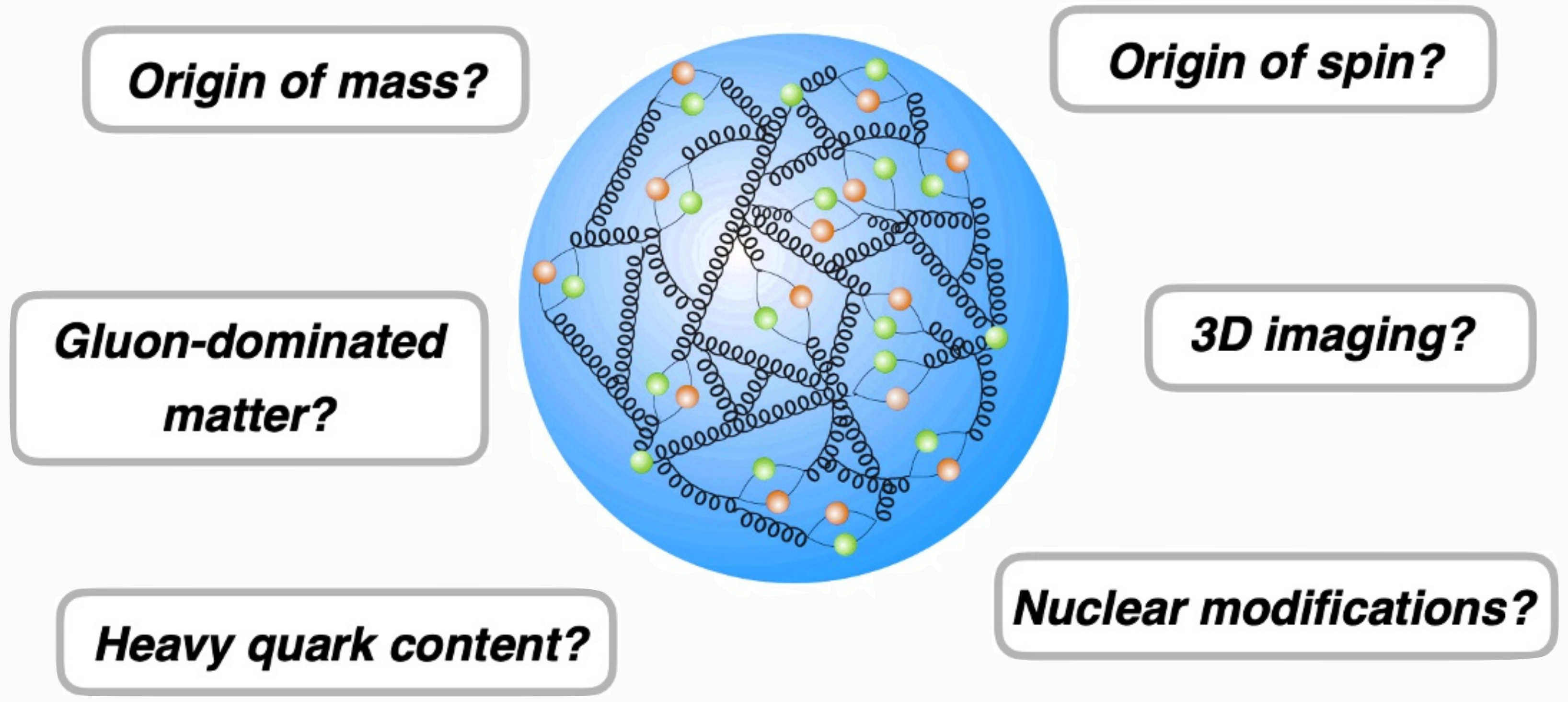
- Our knowledge on hadron is still limited:
  - Spin, mass …
  - How to describe a relativistic moving strong-coupled bound state?
- The language from Feynman: Parton Model in the infinite momentum frame

R. P. Feynman, Conf. Proc. C 690905 (1969)

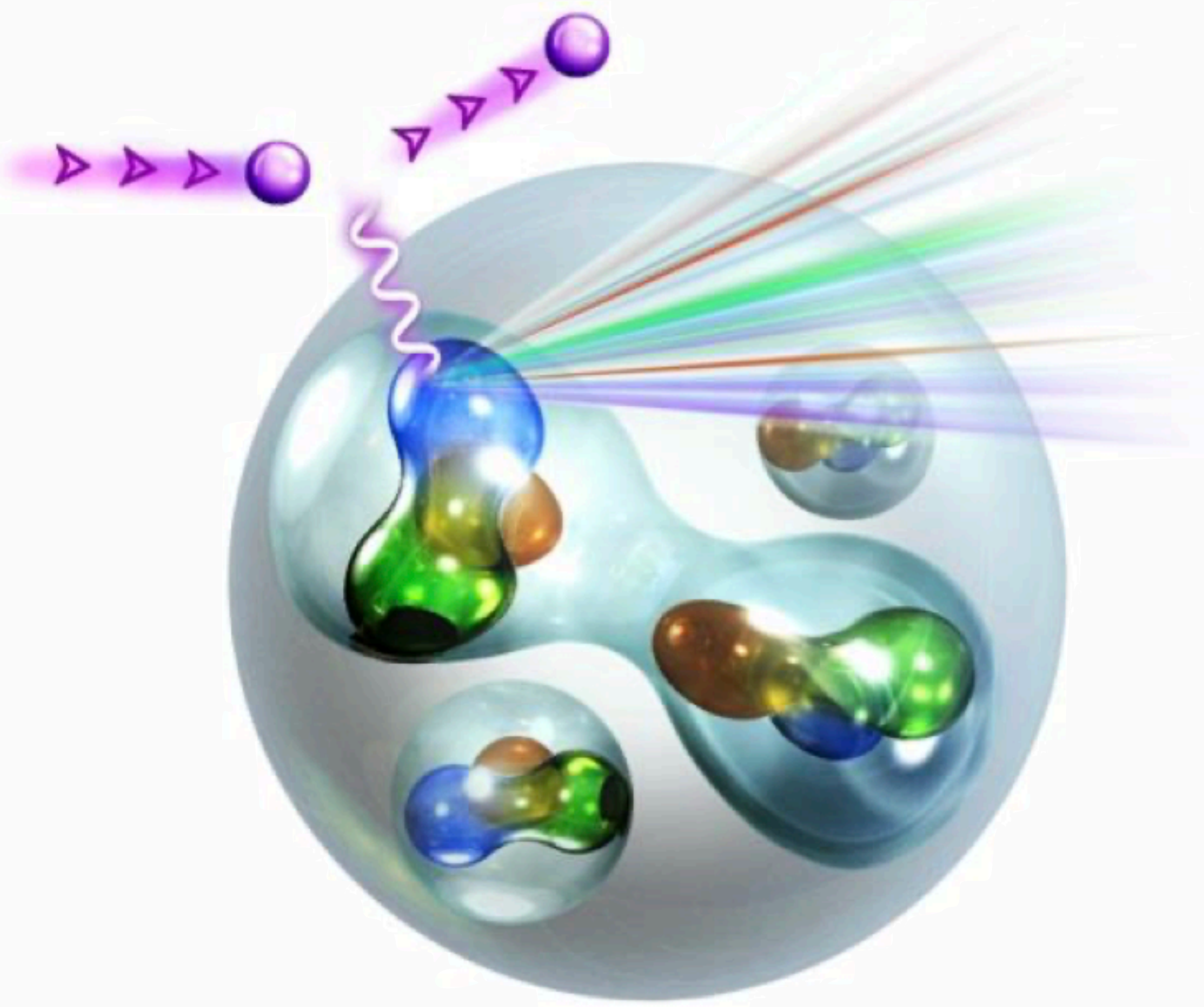
  - Quarks and gluons (partons) are “frozen” in the transverse plane;
  - During a high-energy collision, the struck parton appears like a free particle.

The many faces of the proton

QCD bound state of **quarks** and **gluons**

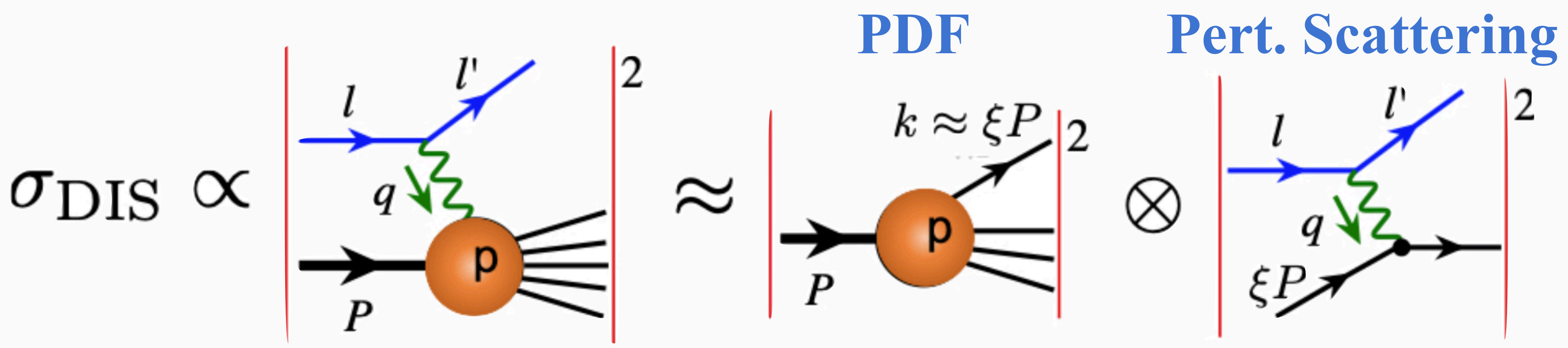


Cr. Juan Rojo



Cr. Dave Gaskell

QCD Factorization



R. Boussarie, et al., ``TMD Handbook”, [arXiv:2304.03302 [hep-ph]]

3D Imaging of Hadron

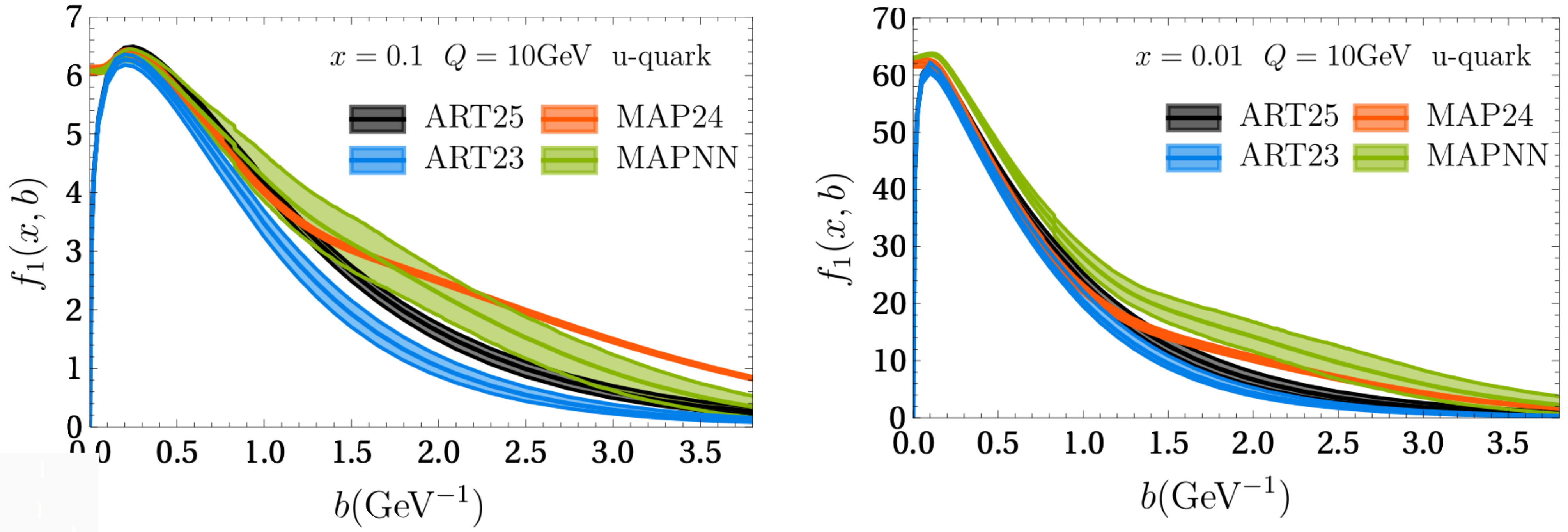
x **is the momentum fraction in the longitudinal (hadron momentum) direction.**

Wigner Distribution / GTMD

$$W(x, \vec{r}_\perp, \vec{k}_\perp)$$

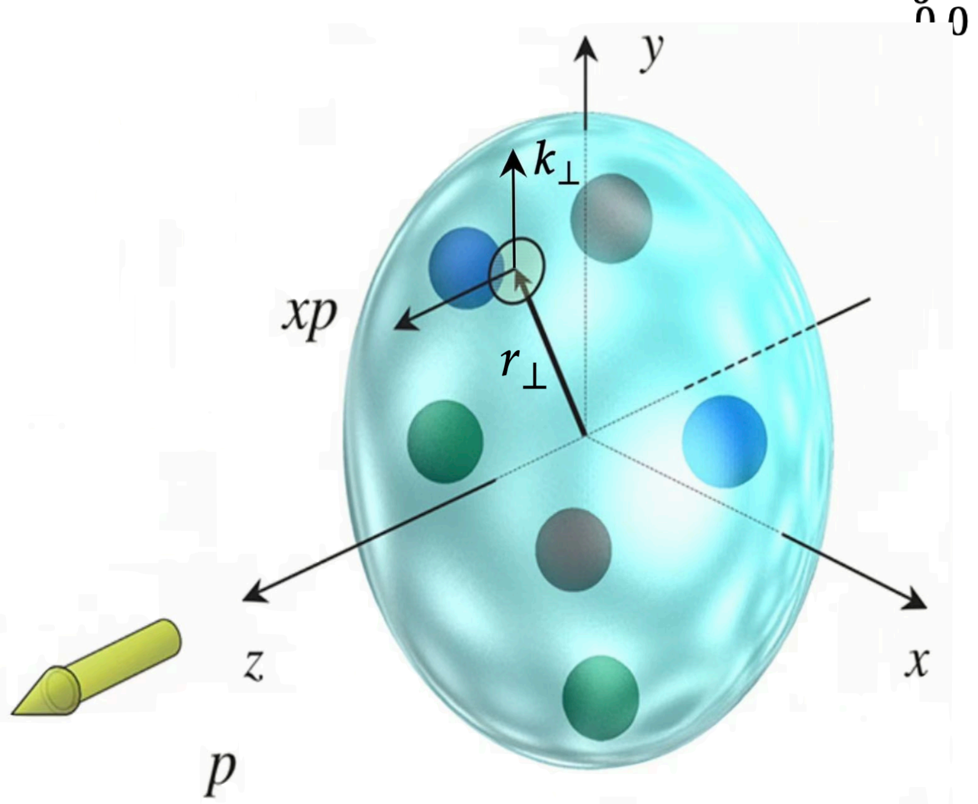
$\int d^2\vec{r}_\perp$ →

Transverse-Momentum-Dependent distributions (TMDs)



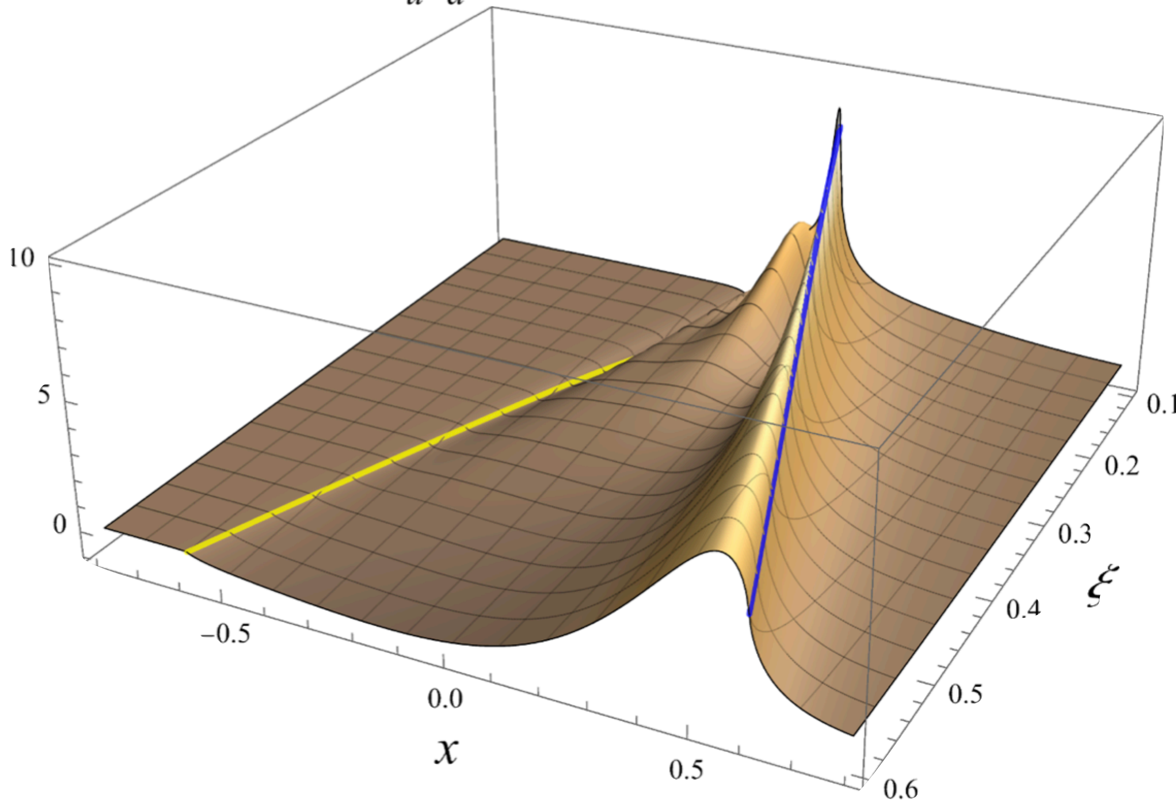
V. Moos, et al., 2503.11201 (2025)

↓ $\int d^2\vec{k}_\perp$



↓ $\int d^2\vec{k}_\perp$

Generalized Parton Distributions (GPDs)

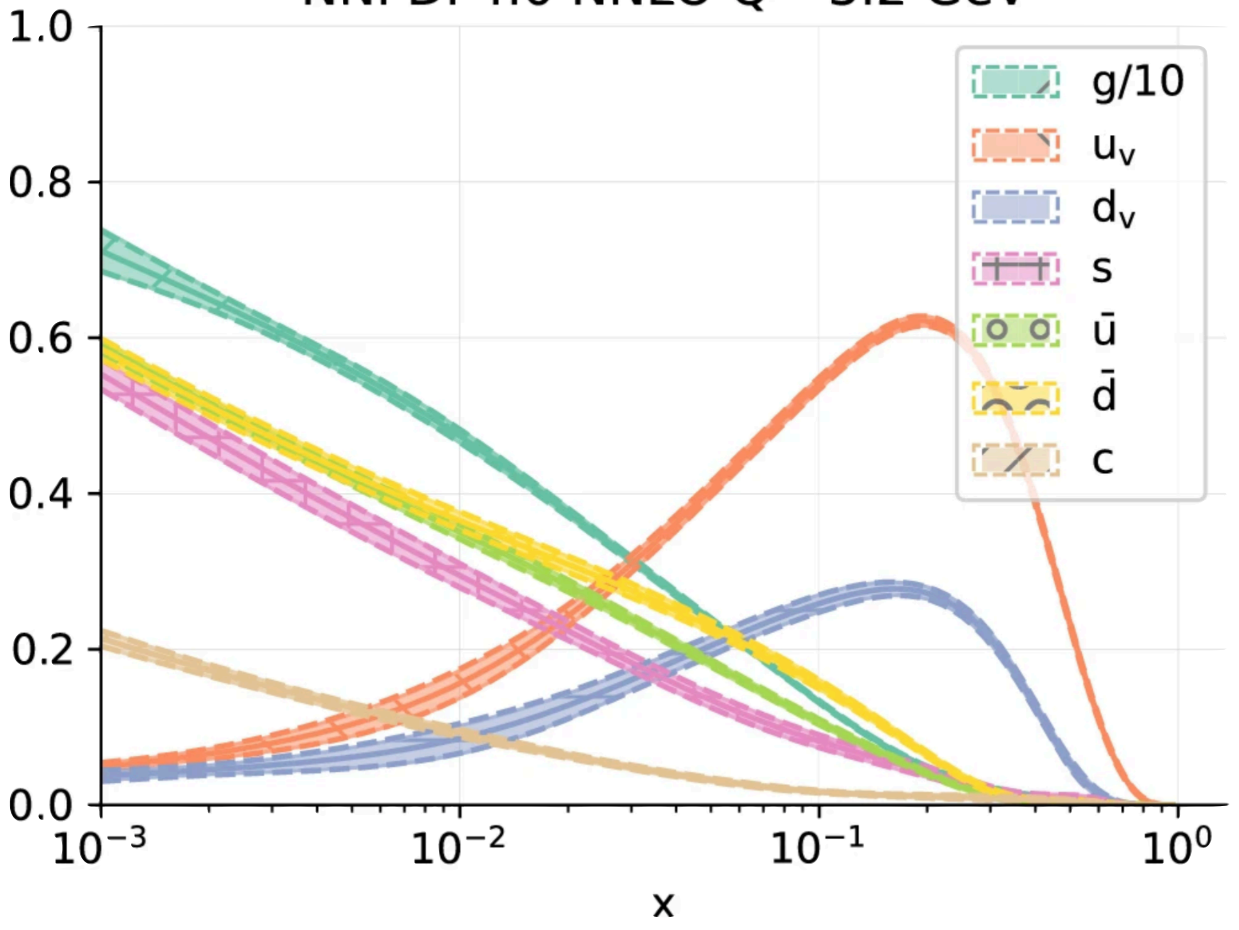


Y. Guo, et al., JHEP 09 (2022)

Latest results of GUMP 1.0 in 2509.08037 [hep-ph]

$\int d^2\vec{r}_\perp$ →

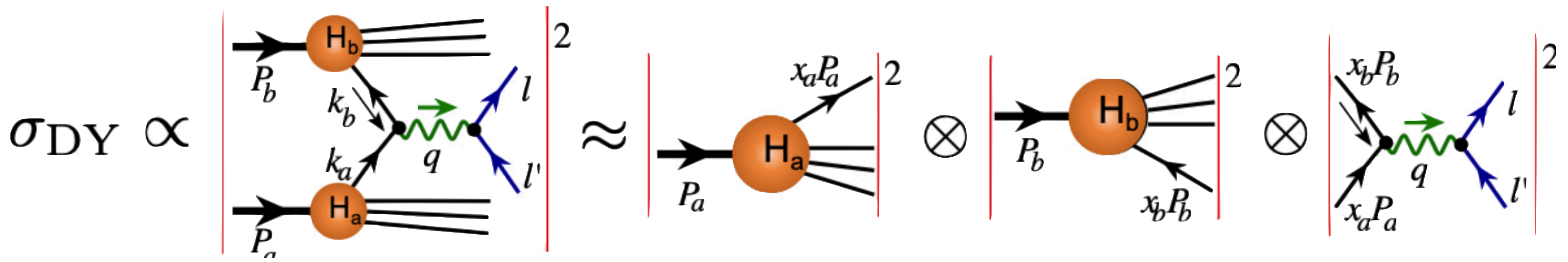
Parton Distribution Functions (PDFs)



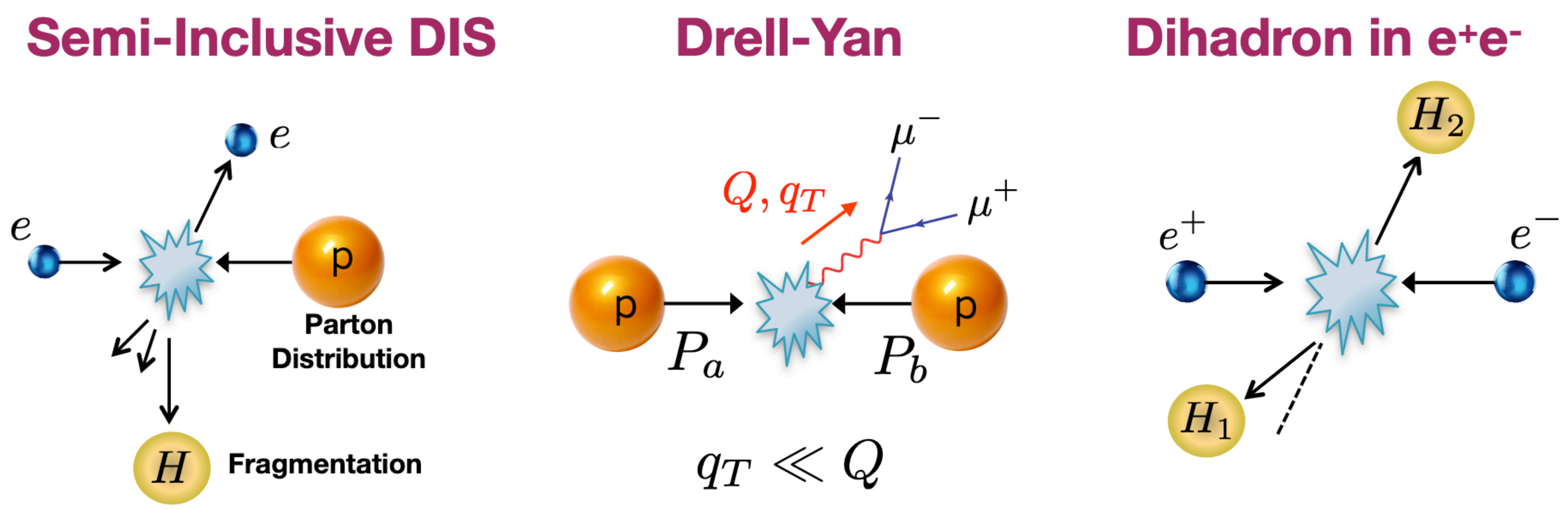
R. D. Ball, et al. [NNPDF], Eur. Phys. J. C 82 (2022)

TMDs in Experiments

- **TMDPDFs**: the distribution densities of finding a parton carrying a longitudinal momentum fraction x and transverse momentum $k_\perp$ in a hadron;
- TMD processes are important processes in high energy collisions, like Drell-Yan process on LHC and Semi-Inclusive DIS on EIC;



$$\frac{d\sigma_{H_a+H_b\to l\bar{l}+X}}{dQ^2\, dY\, d^2\vec{q}_T} = \frac{4\pi\alpha^2}{3N_cQ^2S}\sum_i e_i^2 \int d^2\vec{k}_{a\perp}\, d^2\vec{k}_{b\perp}\,\delta^{(2)}\left(\vec{q}_T-\vec{k}_{a\perp}-\vec{k}_{b\perp}\right) \times f_{1(i/H_a)}\left(x_a,\vec{k}_{a\perp}\right) \times f_{1(\bar{i}/H_b)}\left(x_b,\vec{k}_{b\perp}\right)$$



Leading Quark TMDPDFs — Nucleon Spin — Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity; $h_{1T}^\perp$ = − Pretzelosity

R. Boussarie, et al., 2304.03302 (2023)

Phenomenological Extraction of TMDs

- **Significant progress has been made in the phenomenological parameterizations of TMDs**

  Some of groups will incorporate neural network;

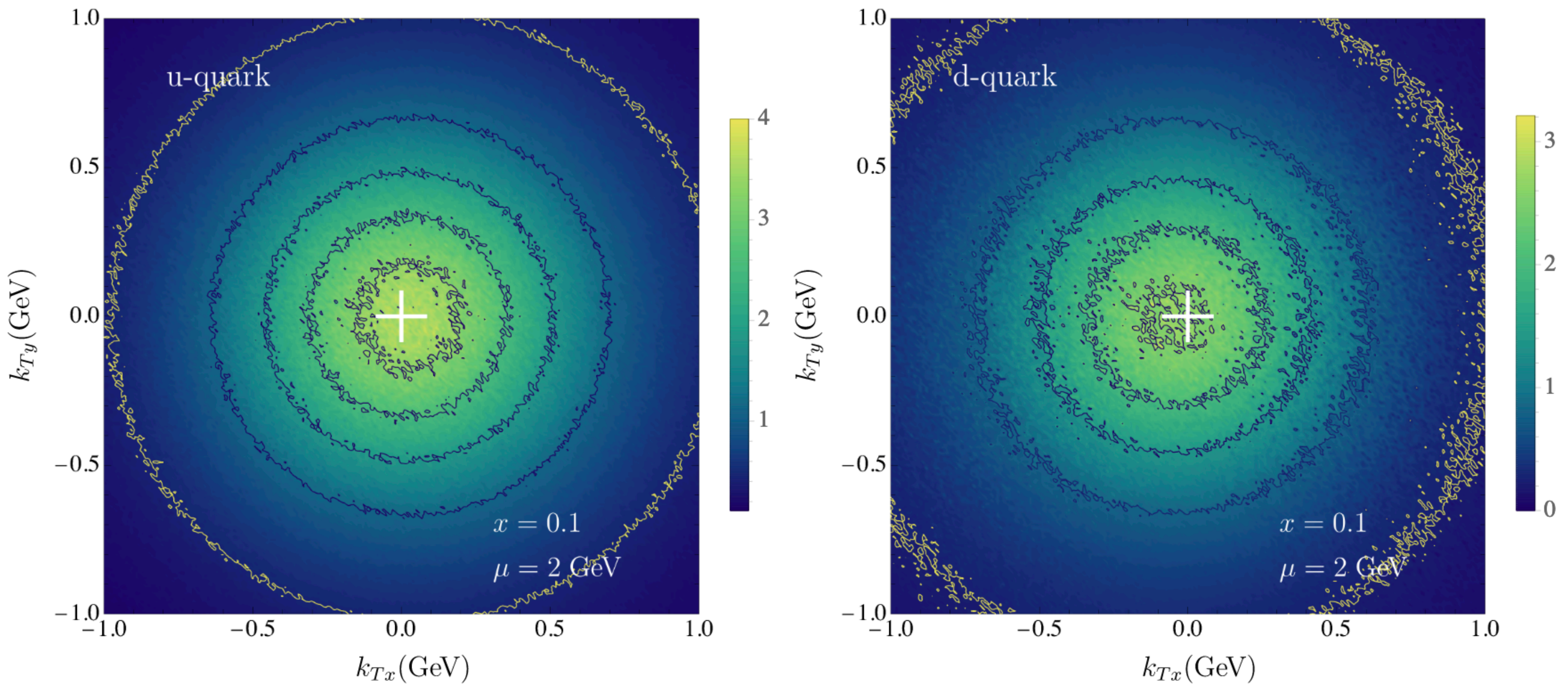
  Some of groups will include lattice data;

  - **Collins-Soper kernel (CS kernel): rapidity evolution kernel of TMDs**

    A. Bacchetta, et al. (MAP) JHEP 08 (2024); V. Moos, et al., 2503.11201 …

  - **Nucleon TMDs**
    - **Unpolarized** *M. Bury, et al., JHEP 10 (2022); A. Bacchetta, et al., JHEP 10 (2022); V. Moos, et al., JHEP 05 (2024) …*
    - **Sivers** *M. Bury, et al., PRL 126 (2021); I. P. Fernando, et al., Phys.Rev.D 108 (2023) …*
    - **Boer-Mulders** *Z. Lu, et al., Phys.Rev.D 81 (2010); X. Liu, et al., Eur.Phys.J.C 81 (2021) …*
  - **Pion TMDs: much less is known about the TMDs of the pion**
    - **Unpolarized** *A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), Phys.Rev.D 107 (2023); P. C. Barry, et al. (JAM) Phys.Rev.D 108 (2023)*

Tomographic scan of the nucleon via quark density function



M. Bury, et al., JHEP 05 (2021)

As the lightest pseudo Nambu-Goldstone boson, the 3D structure of pion will help us understand the strong interaction, such as the origin of chiral-symmetry breaking.

Lattice QCD

- **Path integral formalism**

Wilson link

$$U_\mu(x) = e^{igaA^\mu(x)}$$

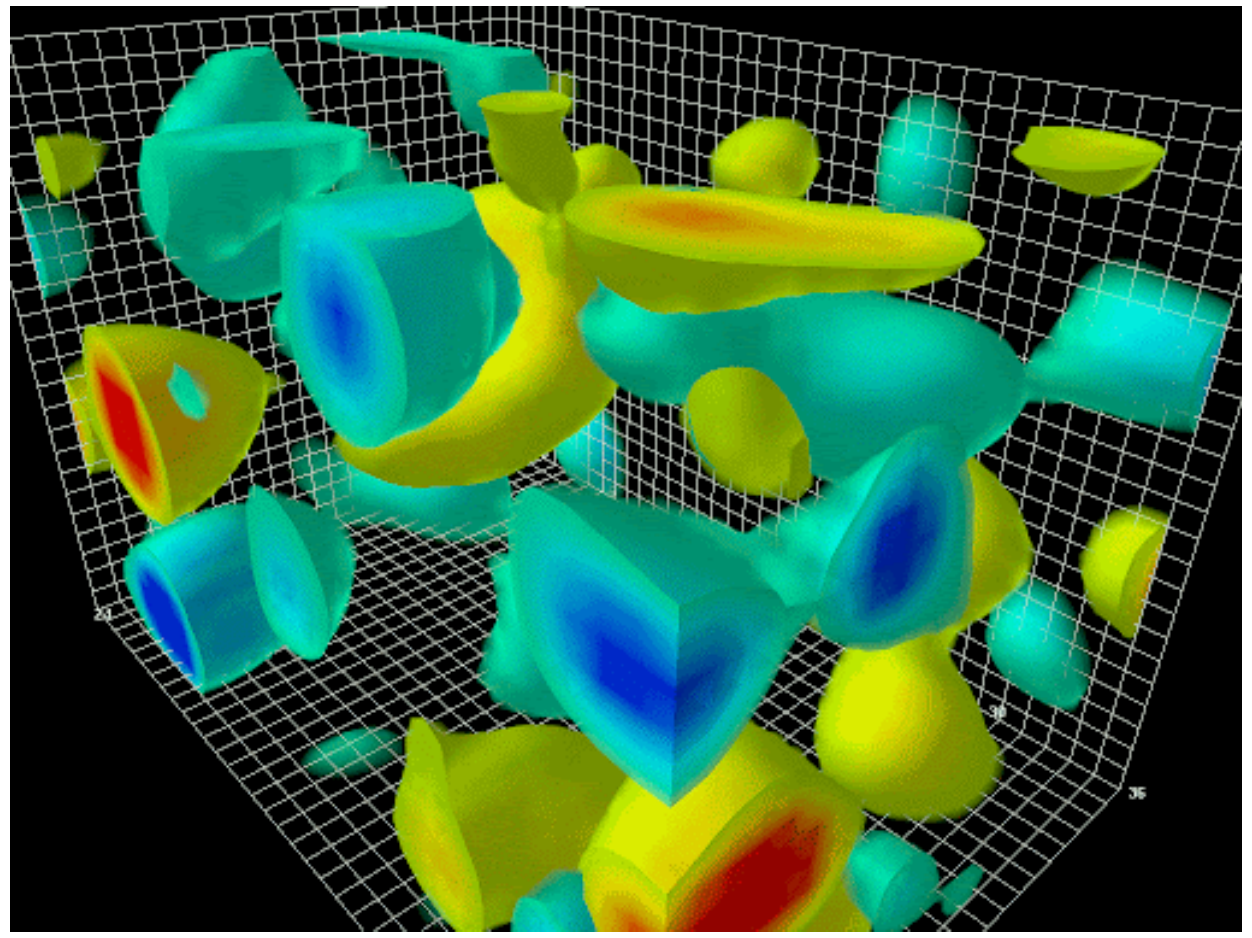
$$Z = \int \mathscr{D}A \; \mathscr{D}\psi \; \mathscr{D}\bar{\psi} \; e^{iS_{\mathrm{QCD}}[A,\psi,\bar{\psi}]} \xrightarrow[t \to -it_{\mathrm{E}}]{} Z_{\mathrm{E}} = \int \mathscr{D}U e^{-S_{\mathrm{E}}^{g}[U]} \int \mathscr{D}\psi \mathscr{D}\bar{\psi} e^{-\bar{\psi}M[U]\psi} = \int \mathscr{D}U \boxed{e^{-S_{\mathrm{E}}^{g}[U]} \det M[U]}$$

Wick rotation

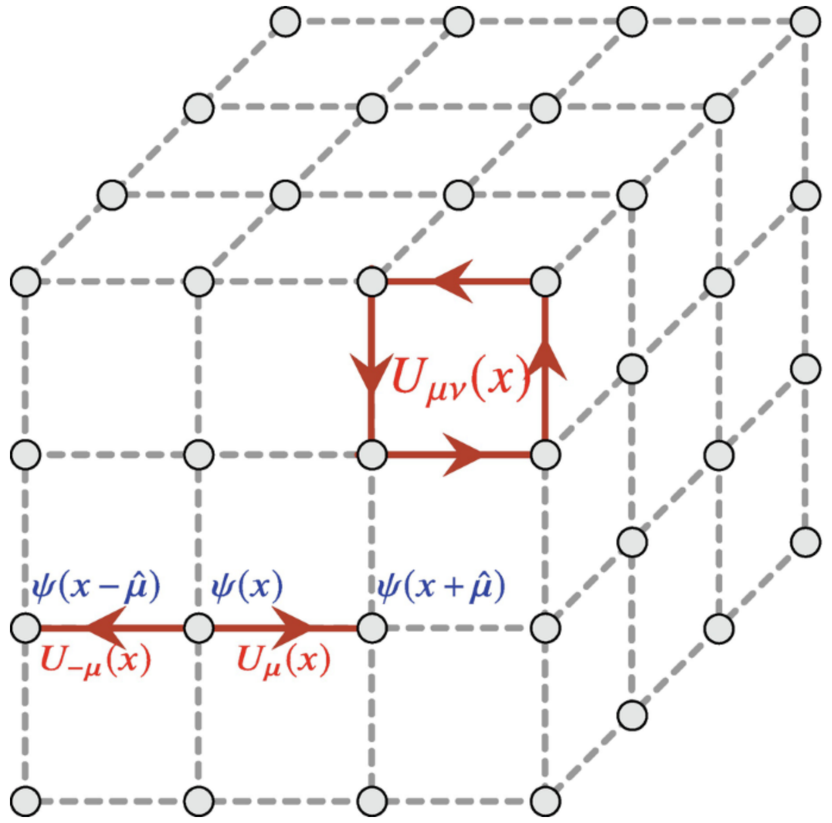
Sampling probability for configuration U

- **Monte Carlo sampling**

$$\langle \hat{O} \rangle = \frac{1}{Z_{\mathrm{E}}} \int \mathscr{D}U \; \mathscr{D}\psi \; \mathscr{D}\bar{\psi} \; \hat{O} \; e^{-S_{\mathrm{E}}^{\mathrm{QCD}}[A,\psi,\bar{\psi}]} = \frac{1}{N} \sum_{i=1}^{N} O[U^{(i)}]$$



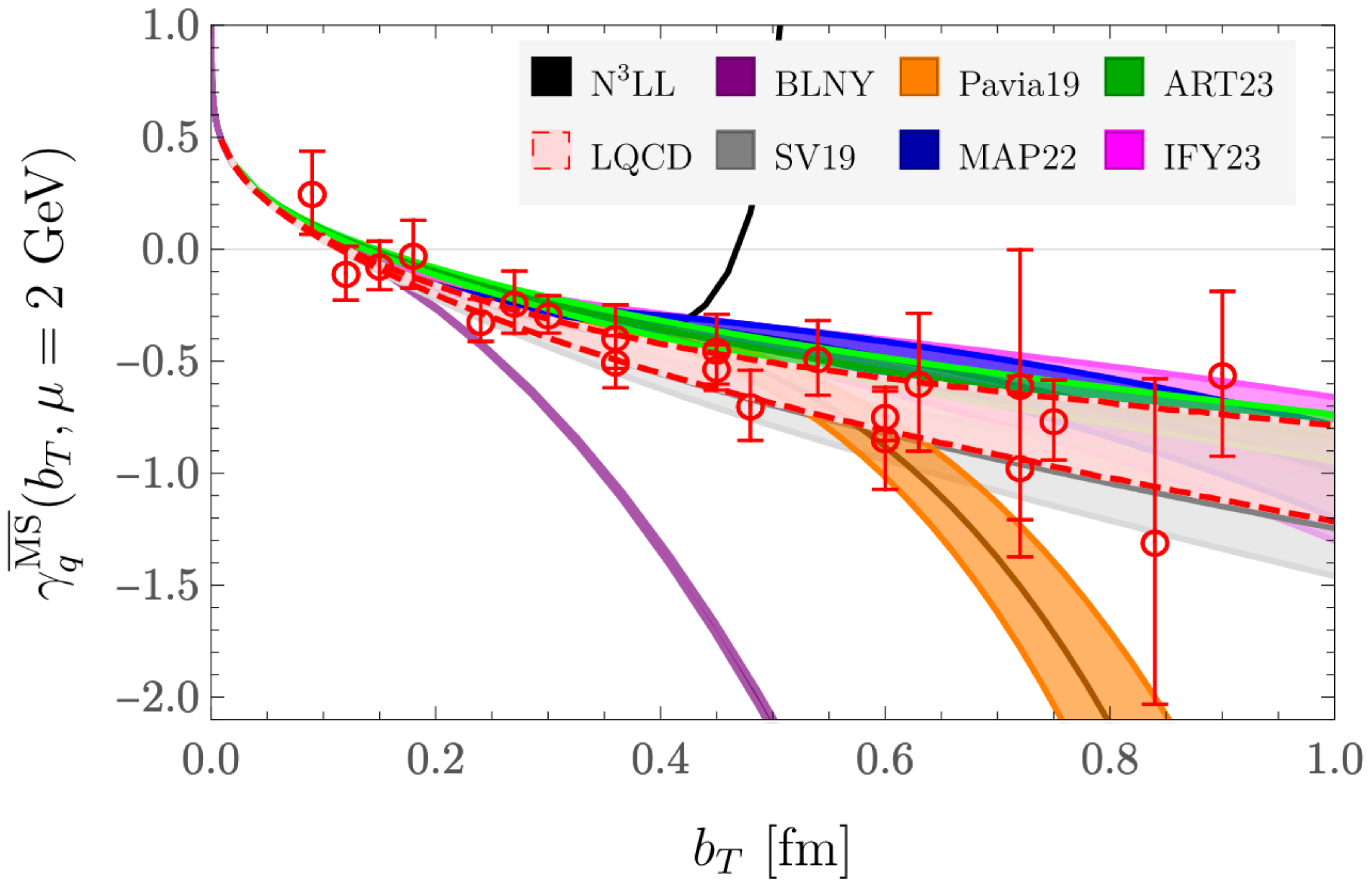
Cr. Derek Leinweber



Cr. Olaf Kaczmarek

Lattice QCD Calculation of TMDs

- **As a first-principles non-perturbative method, Lattice QCD provides independent predictions of TMDs.**
  - **Mellin Moments** *B. Yoon, et al., 1601.05717; B. Yoon, et al., Phys. Rev. D 96 (2017)…*
  - **Large Momentum Effective Theory (LaMET)**
    - **CS kernel** *M. H. Chu, et al. (LPC), JHEP 08 (2023); A. Avkhadiev et al., PRL 132 (2024); D. Bollweg, et al., Phys. Lett. B 852 (2024) …*
    - **Intrinsic soft function** *Q. A. Zhang, et al. (LPC), PRL 125 (2020); M. H. Chu, et al. (LPC), JHEP 08 (2023)*
    - **Unpolarized** *JH, et al. (LPC), Phys.Rev.D 109 (2024)*
    - **Helicity** *D. Bollweg, X. Gao, S. Mukherjee and Y. Zhao, 2505.18430*
    - **Boer-Mulders** *L. Walter, et al. (LPC), 2412.19988; L. Ma, et al. (LPC), 2502.11807*



A. Avkhadiev et al., PRL 132 (2024)

Methodology

Large-Momentum Effective Theory (LaMET)

- TMDPDF is defined from a light-cone correlator in a hadron, which is Lorentz invariant.

Parton model

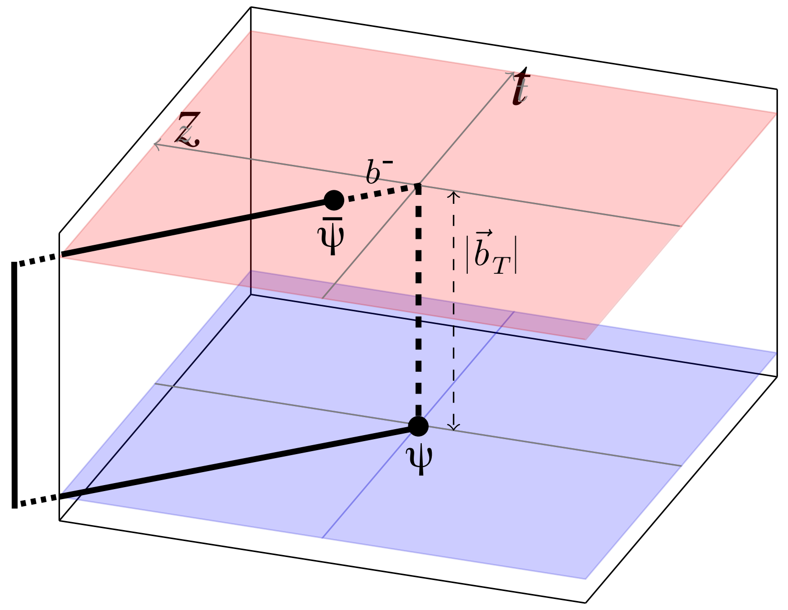
$$f(x,b_\perp,\dots)=\int_{-\infty}^{\infty}\frac{db^-}{2\pi}e^{-ib^-(xP^+)}\left\langle P\left|\bar\psi\left(b^\mu\right)W_\sqsubset\left(b^\mu,0\right)\frac{\gamma^+}{2}\psi(0)\right|P\right\rangle \longleftrightarrow \left\langle |\vec P|=\infty\left|O(t=0)\right||\vec P|=\infty\right\rangle$$

Different orders of limit, but pert.

- Define a quasi distribution with large-momentum states and time-independent operators.

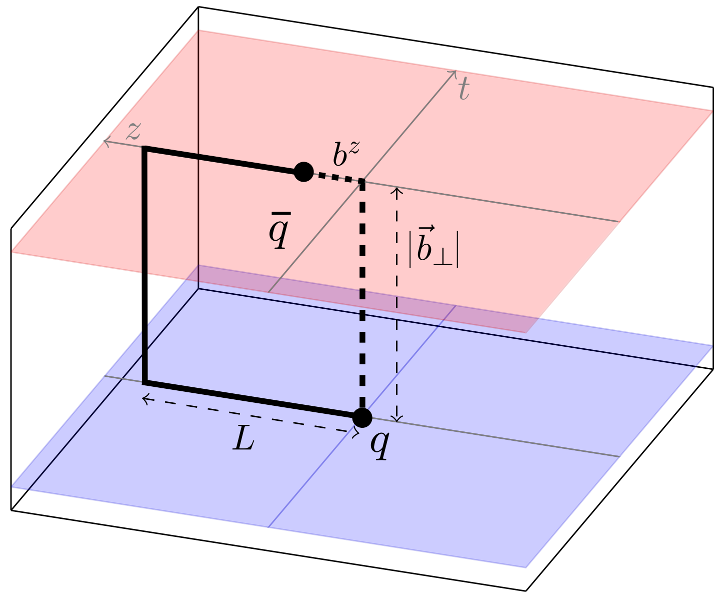
$$\tilde f^0_\Gamma(x,b_\perp,P^z,\mu)=P^z\int\frac{dz}{2\pi}e^{iz(xP^z)}\frac{1}{2P^t}\langle P|\bar\psi_0\left(z,b_\perp\right)W_\sqsubset\left(z,b_\perp;0\right)\Gamma\psi_0(0)|P\rangle,\quad \Lambda_{\rm QCD}\ll|\vec P|\ll\frac{\pi}{a}$$

Large-momentum expansion



X. Ji, Phys.Rev.Lett. 110 (2013)
X. Ji, et al., Rev.Mod.Phys. 93 (2021)
X. Ji, Nucl. Phys. B 1007 (2024)

Lorentz boost & Matching



Light-cone distribution:
Critical point with infinite correlation length
Cannot be directly calculated on the lattice

Quasi distribution:
Directly calculable on the lattice

$H_f\left(x,P^z;\mu\right)=|C_{\rm TMD}(xP^z;\mu)|^2$ is the TMD hard kernel for matching.

- LaMET enables us to obtain the precision-controlled x-distribution of TMDs in $x\in[x_{\min},x_{\max}]$.

Collins-Soper scale: $\zeta\sim 2(xP^+)^2$

$$\sqrt{S_I\left(b_\perp;\mu\right)}\cdot\tilde f_\Gamma\left(x,b_\perp,P^z;\mu\right)=f\left(x,b_\perp;\mu,\zeta\right)H_f\left(x,P^z;\mu\right)\exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\rm MS}}\left(b_\perp;\mu\right)\right]+\text{Power corrections}$$

Soft Function

$$\sqrt{S_I(b_\perp;\mu)}\cdot\tilde{f}_\Gamma(x,b_\perp,P^z;\mu)=f(x,b_\perp;\mu,\zeta)\,H_f(x,P^z;\mu)\exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}(b_\perp;\mu)\right]+\text{Power corrections}$$

After regularization, the rapidity evolution is controlled by Collins-Soper scale: $\zeta = 2(xP^+)^2 e^{-2y_n}$

- The soft gluon radiation will lead to the existence of soft functions;

- Due to the gluon radiation in the collinear mode, the soft function contains the well-known rapidity divergence;

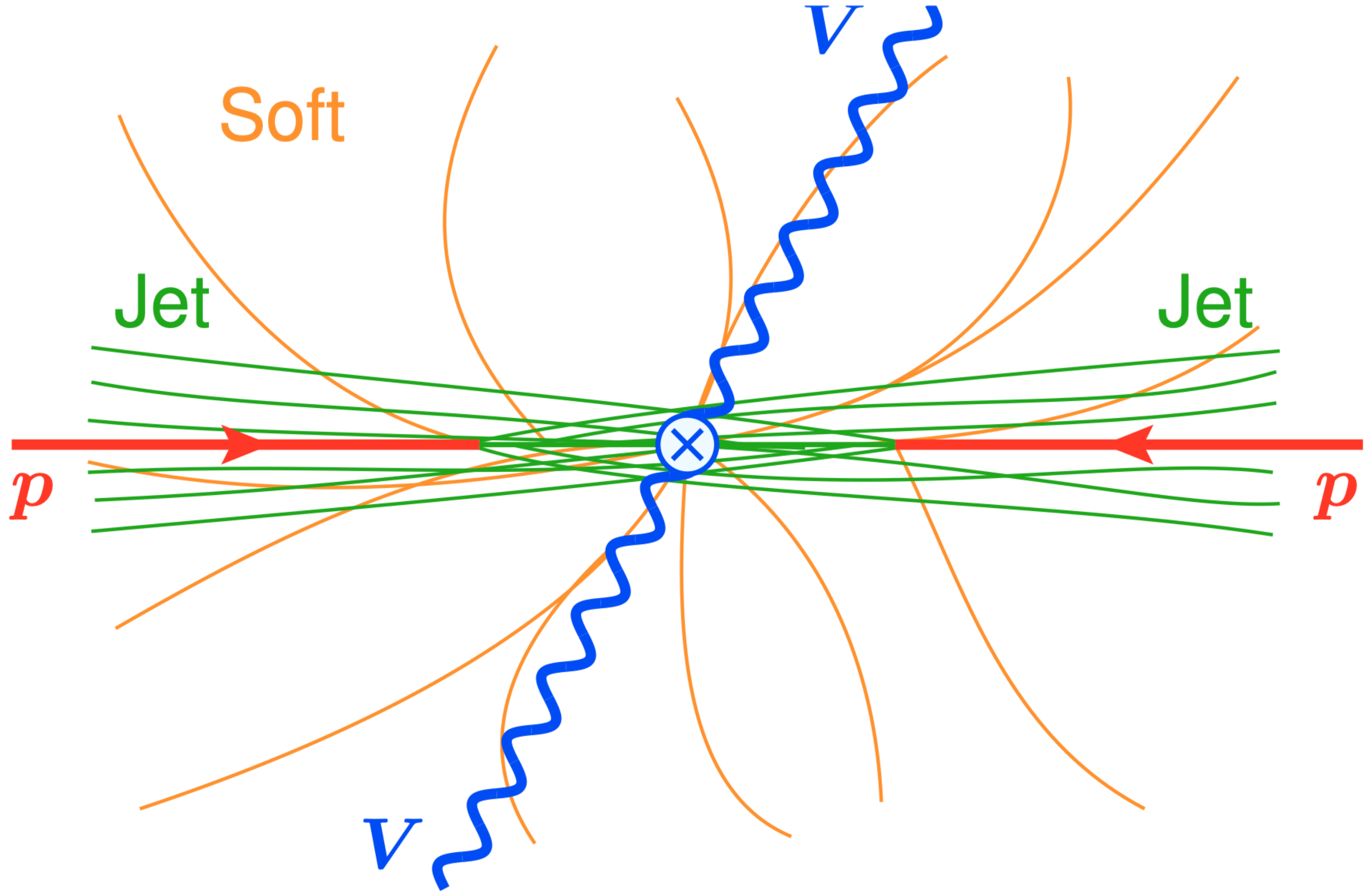
$$\underbrace{\int_{q_T}^{Q}\frac{\mathrm{d}k}{k}}_{\text{full}}=\lim_{\tau\to 0}[\underbrace{\int_0^Q\frac{\mathrm{d}k}{k}R_c(k,\tau)}_{\text{collinear}}+\underbrace{\int_{q_T}^{\infty}\frac{\mathrm{d}k}{k}R_s(k,\tau)}_{\text{soft}}]=\ln\frac{Q}{q_T}$$

- The soft function can be separated into two parts:
  - Rapidity evolution kernel: CS kernel $\gamma^{\overline{\mathrm{MS}}}(b_\perp;\mu)$
  - Rapidity independent part: intrinsic soft function $S_I(b_\perp;\mu)$

- CS kernel can be extracted from the rapidity evolution of TMDs

$$\gamma^{\overline{\mathrm{MS}}}(b_\perp,P_1,P_2;\mu)=\frac{1}{\ln(P_2/P_1)}\ln\frac{H_f(x,\bar{x},P_1;\mu)\,\tilde{f}_{\gamma^t}(x,b_\perp,P_2;\mu)}{H_f(x,\bar{x},P_2;\mu)\,\tilde{f}_{\gamma^t}(x,b_\perp,P_1;\mu)}$$

$H_f(x,P^z;\mu)=|C_{\mathrm{TMD}}(xP^z;\mu)|^2$ is the TMD hard kernel for matching.



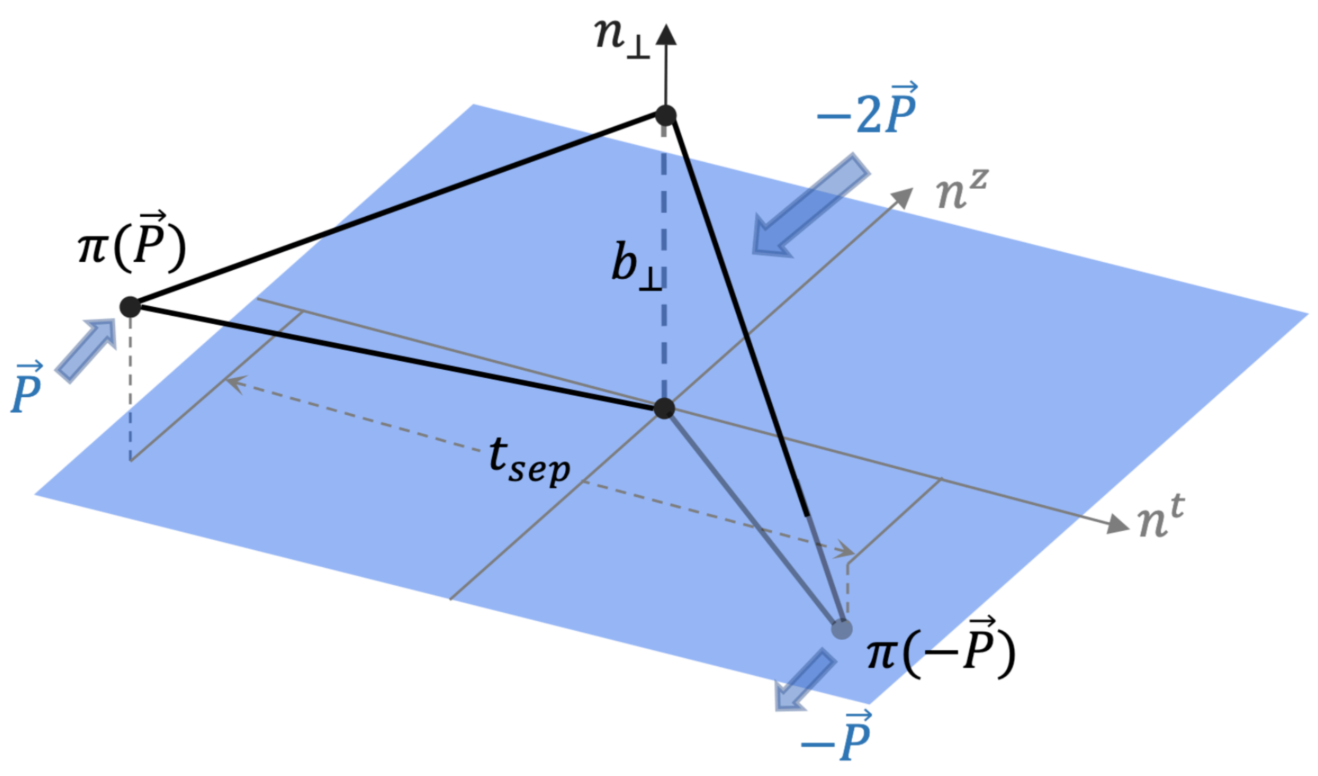
M. Ebert, PhD Thesis (2017)

Soft Function

- **The intrinsic soft function cannot be directly calculated on lattice because of two light-like Wilson lines in different directions**

  - **Fortunately, it can be extracted from the meson form factor** *X. Ji, et al., Nucl.Phys.B 955 (2020)*

$$F\left(b_\perp, P_1, P_2, \Gamma, \Gamma'\right) \equiv -4N_c \frac{\left\langle P_2 \right| \bar{q}\left(b_\perp\right)\Gamma q\left(b_\perp\right)\bar{q}(0)\Gamma' q(0)\left| P_1 \right\rangle}{f_\pi^2 (P_1 \cdot P_2)}$$



Q. A. Zhang, et al., Phys. Rev. Lett. 125 (2020)

  - **The form factor satisfies the factorization formula**

$$F(b_\perp, P^z) = \int dx_1 dx_2\ H_F(x_1, x_2, P^z; \mu)\ \phi^\dagger(x_1, b_\perp, y_n; \mu, \zeta_1, \bar{\zeta}_1)\ \phi(x_2, b_\perp, -y_n; \mu, \zeta_2, \bar{\zeta}_2)$$

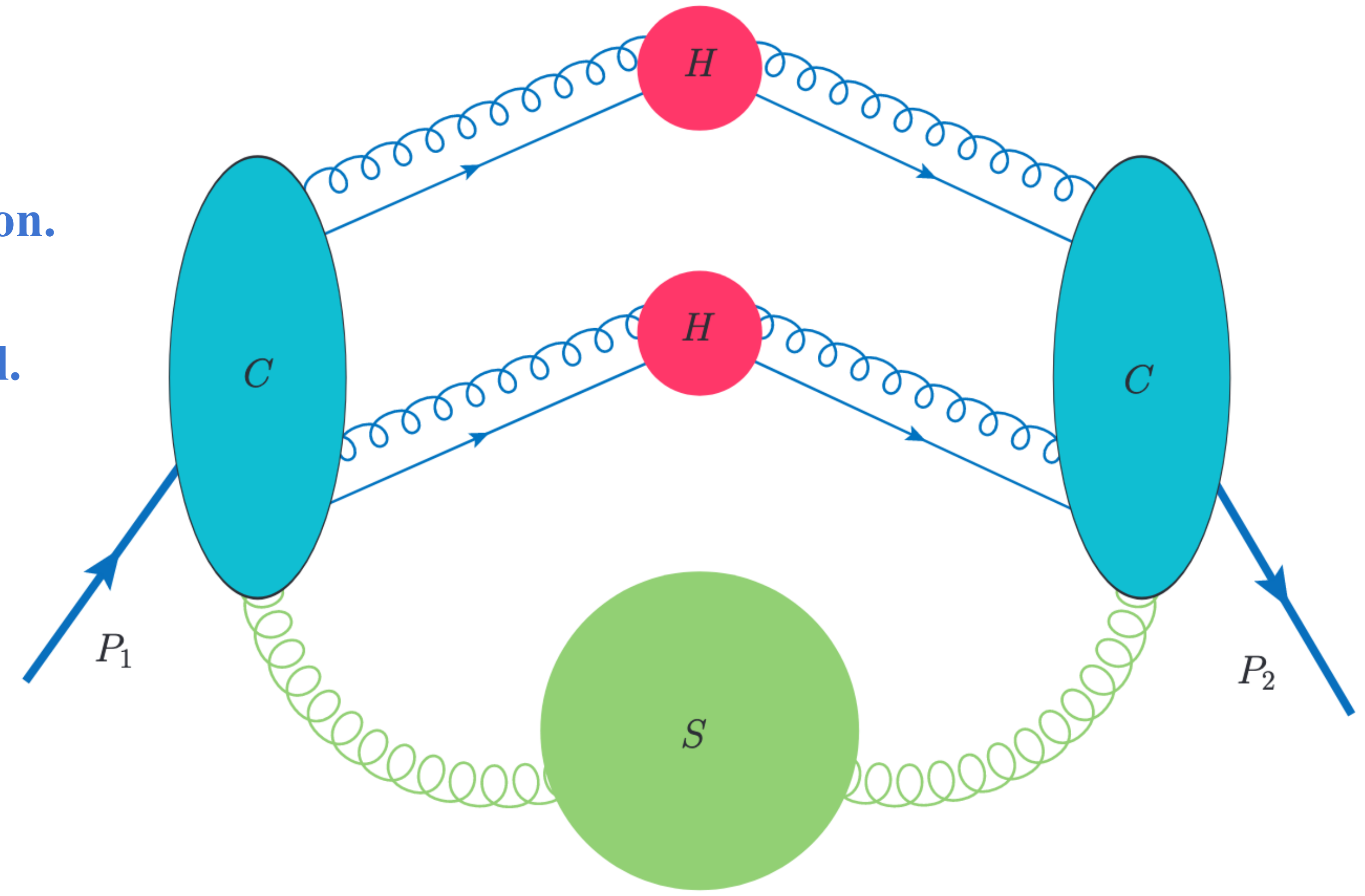
$\phi(x, b_\perp, \ldots) = \int_{-\infty}^{\infty} \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \left\langle 0 \left| \bar{\psi}\left(b^\mu\right) W_\sqsubset \left(b^\mu, 0\right) \gamma^+ \gamma^5 \psi(0) \right| P \right\rangle$ **is TMD wave function.**

$H_F(x_1, x_2, P^z; \mu) = C_{\text{Sud}}(x_1, x_2, P^z; \mu) \cdot C_{\text{Sud}}(\bar{x}_1, \bar{x}_2, P^z; \mu)$, **where C_{Sud} is the Sudakov kernel.**

  - **Therefore, the intrinsic soft function can be extracted via**

$$S_I(b_\perp; \mu) = \frac{F(b_\perp, P^z)}{\int dx_1 dx_2 H_F(x_1, x_2, P^z; \mu) \tilde{\Phi}^\dagger(x_1) \tilde{\Phi}(x_2)} \text{ with } \tilde{\Phi}(x) \equiv \frac{\tilde{\phi}_\Gamma\left(x, b_\perp, P^z; \mu\right)}{H_\phi\left(x, \bar{x}, P^z; \mu\right)}$$

$\tilde{\phi}_\Gamma\left(x, b_\perp, P^z; \mu\right)$ **is quasi-TMD wave function.**



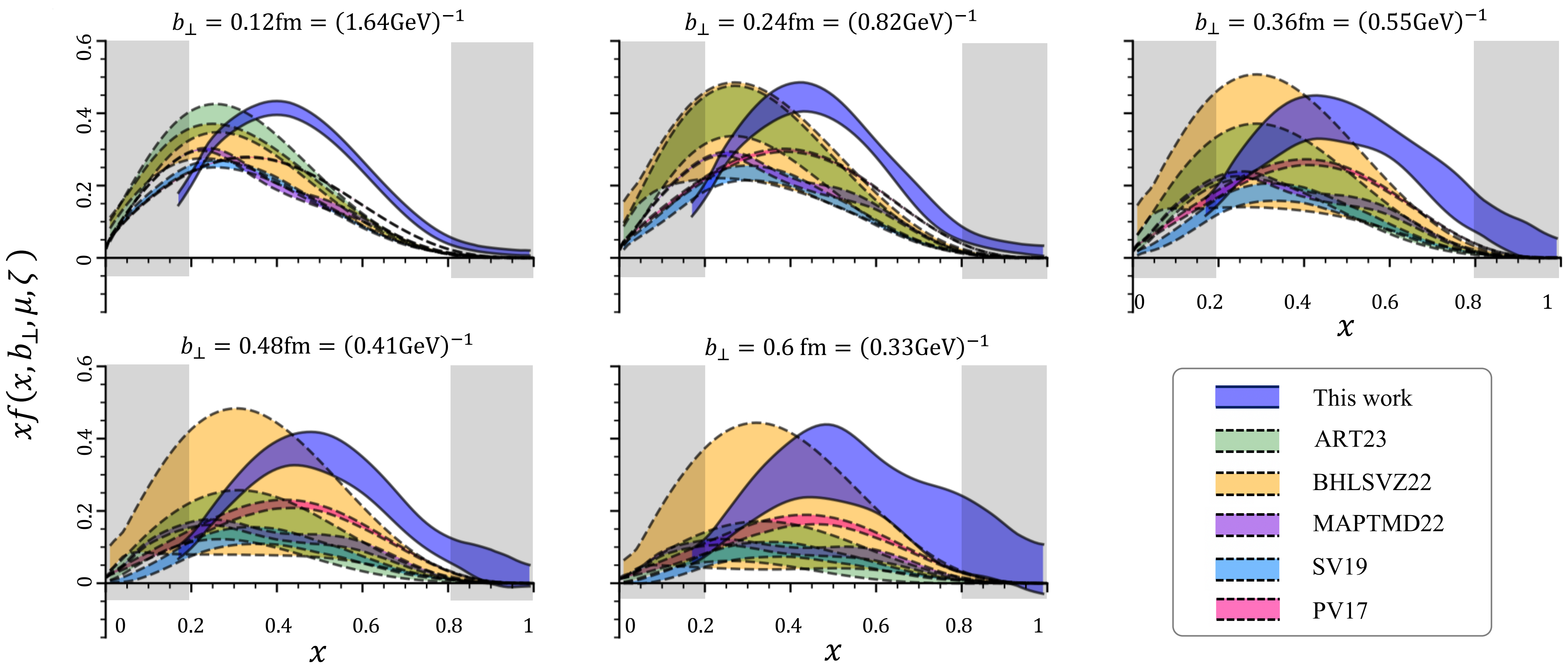
Z. F. Deng, et al., JHEP 09 (2022)

Unpolarized TMD via LaMET

- In recent years, a lot of improvements of renormalization and matching has been developed in LaMET;

Y. Su, et al., Nucl. Phys. B 991 (2023); R. Zhang, et al., Phys. Lett. B 844 (2023); X. Ji, et al., 2410.12910 [hep-ph]

Unpolarized nucleon TMDPDF



JH, et al. (LPC), Phys.Rev.D 109 (2024)

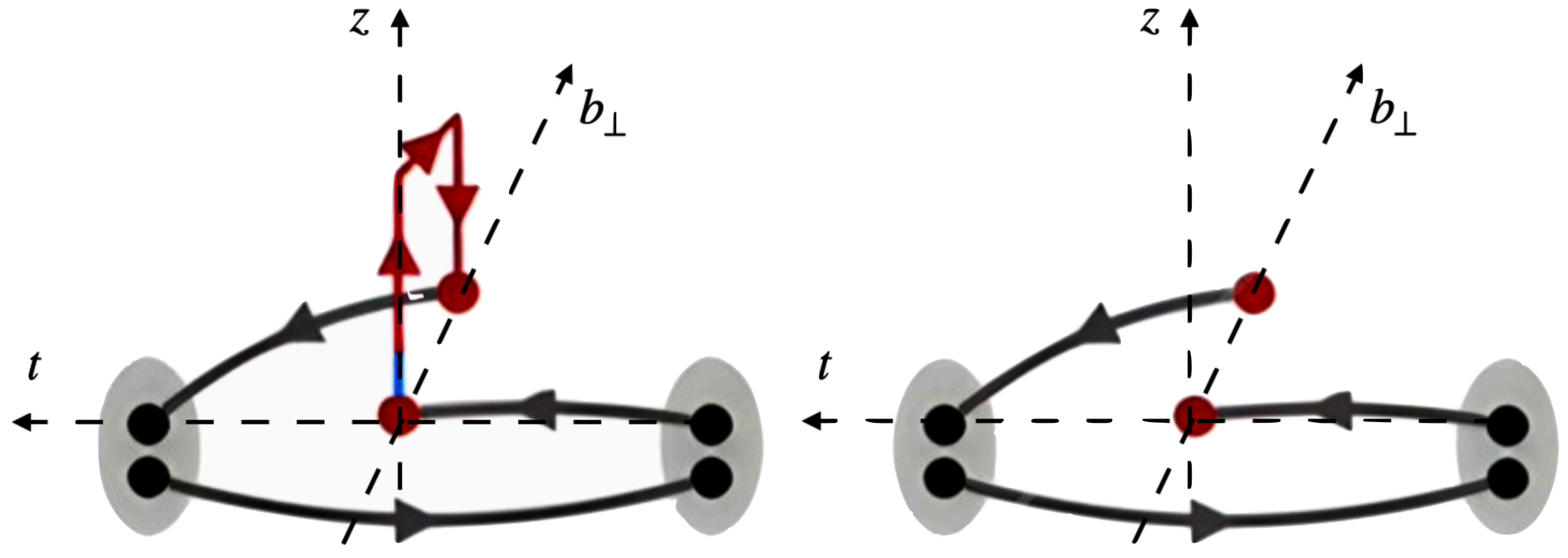
- $a = 0.12$ fm
- $P^z_{\max} = 2.58$ GeV
- **Physical limit of m_π^{val}**
- **N3LL matching**
- **NLO soft function**

- **Existing lattice calculations of the nucleon TMD still suffer from some systematics:**
  - **Discretization effects;**
  - **Excited-state contamination;**
  - **Hadron momentum is not large enough …**
- **Due to the bad signal-to-noise ratio (SNR), it is hard to probe the large $b_\perp$ region.**

Coulomb Gauge Method

- **Define a quasi distribution in CG without Wilson line:** *X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)*; *Y. Zhao, PRL 133 (2024)*

$$\tilde{f}^{0}_{\rm CG}(x, b_\perp, P^z, \mu) = P^z \int \frac{dz}{2\pi} e^{iz(xP^z)} \frac{1}{2P^t} \langle P | \bar{\psi}_0\left(z, b_\perp\right) \Gamma \psi_0(0) \Big|_{\vec{\nabla}\cdot\vec{A}=0} | P \rangle$$



- **Why choose CG?**

X. Ji, Y. S. Liu, Y. Liu, J. H. Zhang and Y. Zhao, RMP 93 (2021)

  - $\vec{\nabla}\cdot\vec{A} = 0$ **becomes** $A^+ = 0$ **in the infinite boost, so the quasi distribution in CG belongs to the universality class in LaMET;**

Wilson line on light-cone

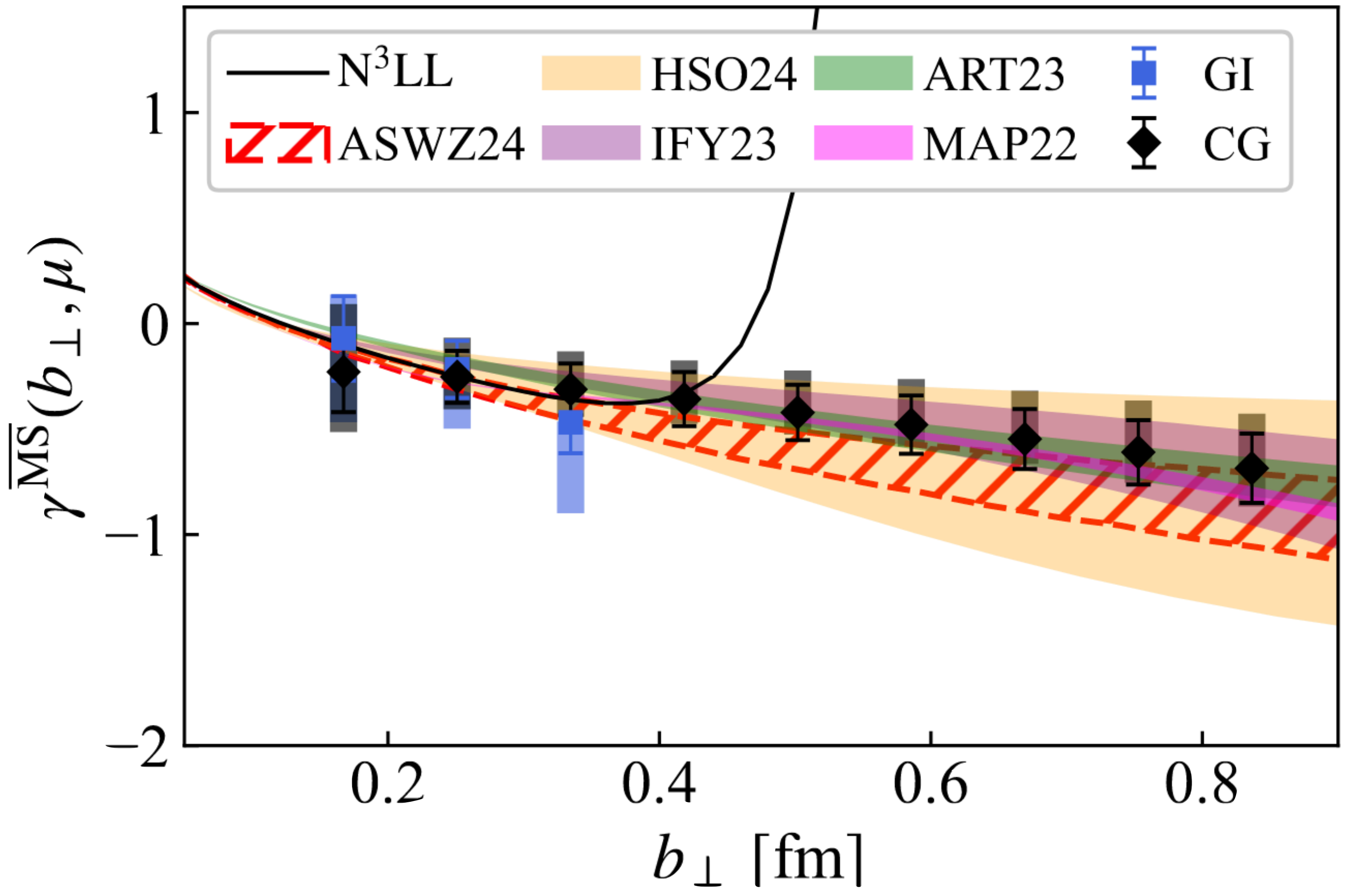
$$\psi_C(x) = e^{-ie\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A}}\psi_0(x) \xrightarrow{P^z\to\infty} e^{-ie\frac{1}{(\nabla^+)^2}\nabla^+ A^+}\psi_0(x) = e^{-ie\frac{1}{\nabla^+ \pm 0}A^+}\psi_0(x) = e^{-ie\int_{\pm\infty}^{x^-} dy^- A^+(y^-, x_\perp)}\psi_0(x)$$

  - **No linear divergence from the Wilson link, improving the SNR significantly;**
  - **Simplified renormalization** $\bar{\psi}_0(z, b_\perp)\Gamma\psi_0(0) = Z_\psi(a)\left[\bar{\psi}(z, b_\perp)\Gamma\psi(0)\right]$**;**

$$\tilde{h}_{\gamma^t}\left(z, b_\perp, P^z; \mu\right) = \frac{\tilde{h}^0_{\gamma^t}\left(z, b_\perp, P^z; a\right)}{\tilde{\varphi}^0_{\gamma^t\gamma_5}\left(z=0, b_\perp, P^z=0; a\right)}$$

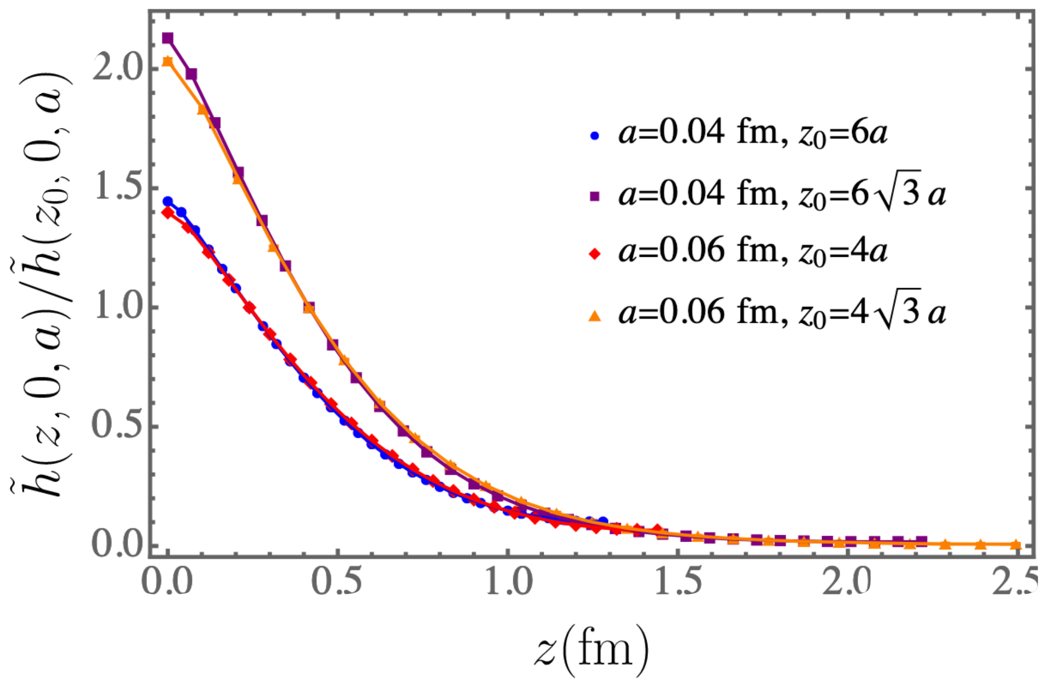
  - **Larger off-axis momenta (3D rotational symmetry).**

Collins-Soper kernel in CG v.s. GI

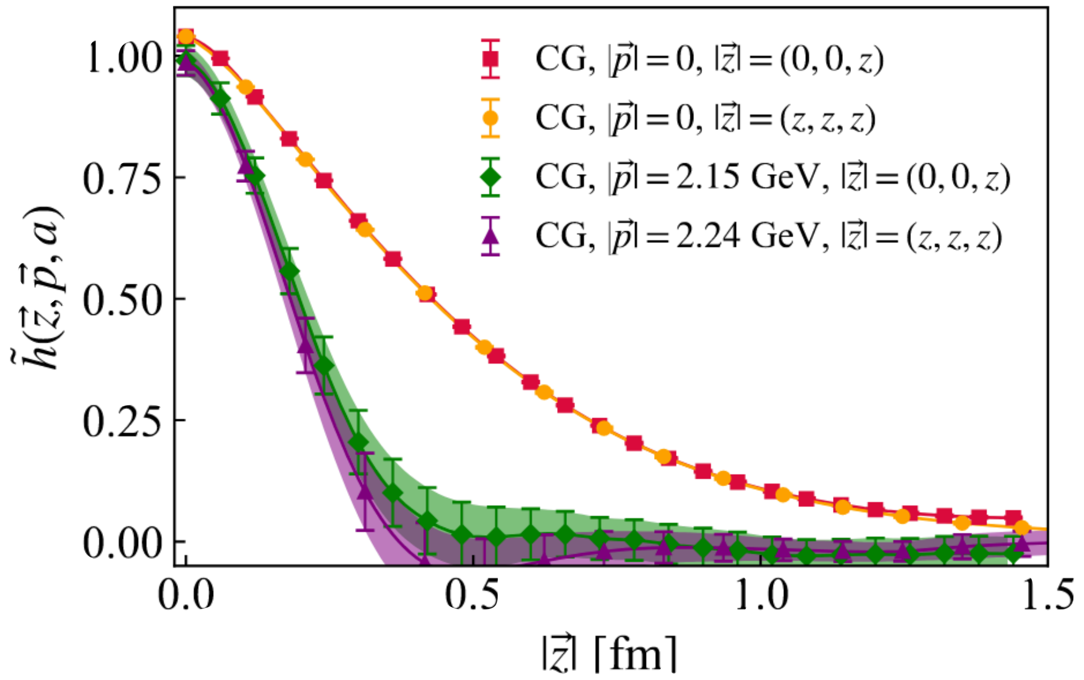


D. Bollweg, et al., Phys.Lett.B 852 (2024)

No linear divergence



3D rotational symmetry



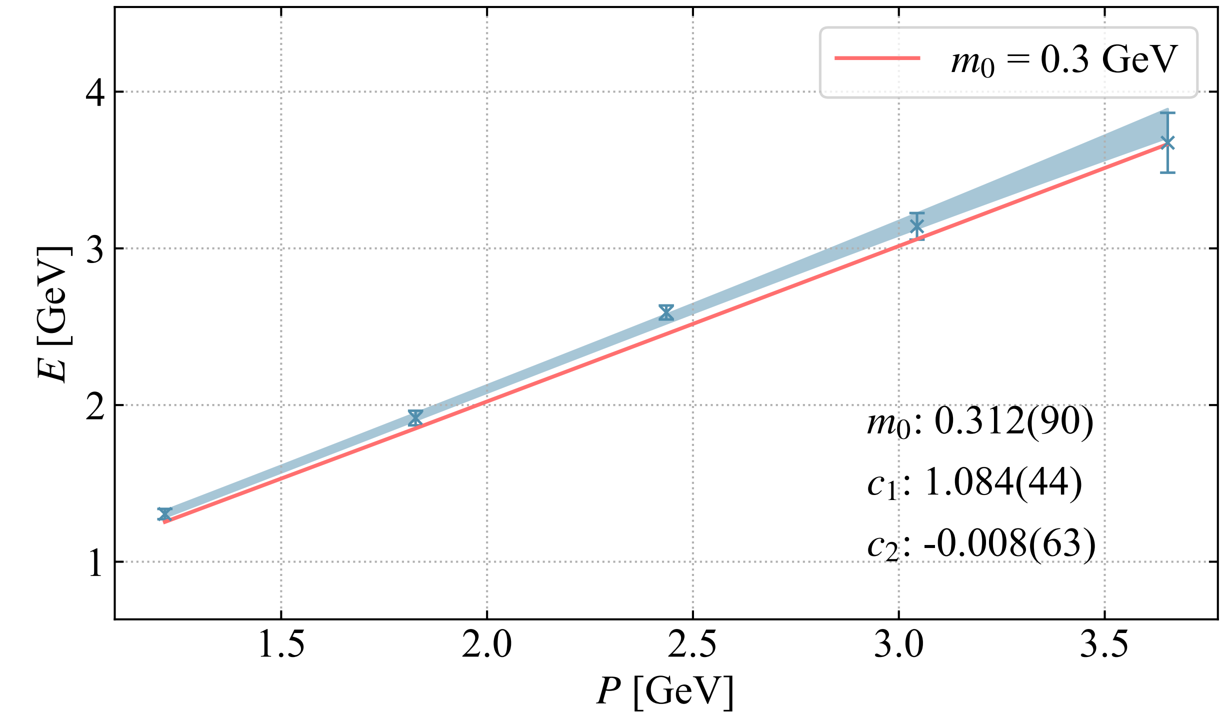
X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)

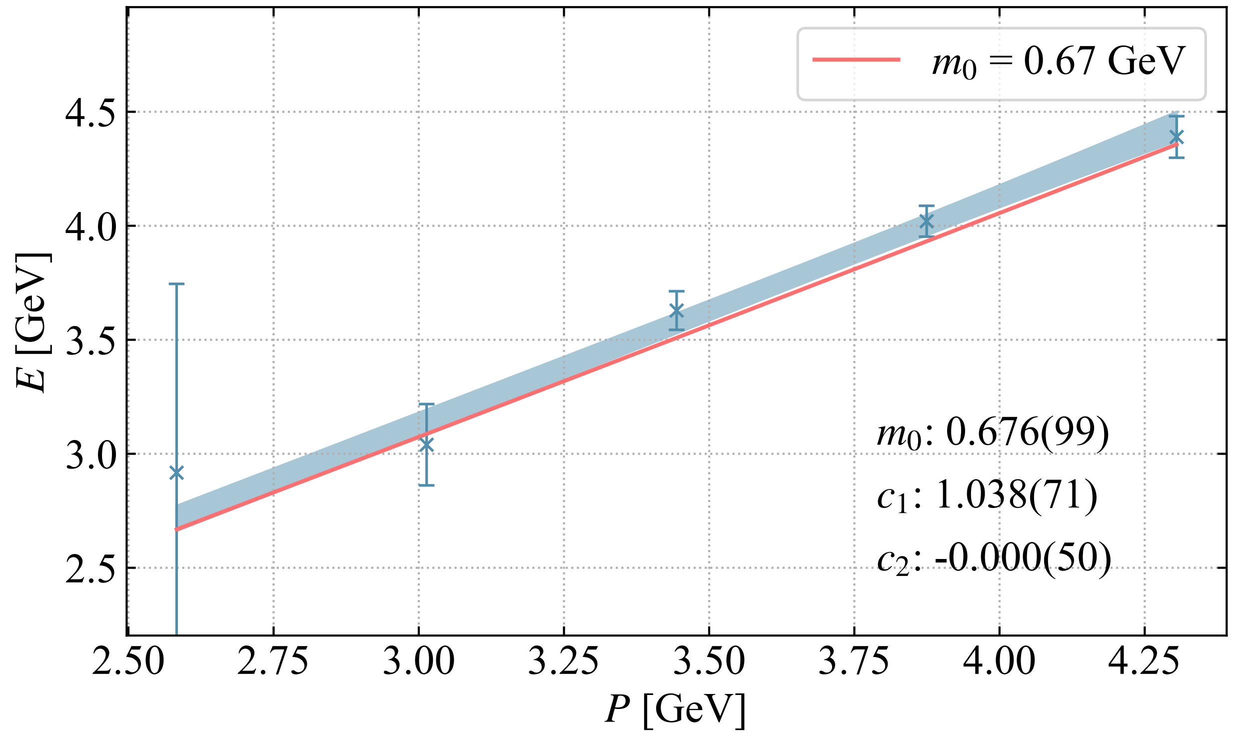
Numerical Results

Lattice Setup

- 2+1 flavor HISQ ensemble by HotQCD with volume $L_s \times L_t = 48^3 \times 64$;
- Lattice spacing is $a = 0.06$ fm;
- Pion mass of sea quark: $m_\pi^{\text{sea}} = 160$ MeV;
- Pion mass of valence quark for quasi-TMD: $m_\pi^{\text{val}} = 300$ MeV;
- Off-axis ($\vec{n} = (1,1,0)$) hadron momenta for quasi-TMD: 1.83 GeV, 2.43 GeV and 3.04 GeV;
- Pion mass of valence quark for qTMDWF and meson form factor: $m_\pi^{\text{val}} = 670$ MeV;
- On-axis hadron momenta for qTMDWF: 3.44 GeV, 3.87 GeV and 4.30 GeV;
- On-axis hadron momenta for meson form factor: 2.58 GeV.

Dispersion relation: $E^2 = m_0^2 + c_1\, P^2 + c_2\, a^2 P^4$





Quasi-TMD | TMDPDF | Hard kernel

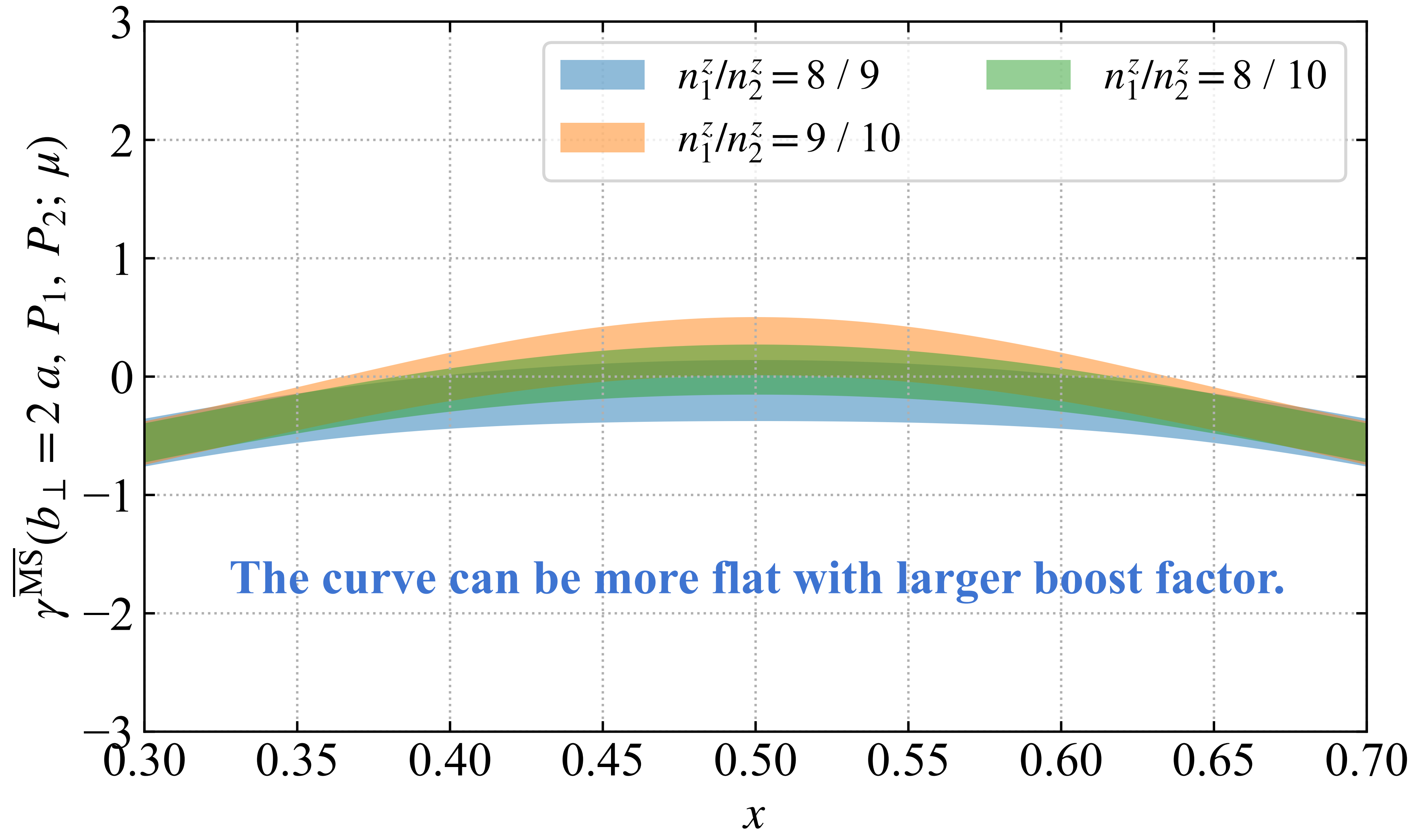
$$\sqrt{S_I(b_\perp;\mu)} \cdot \tilde{f}_\Gamma(x, b_\perp, P^z;\mu) = f(x, b_\perp;\mu,\zeta)\, H_f(x, P^z;\mu) \exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\text{MS}}}(b_\perp;\mu)\right] + \text{Power corrections}$$

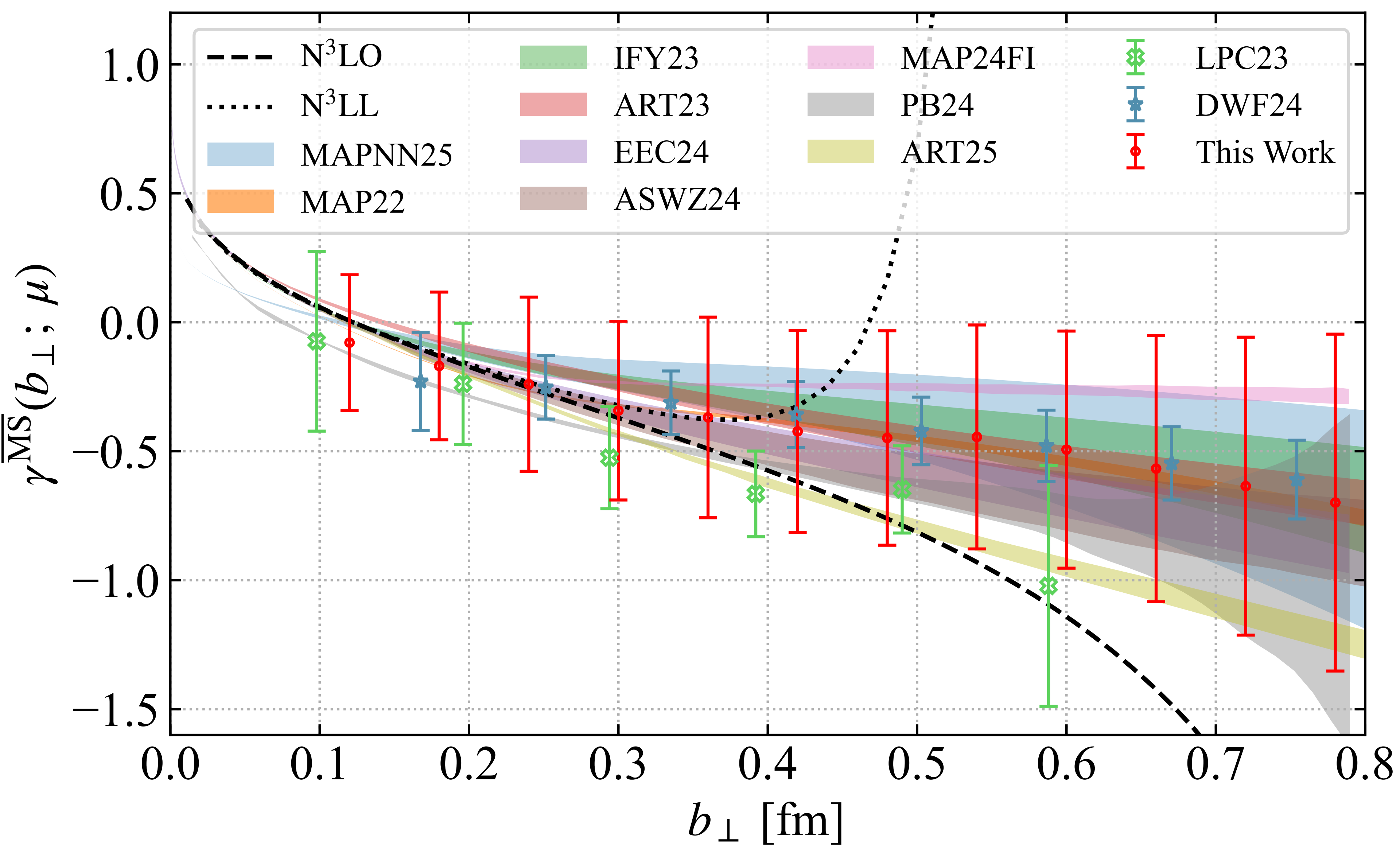
Intrinsic soft function ← Quasi-TMDWF / Meson Form Factor → CS kernel

Collins-Soper Kernel

$$\gamma^{\overline{\mathrm{MS}}}(b_\perp, P_1, P_2; \mu) = \frac{1}{\ln\left(P_2/P_1\right)} \ln \frac{H_\phi\left(x, \bar{x}, P_1; \mu\right) \tilde{\phi}_{\gamma^z\gamma^5}\left(x, b_\perp, P_2; \mu\right)}{H_\phi\left(x, \bar{x}, P_2; \mu\right) \tilde{\phi}_{\gamma^z\gamma^5}\left(x, b_\perp, P_1; \mu\right)}$$

$H_\phi(x, \bar{x}, P^z; \mu) = C_{\mathrm{TMD}}(xP^z; \mu) \cdot C_{\mathrm{TMD}}(\bar{x}P^z; \mu)$ **is the TMD hard kernel for matching.**





- **DWF24 is another lattice calculation using the CG method on chirally symmetric domain-wall fermion configurations;**
- **There is a notable tension among recent results in phenomenology (MAP24FI & ART25);**
- **Both remain consistent with this work due to the large uncertainty;**
- **The large uncertainty is mainly caused by the small Lorentz boost factor at such a heavy pion mass ($m_\pi^{\mathrm{val}} = 670$ MeV);**

Pion Form Factor

- The form factor is defined as

$$F\left(b_\perp, P^z, \Gamma\right) \equiv -4N_c \frac{\langle -P^z \left| \bar{q}\left(b_\perp\right)\Gamma q\left(b_\perp\right)\bar{q}(0)\Gamma q(0) \right| P^z\rangle}{f_\pi^2\left((P^t)^2+(P^z)^2\right)}$$

We choose $\Gamma \in \{\gamma^\perp, \gamma^\perp\gamma^5\}$ to get leading-twist contribution, then take the Fierz rearrangement.

$$F(b_\perp, P^z) = \frac{1}{4}\left[F(b_\perp, P^z, \Gamma=\gamma^x\gamma^5) + F(b_\perp, P^z, \Gamma=\gamma^y\gamma^5) + F(b_\perp, P^z, \Gamma=\gamma^x) + F(b_\perp, P^z, \Gamma=\gamma^y)\right]$$

- It can be extracted from the ratio

$$R_F\left(b_\perp, P^z, \Gamma\right) \equiv -4N_c \frac{\langle -P^z \left| \bar{q}\left(b_\perp\right)\Gamma q\left(b_\perp\right)\bar{q}(0)\Gamma q(0) \right| P^z\rangle}{\langle 0 \,|\, \bar{q}(0)\gamma^\mu\gamma^5 q(0) \left| P^z\rangle \langle -P^z \right| \bar{q}(0)\gamma_\mu\gamma^5 q(0) \,|\, 0\rangle}$$

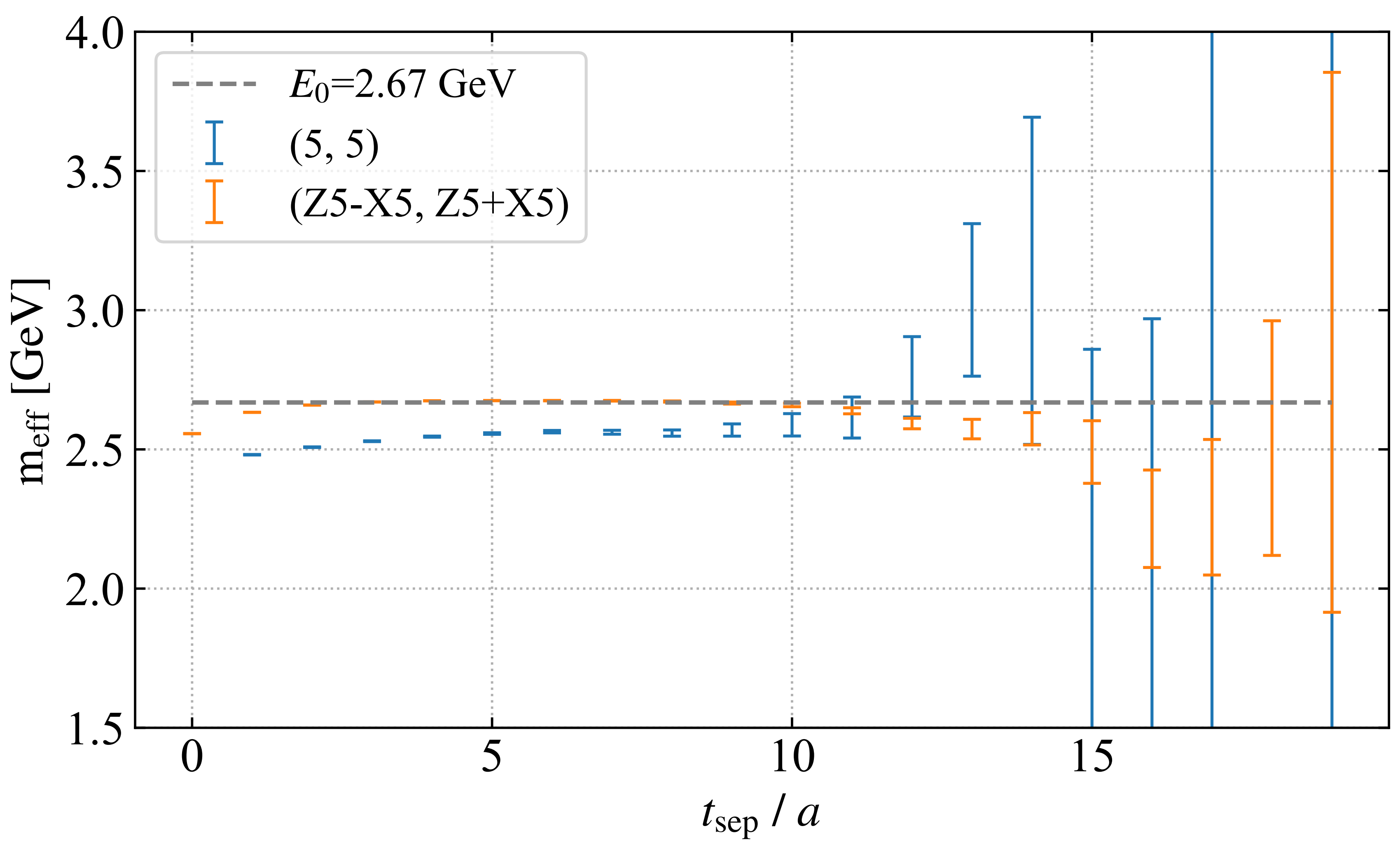
- The ratio in terms of correlators on lattice

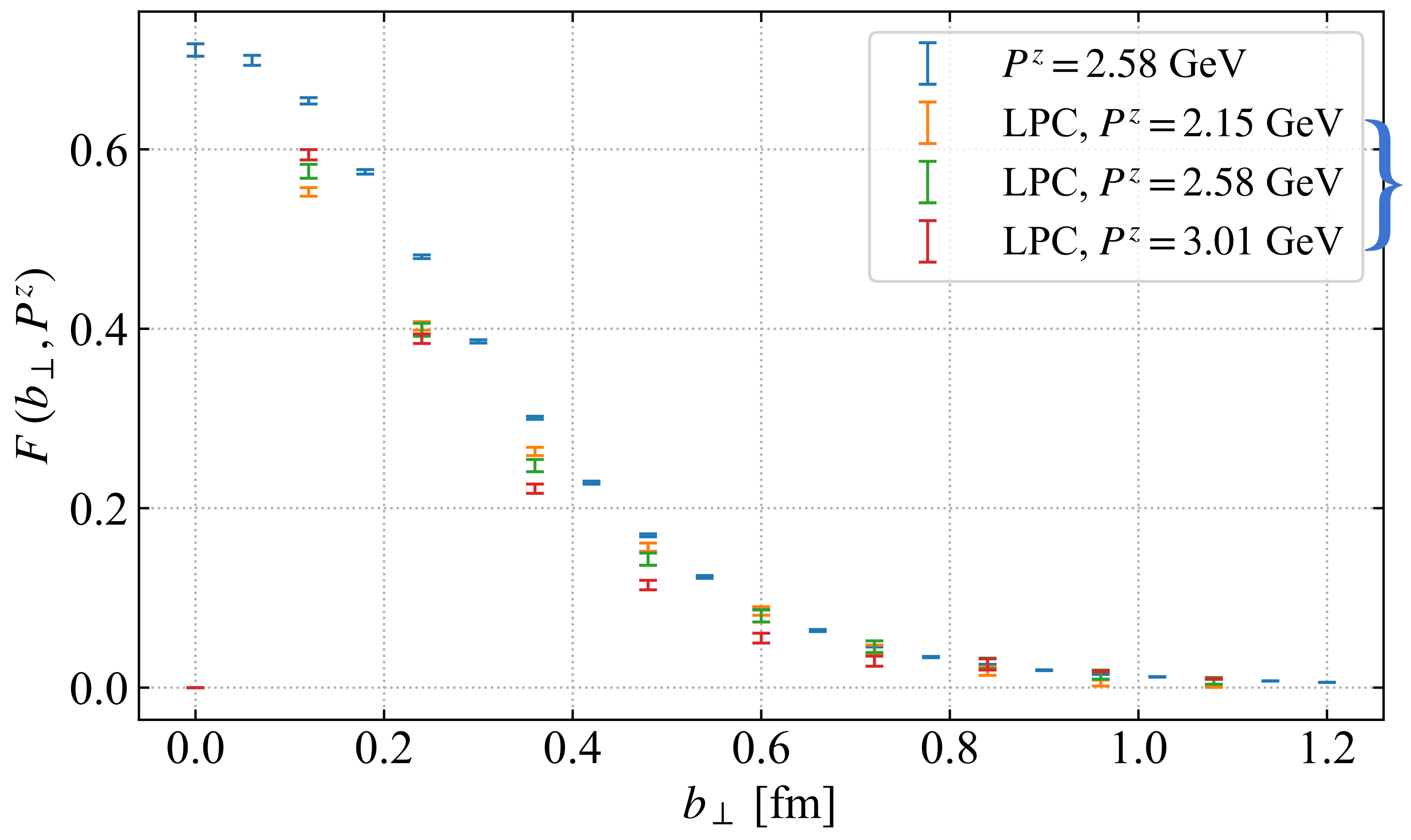
$$R_F\left(t_{\rm sep}, \tau\right) = \frac{-4N_c}{1+(P^t/P^z)^2} \frac{C_{\rm F}\left(t_{\rm sep}, \tau\right)}{\left|C_{\rm 2pt}\left(t_{\rm sep}/2\right)\right|^2}$$

The 2pt is calculated using the new interpolating operator.

Kinematically-enhanced interpolating operator

R. Zhang, et al., Phys. Rev. D 112 (2025)



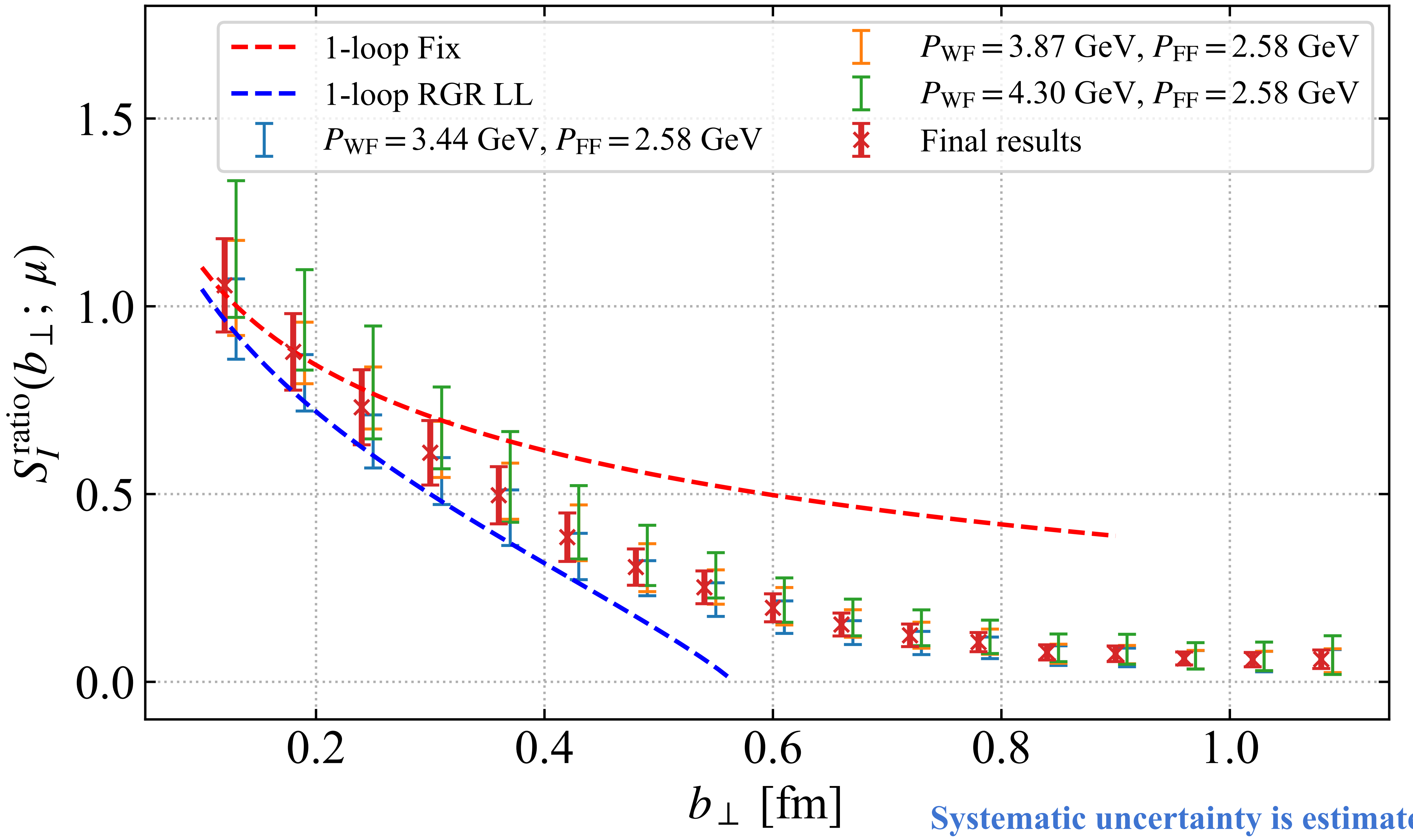


$a = 0.06$ fm

$a = 0.12$ fm

Intrinsic Soft Function

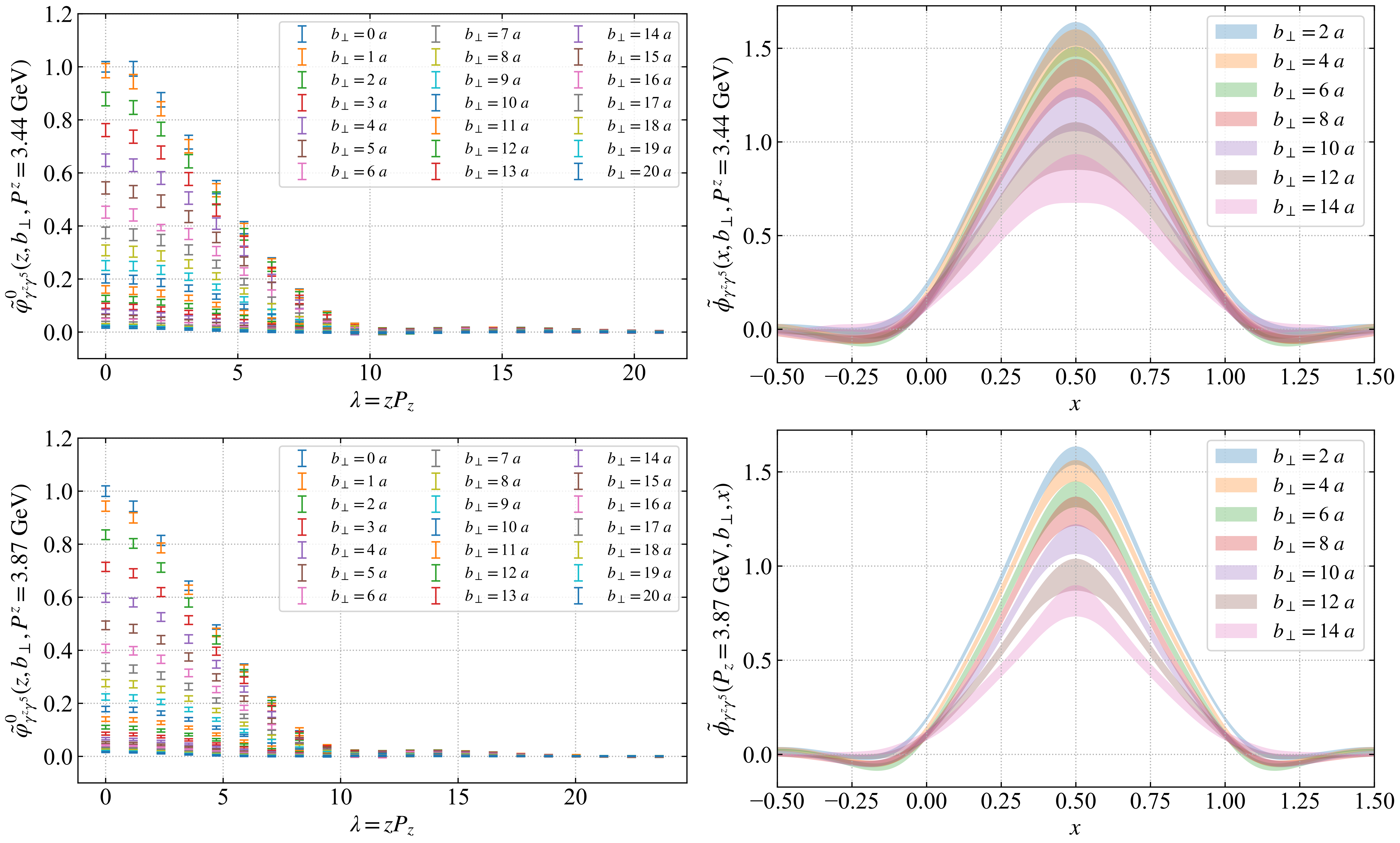
$$S_I(b_\perp;\mu) = \frac{F(b_\perp, P^z)}{\int dx_1 dx_2 H_F(x_1, x_2, P^z;\mu)\tilde{\Phi}^\dagger(x_1)\tilde{\Phi}(x_2)} \text{ with } \tilde{\Phi}(x) \equiv \frac{\tilde{\phi}_\Gamma\left(x, b_\perp, P^z;\mu\right)}{H_\phi\left(x, \bar{x}, P^z;\mu\right)}$$



Systematic uncertainty is estimated by the spread of the mean values between these three momentum pairs.

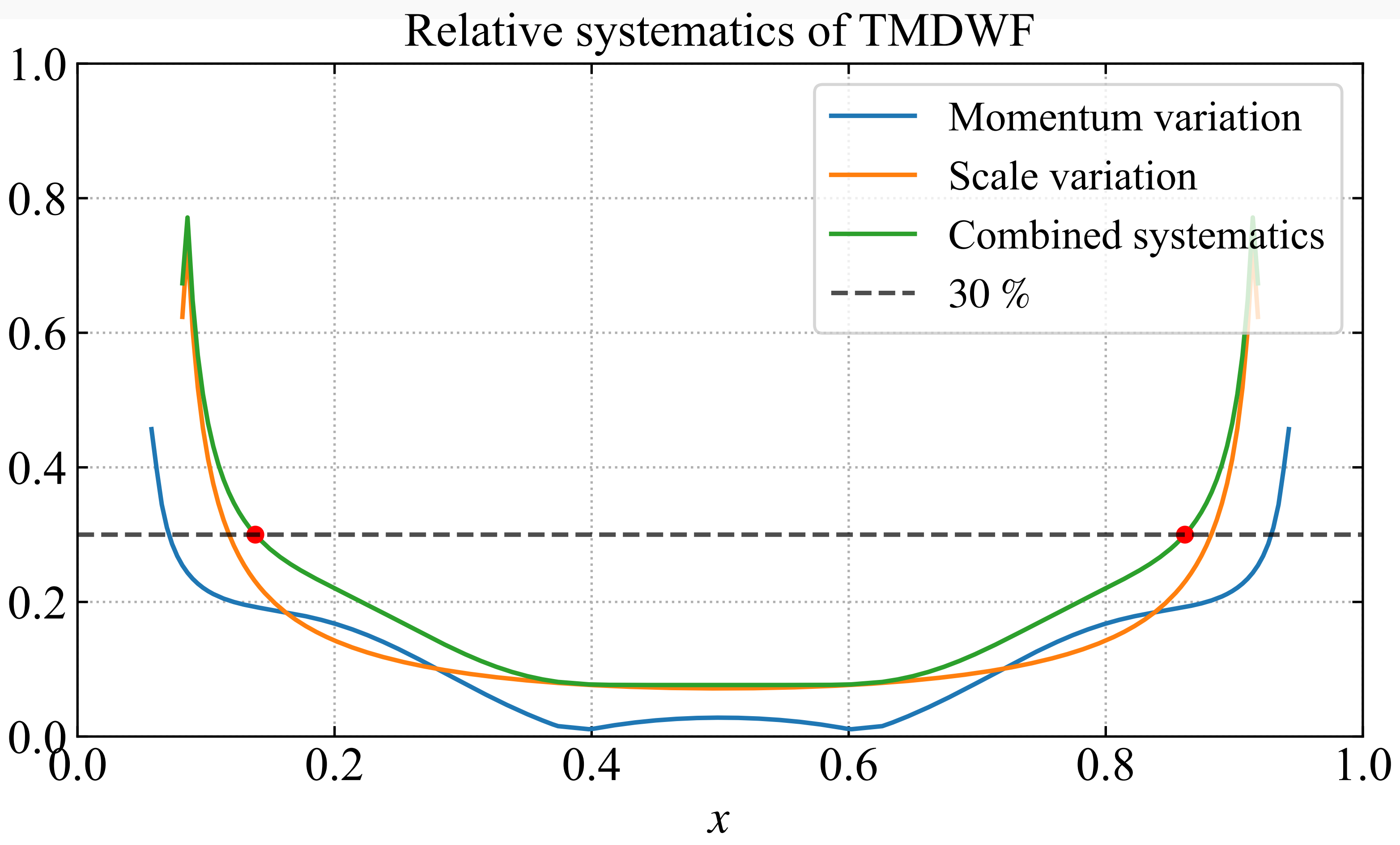
- **Our lattice results are consistent with the perturbation theory in the small $b_\perp$ regime;**
- **Different momenta of quasi-TMDWF give consistent results;**
- **Thanks to the absence of linear divergence, our final results of the intrinsic soft function can go beyond $b_\perp \sim 1$ fm.**

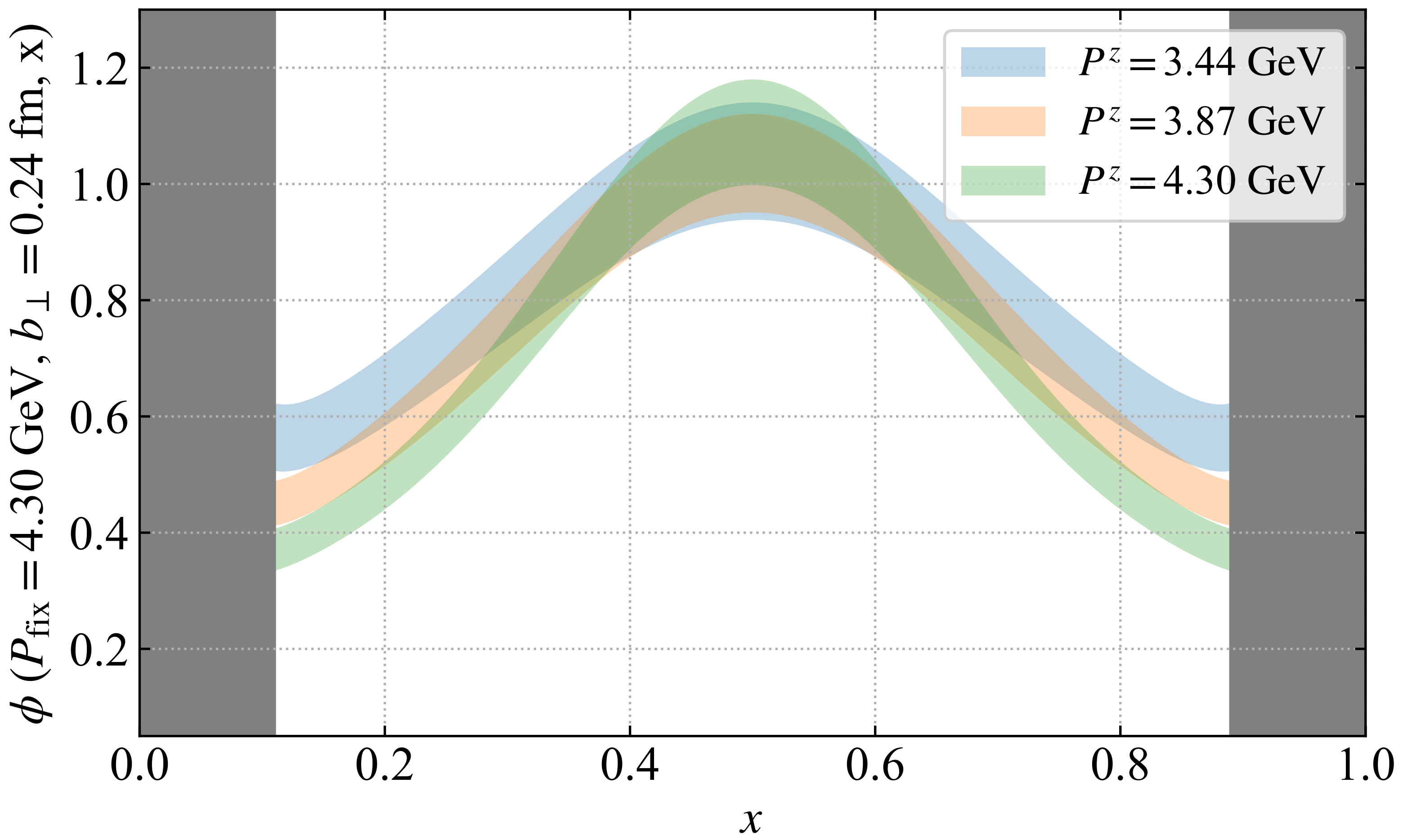
Pion quasi-TMD Wave Function

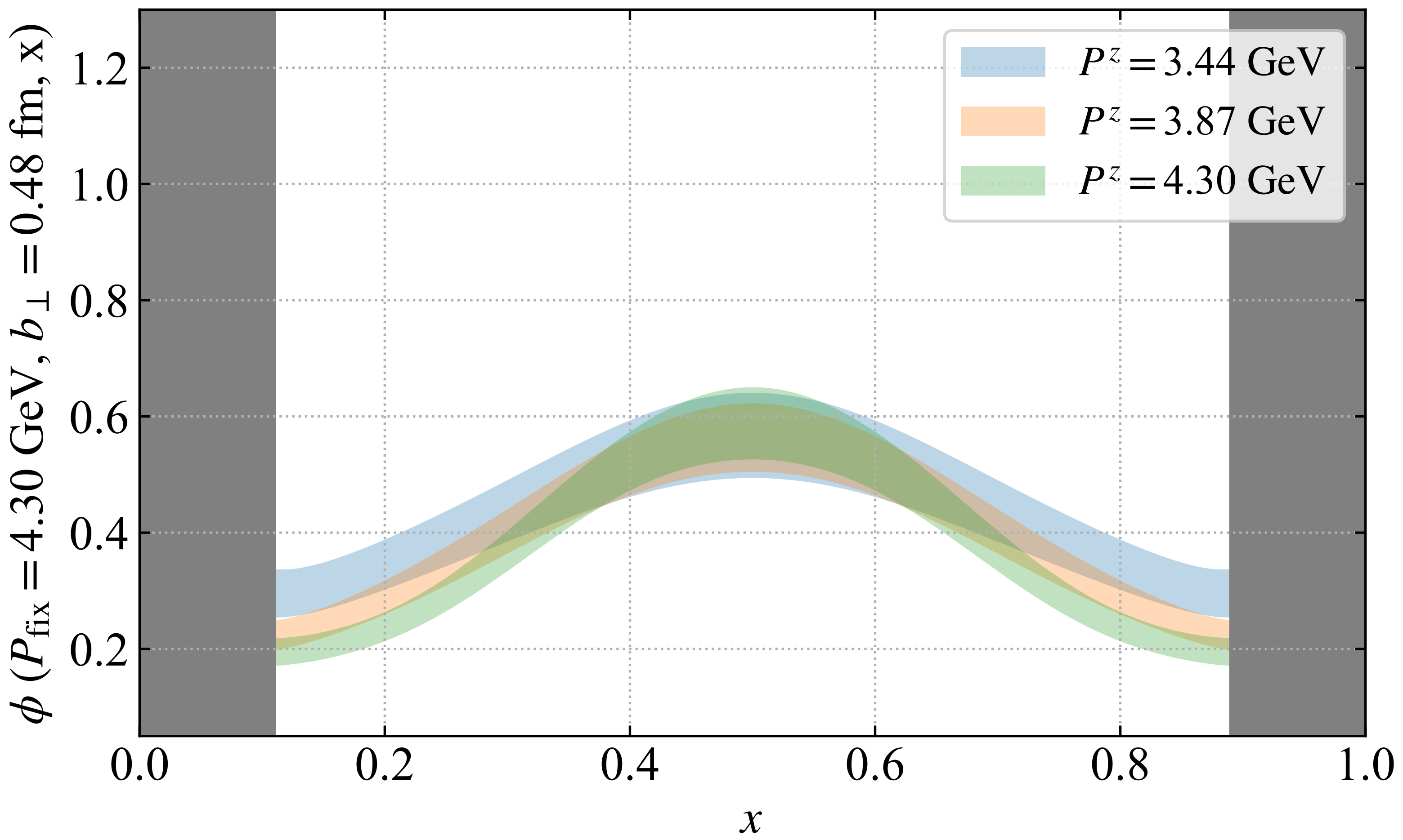


- We did a discretized Fourier transform because of the good convergence in the λ-space;
- The CG matrix elements decay to zero with the error bars remain almost constant, making the FT easy to be under control.

Pion TMD Wave Function



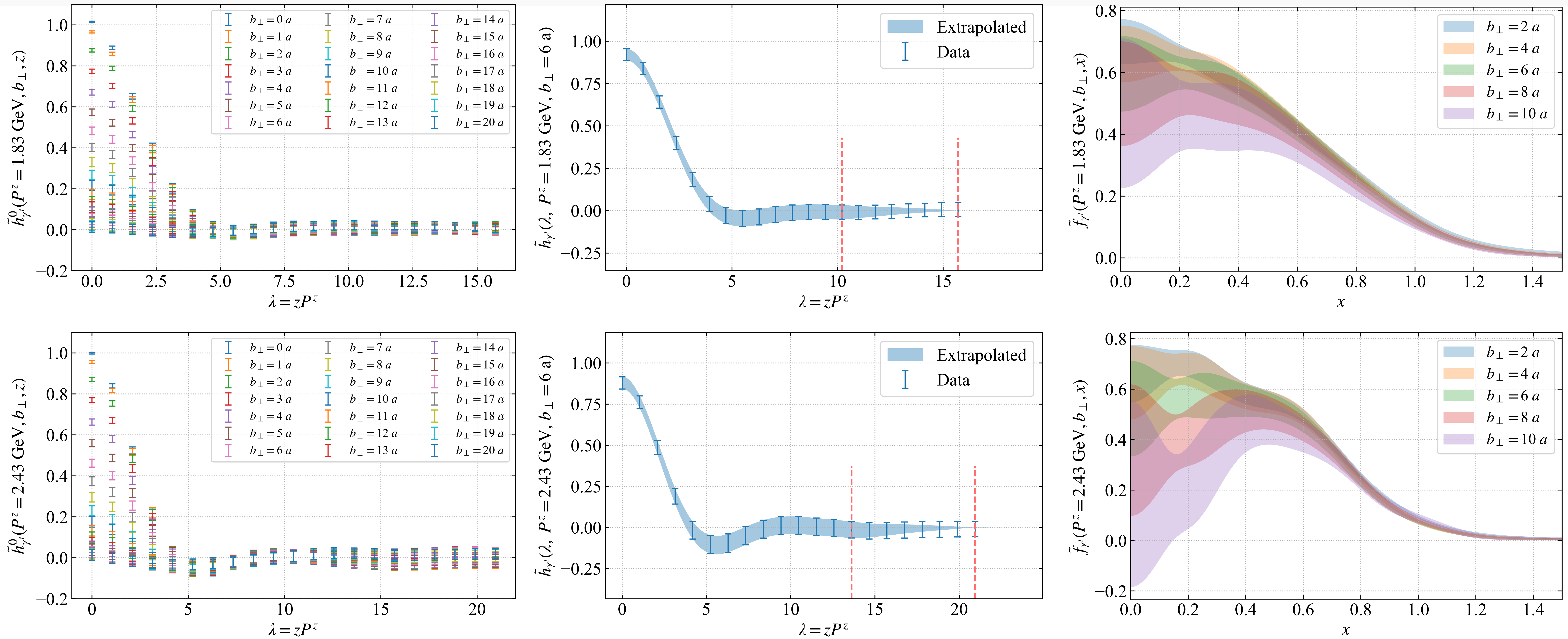




$$\sqrt{S_I\left(b_\perp;\mu\right)}\cdot\tilde{\phi}_\Gamma\left(x,b_\perp,P^z;\mu\right)=\phi(x,b_\perp;\mu,\zeta,\bar{\zeta})H_\phi\left(x,\bar{x},P^z;\mu\right)\exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\rm MS}}\left(b_\perp;\mu\right)\right]+\text{Power corrections}$$

- **The variation between different momenta remains mild in the moderate x region, demonstrating the validity of power expansion in large P^z;**
- **The combined systematics are estimated from two sources:**
  - **Momentum variation: spread of central values between three momenta / mean of central values of three momenta**
  - **Vary the initial scale in the RG resummation of matching kernel by a factor of $\sqrt{2}$;**
- **The $30\,\%$ combined systematics are used to quantify the moderate x region that LaMET can make reliable predictions;**
- **The convergence between three momenta near the endpoint regions can be improved with larger Lorentz boost factor.**

Pion quasi-TMD Beam Function



- **The CG matrix elements decay to zero with the error bars remain almost constant, making the FT easy to be under control.**

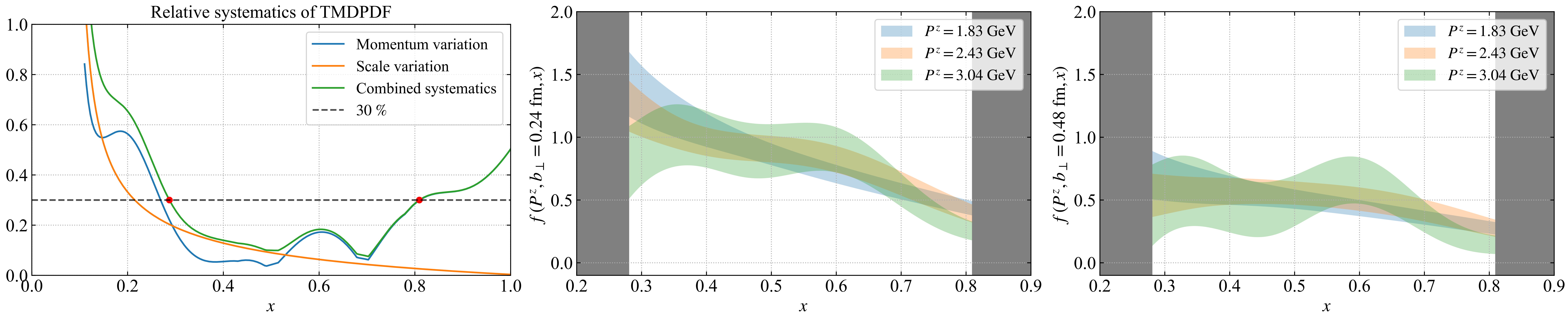
- **To remove the non-physical oscillation, we apply the extrapolation to make error bars converge to zero smoothly.**

A recent paper on asymptotic analysis in LaMET: *J. W. Chen, et al., 2505.14619*

- **Since quasi-TMD (in moderate x) is insensitive to the extrapolation strategies, the non-fit extrapolation is adopted here:**

$\tilde{h}^{\text{ext}} = w \cdot \tilde{h} + (1 - w) \cdot 0$, **where the weight $w(z)$ linearly decays from 1 to 0 within two red dashed lines below.**

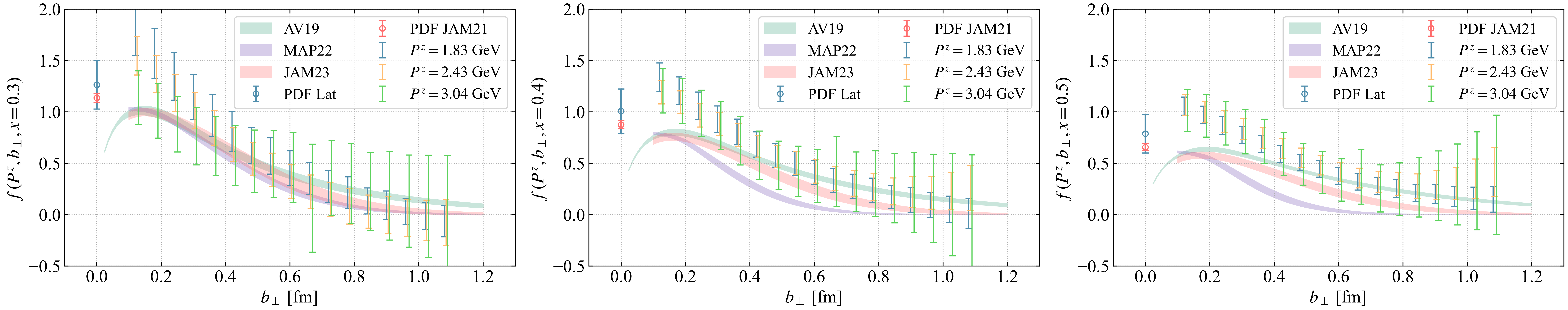
Pion TMDPDF in the x Space



$$\sqrt{S_I(b_\perp;\mu)} \cdot \tilde{f}_\Gamma(x, b_\perp, P^z;\mu) = f(x, b_\perp;\mu,\zeta)\, H_f(x, P^z;\mu) \exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\text{MS}}}(b_\perp;\mu)\right] + \text{Power corrections}$$

- **The variation between different momenta remains mild in the moderate x region, demonstrating the validity of power expansion in large P^z;**
- **The combined systematics are estimated from two sources:**
  - **Momentum variation: spread of central values between three momenta / mean of central values of three momenta**
  - **Vary the initial scale in the RG resummation of matching kernel by a factor of $\sqrt{2}$;**
- **The 30 % combined systematics are used to quantify the moderate x region that LaMET can make reliable predictions;**

Pion TMDPDF in the $b_\perp$ Space

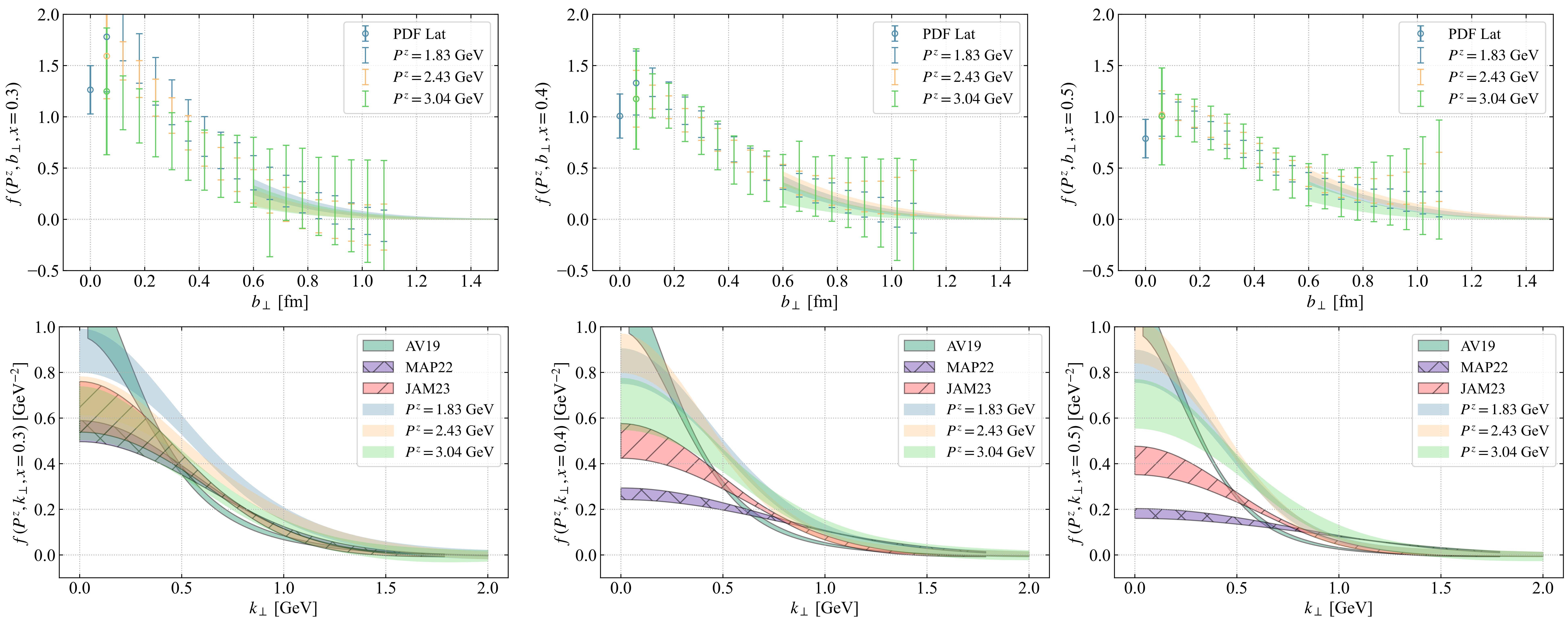


A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), Phys.Rev.D 107 (2023); P. C. Barry, et al. (JAM) Phys.Rev.D 108 (2023)

- **Thanks to the absence of linear divergence, we can calculate pion TMDPDF up to** $b_\perp > 1$ fm**;**
- **When x gets larger, the amplitude of TMDPDF is decreasing, while the the transverse correlation length stays roughly the same;**
- **When x gets closer to $x = 0.5$, we can find that the variance across different momenta becomes smaller, indicating the suppression of power correction;**
- **Global analysis has a better control at relative small , where we saw consistency;**
- **Lattice provides predictions at relative large , where experimental data gives less constraints.**

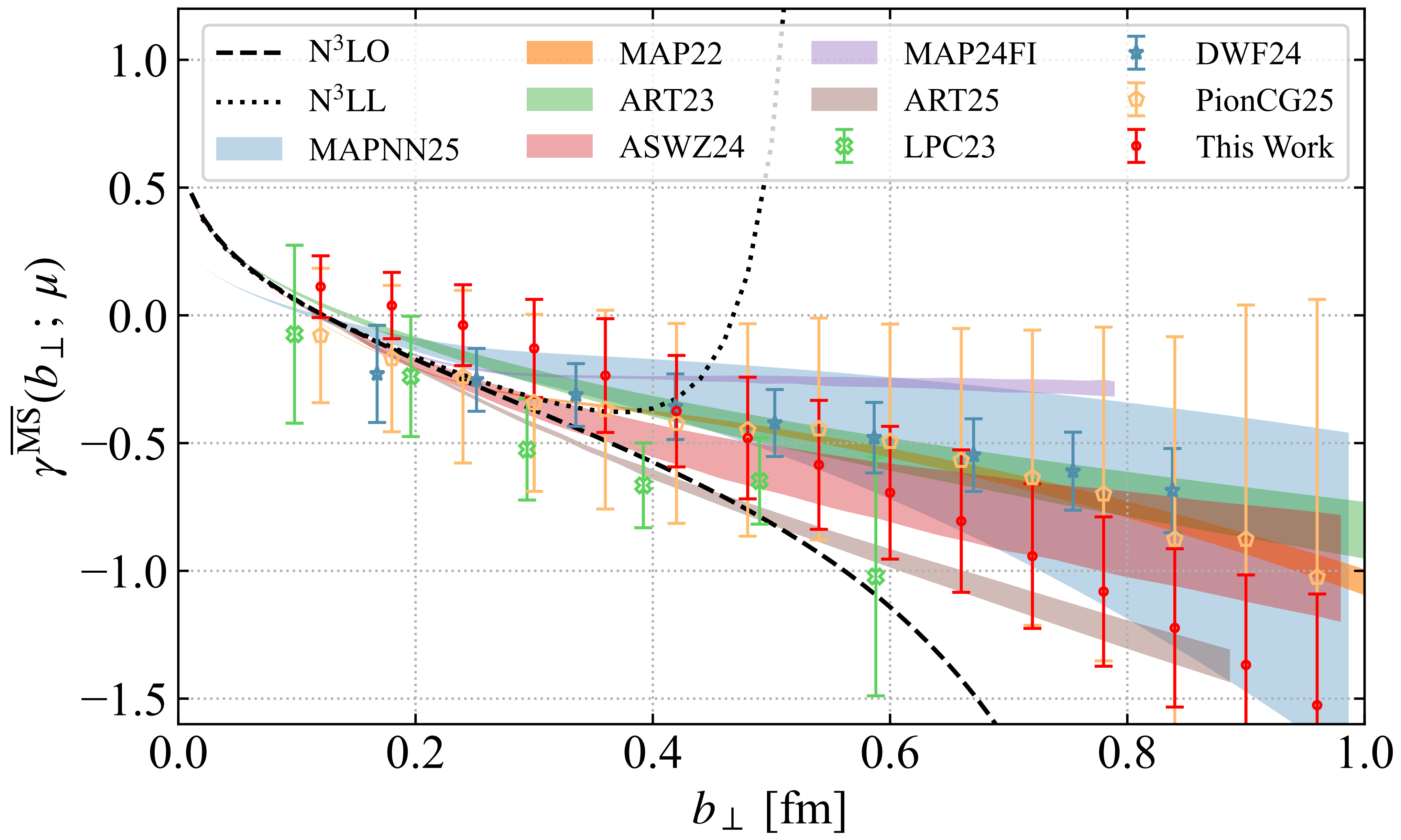
Pion TMDPDF in the $k_\perp$ Space

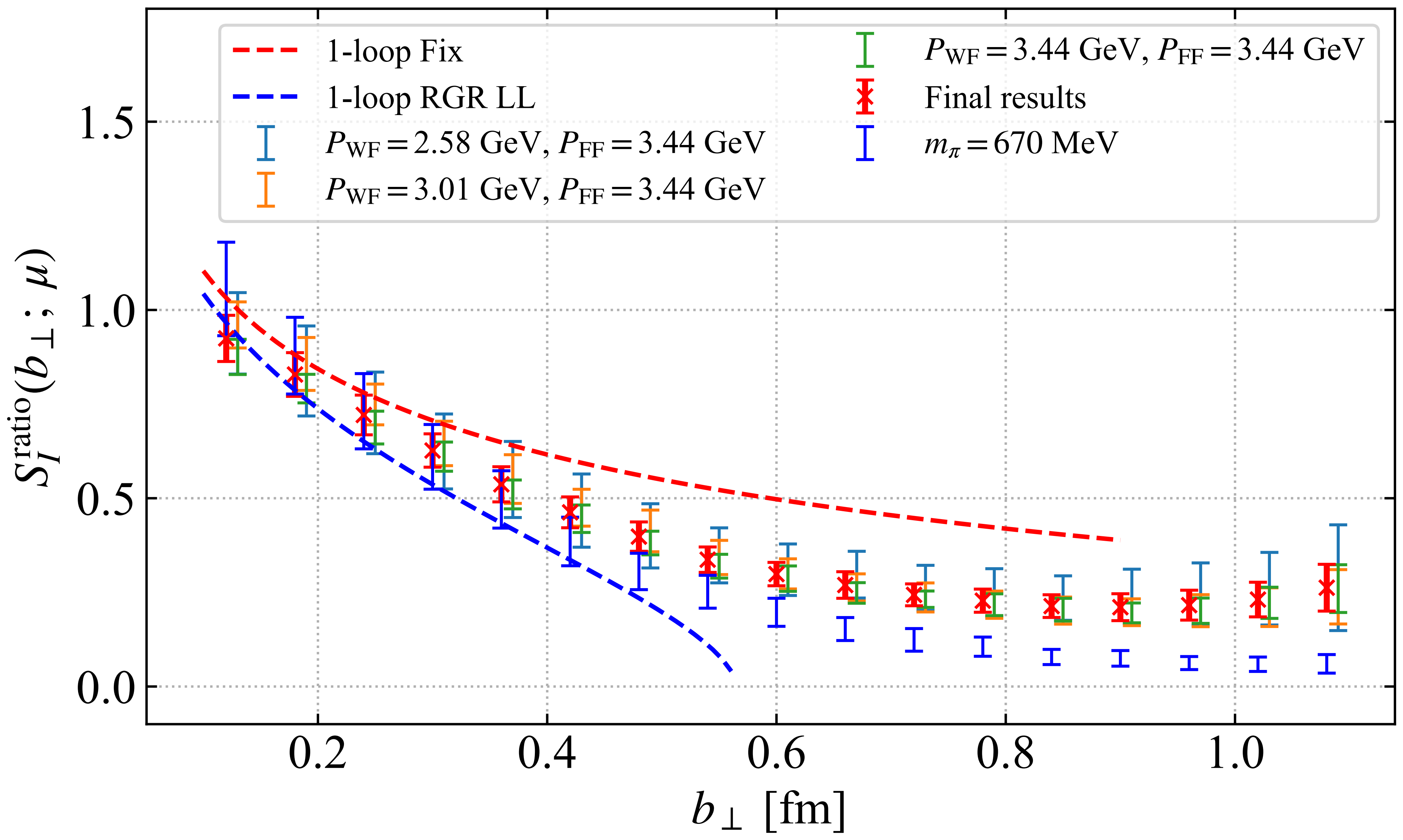
- **We can give the $k_\perp$-dependence thanks to the good SNR in CG;**
- **Extrapolate the large $b_\perp$ using a simple Gaussian form:** $f(b_\perp) = Ae^{-mb_\perp^2}$ **;**
- **Fourier transform to the $k_\perp$ space:** $\tilde{f}\left(k_\perp\right) = \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{\vec{b_\perp}\cdot\vec{k}_\perp} f\left(b_\perp\right) = \int \frac{d\left|b_\perp\right|}{2\pi} \left|b_\perp\right| \cdot J_0\left(\left|b_\perp\right| \cdot \left|k_\perp\right|\right) \cdot f\left(b_\perp\right)$.



A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), Phys.Rev.D 107 (2023); P. C. Barry, et al. (JAM) Phys.Rev.D 108 (2023)

Preliminary Results with mpi=300





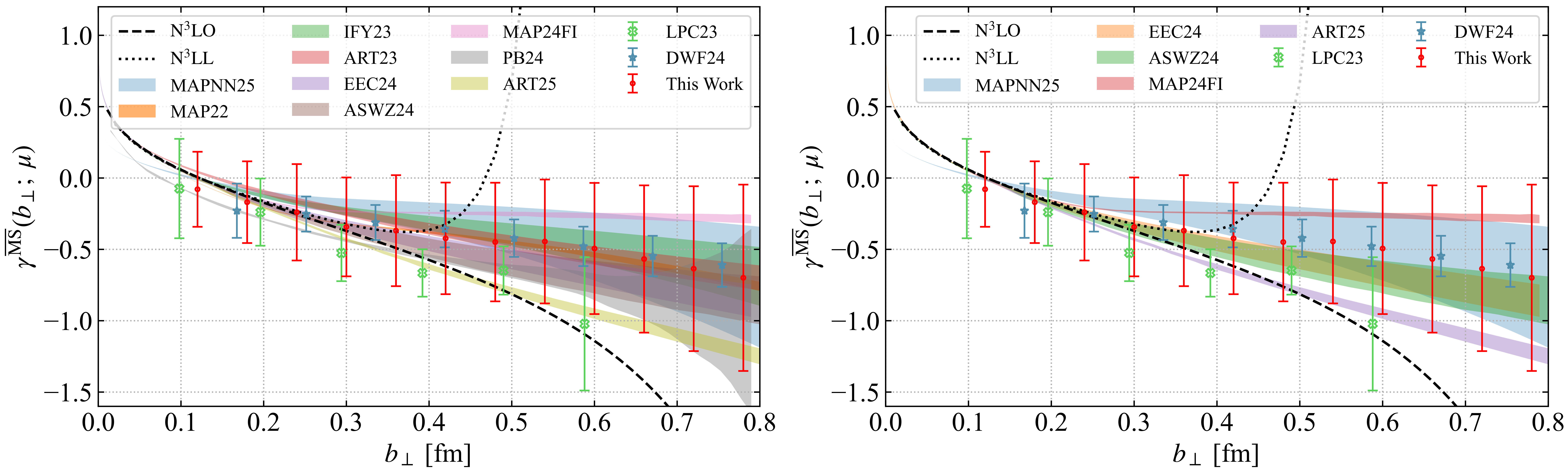
Summary

Summary

- This is the first lattice calculation of the pion unpolarized TMDPDF within LaMET framework;
- The novel CG method is employed to remove the linear divergence, so that to have a good SNR up to $b_\perp > 1$ fm;
- The soft function is extracted at NLL factorization using RG resummation, the results show consistency with perturbation theory;
- The TMDs, including CS kernel, intrinsic function, TMDWF and TMDPDF are calculated using the same lattice ensemble, and the results show consistency with existing studies, including phenomenology and lattice calculations;
- The outcome of this study highlights the efficacy of the CG quasi-TMD approach in probing the transverse momentum structure of hadrons;
- In the future work, we will apply the CG quasi-TMD approach on nucleon, and the lattice systematics like discretization effects and non-physical pion mass will be investigated in detail.

Backup

CS Kernel



N^{3}LO
N^{3}LL
MAPNN25
MAP22
IFY23
ART23
EEC24
ASWZ24
MAP24FI
PB24
ART25
LPC23
DWF24
This Work
$\gamma^{\overline{\mathrm{MS}}}(b_\perp;\mu)$
$b_\perp$ [fm]
N^{3}LO
N^{3}LL
MAPNN25
EEC24
ASWZ24
MAP24FI
ART25
LPC23
DWF24
This Work
$\gamma^{\overline{\mathrm{MS}}}(b_\perp;\mu)$
$b_\perp$ [fm]

Soft Function in TMD Factorization

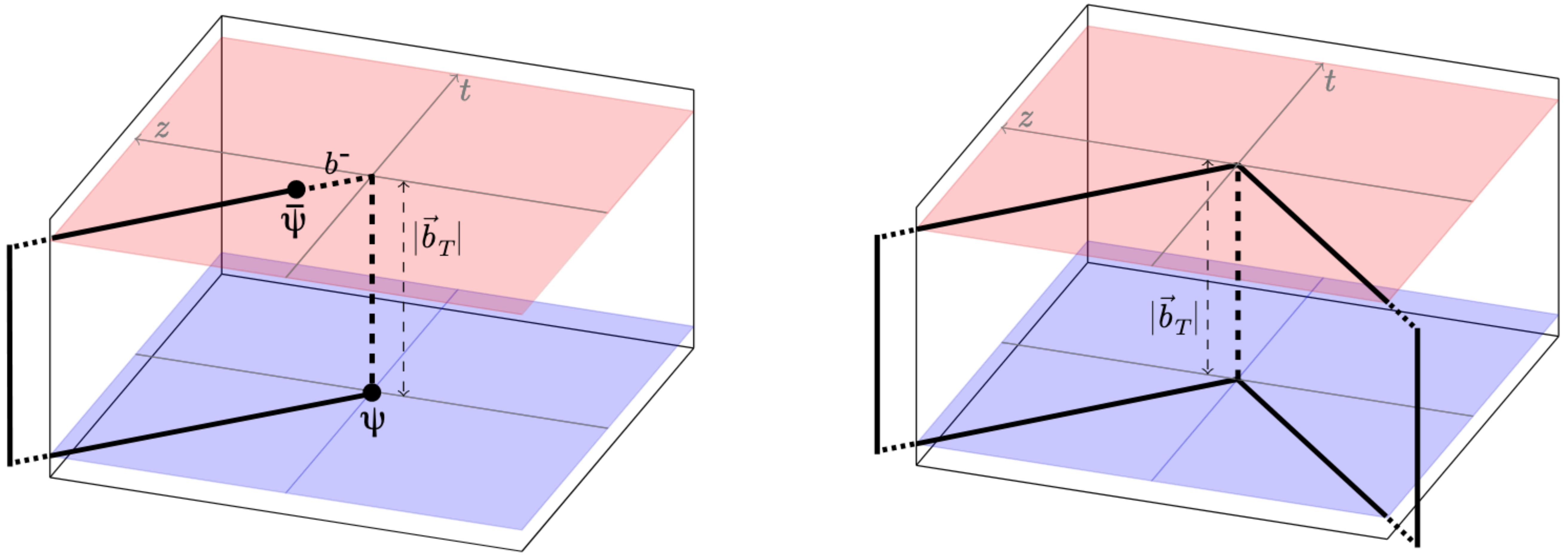
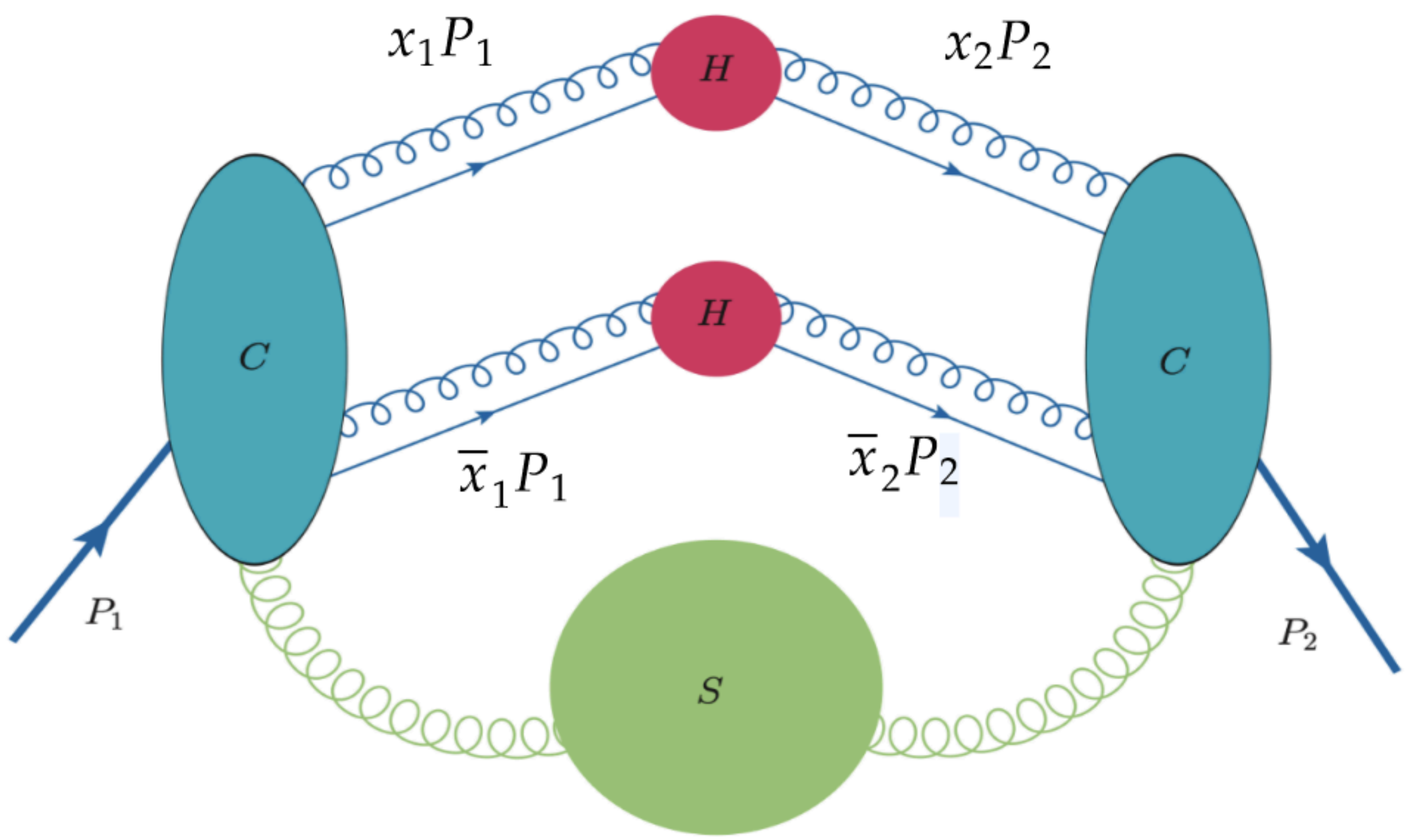


Figure 2.1: Graphs of the Wilson line structure $W_{\sqsubset}(b^\mu, 0)$ of the unsubtracted TMD PDF $f_{i/p}^{0\,(\mathrm{u})}$ (left) and of $W_{\gtrless}(b_T)$ for the soft function $S^0_{n_a n_b}$ (right), defined in Eqs. (2.37) and (2.38). The Wilson lines (solid) extend to infinity in the directions indicated. Adapted from [106].

R. Boussarie, et al., 2304.03302 (2023)

Sudakov Kernel



Z. F. Deng, et al., JHEP 09 (2022)

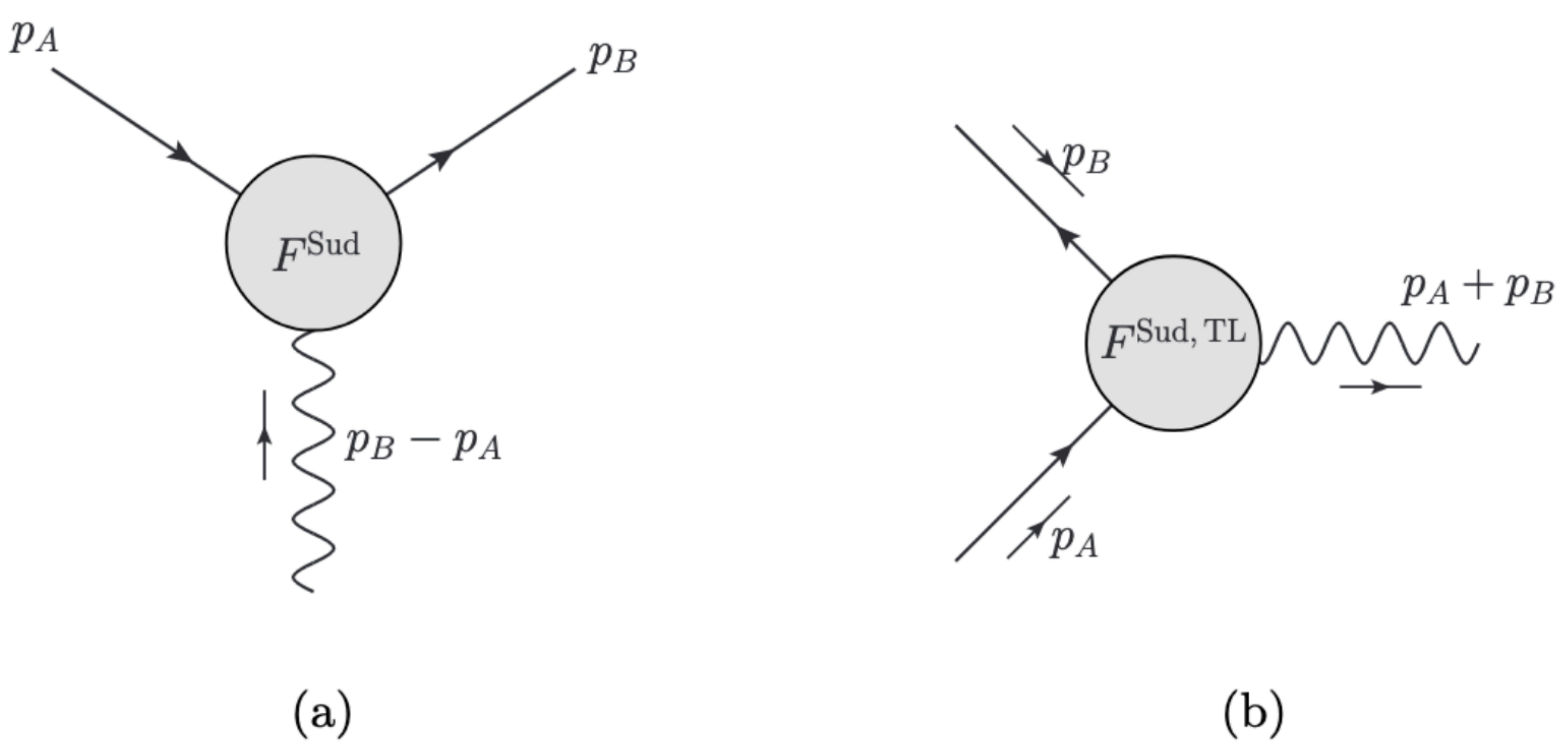


FIG. 1. Graphs for the space-like (a) and time-like (b) quark form factor.

J. Collins and T. C. Rogers, Phys. Rev. D 96 (2017)

$H_F(x_1, x_2, P^z; \mu) = C_{\rm Sud}(x_1, x_2, P^z; \mu) \cdot C_{\rm Sud}(\bar{x}_1, \bar{x}_2, P^z; \mu)$, **where $C_{\rm Sud}$ is the Sudakov kernel.**

$$p_A = (x_1P_1, 0, 0, x_1P_1), \quad p_B = (x_2P_2, 0, 0, -x_2P_2)$$

$$Q^2 = -(p_B - p_A)^2 = 4x_1x_2P_1P_2$$

$$\bar{Q}^2 = 4\bar{x}_1\bar{x}_2P_1P_2$$

Gauge Fixing in Lattice QCD

Continuous Theory

$$F_{\mathrm{CG}}[A,\Omega] \equiv \frac{1}{2}\sum_{\mu=1}^{3}\int d^4x A^a_{\Omega\mu}(x)A^{\mu a}_{\Omega}(x)$$

$$\begin{aligned}\delta F_{\mathrm{CG}}[A,\Omega] &= -\sum_{\mu=1}^{3}\int d^4x(D^{\Omega}_{\mu ab}\theta_b)A^{\mu a}_{\Omega}\\ &= -\sum_{\mu=1}^{3}\int d^4x(\partial_\mu\theta_a - gf^{cab}A^c_{\Omega\mu}\theta_b)A^{\mu a}_{\Omega}\\ &= \sum_{\mu=1}^{3}\int d^4x\theta_a(\partial_\mu A^{\mu a}_{\Omega})\end{aligned}$$

$$* \; A_{\Omega\mu}(x) \equiv \Omega^\dagger(x)A_\mu(x)\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$$

Lattice Theory

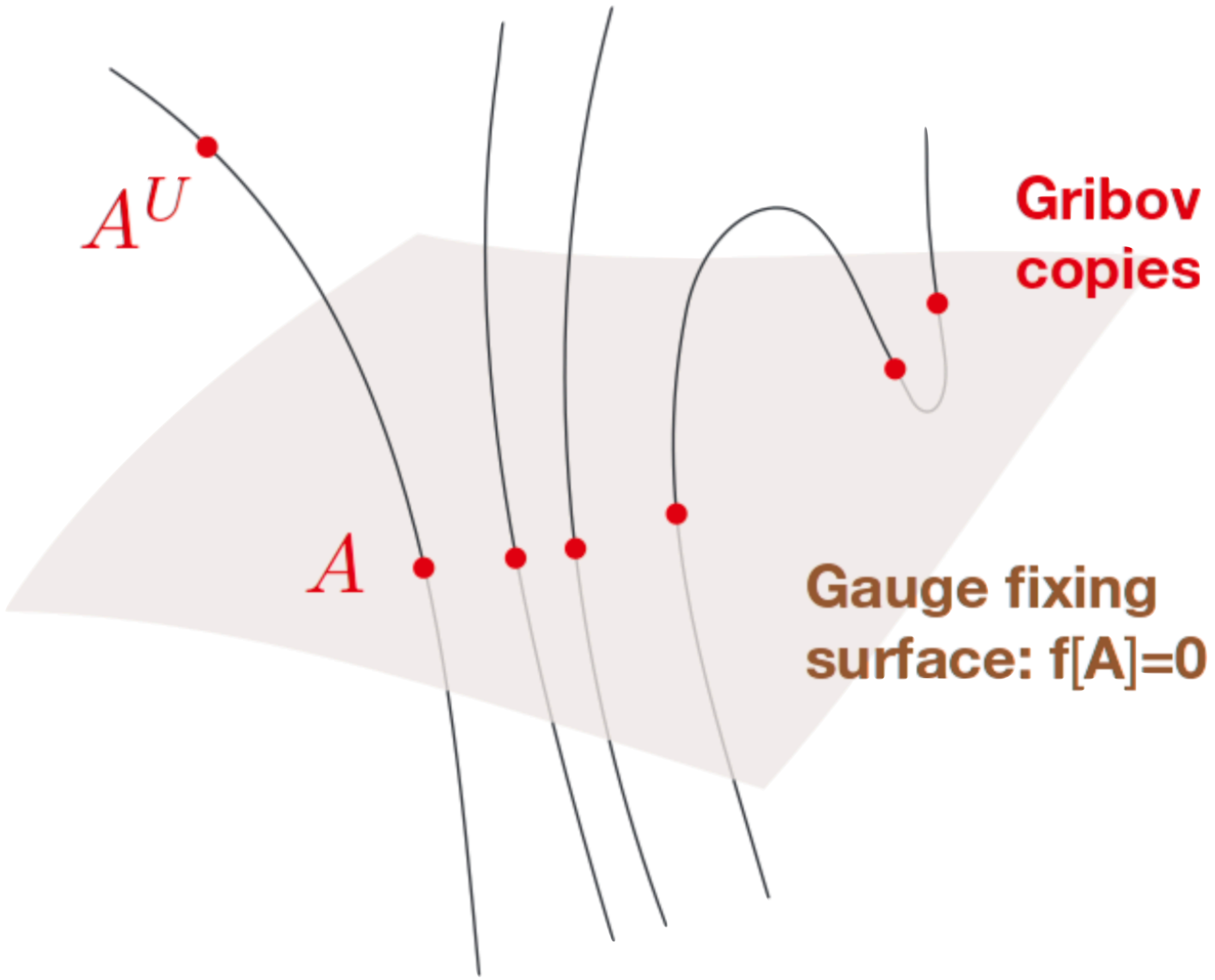
$$F_{\mathrm{CG}}[U,\Omega] \equiv -\Re\left[\mathrm{Tr}\sum_x\sum_{\mu=1}^{3}\Omega^\dagger(x+\hat{\mu})U_\mu(x)\Omega(x)\right]$$

Find stationary points of the functional value.

Gauge fixing criterion in this work: variation of functional satisfies $\delta F/F < 10^{-8}$.

Gribov Copies

The gauge fixing condition may have many solutions in Lattice QCD.



Ph. D. Thesis of Diego Fiorentini

Criteria of Gauge Fixing

- **Variation of the functional**

$$\delta F/F < 10^{-8}$$

- **Residual gradient of the functional**

$$\theta^G \equiv \frac{1}{V}\sum_x \theta^G(x) \equiv \frac{1}{V}\sum_x \mathrm{Tr}\left[\Delta^G(x)\left(\Delta^G\right)^\dagger(x)\right],\ \Delta^G(x) \equiv \sum_\mu \left(A_\mu^G(x) - A_\mu^G(x-\hat{\mu})\right)$$