

Towards Precision-Controlled Partonic Structures from First Principles

Doctoral Defense

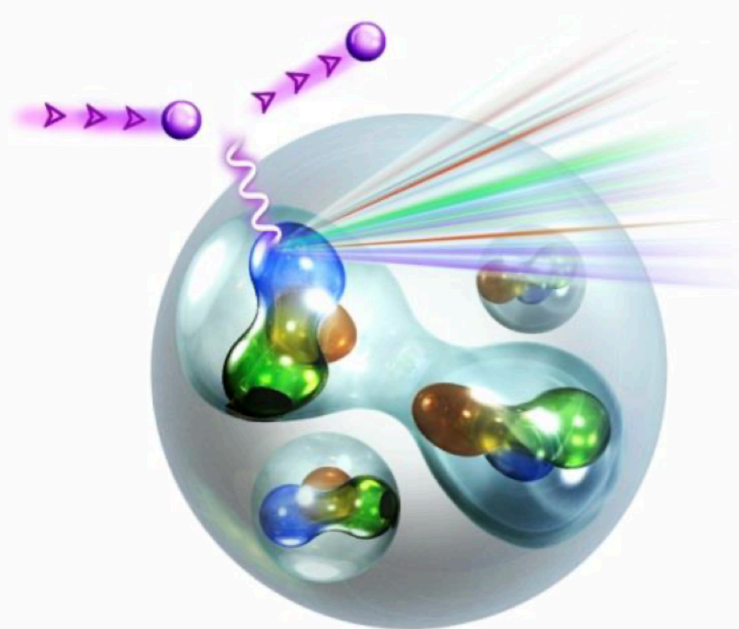
Advisor: Xiangdong Ji

Jinchen He

2026/4/8

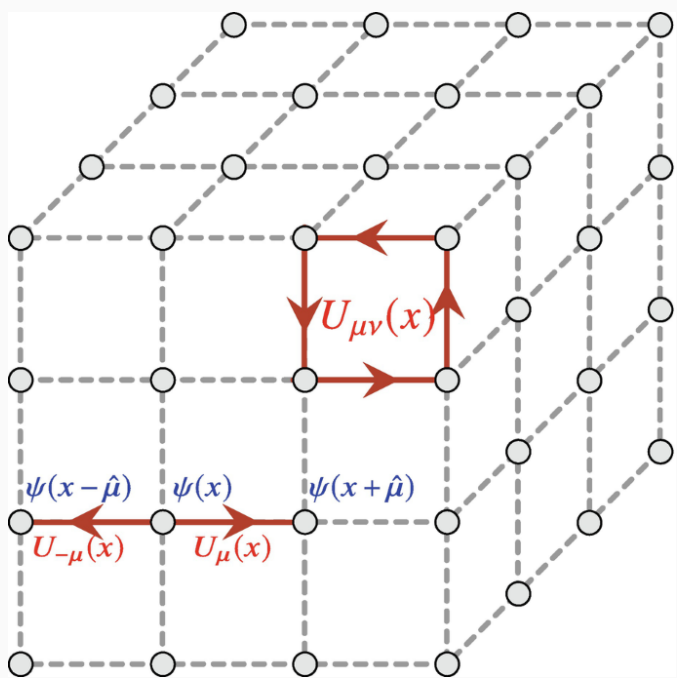
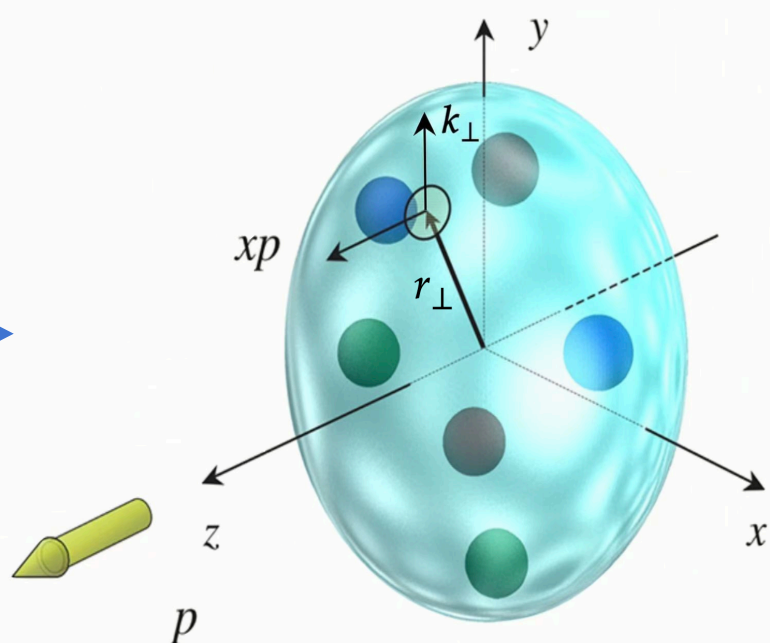
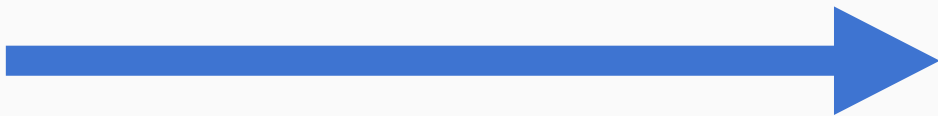


Lattice Calculation of Partonic Physics



Cr. Dave Gaskell

Collinear to TMDs



Cr. Olaf Kaczmarek



OLCF



NERSC



LCRC

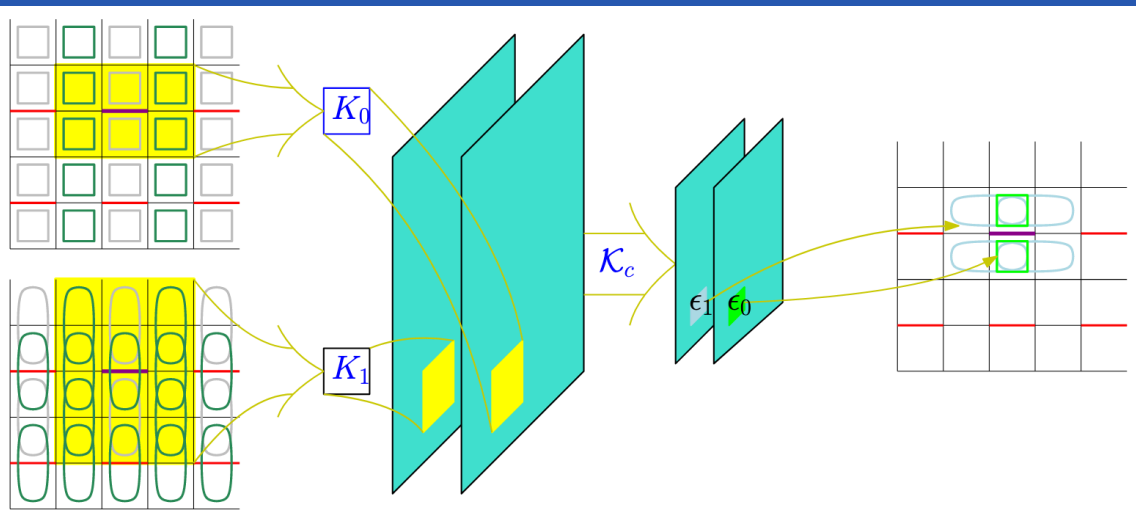


ALCF

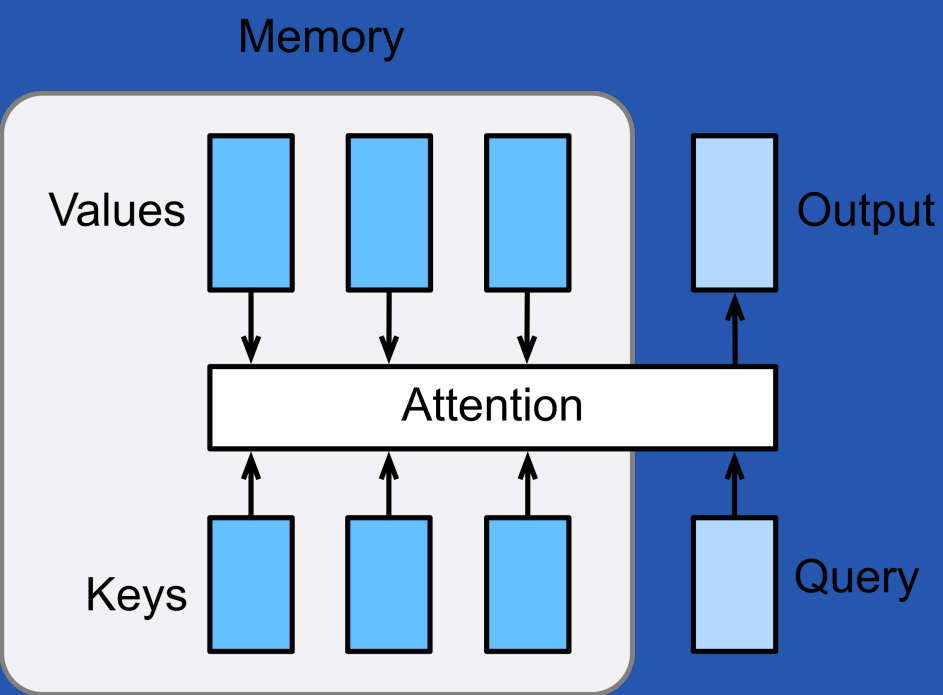


FNAL

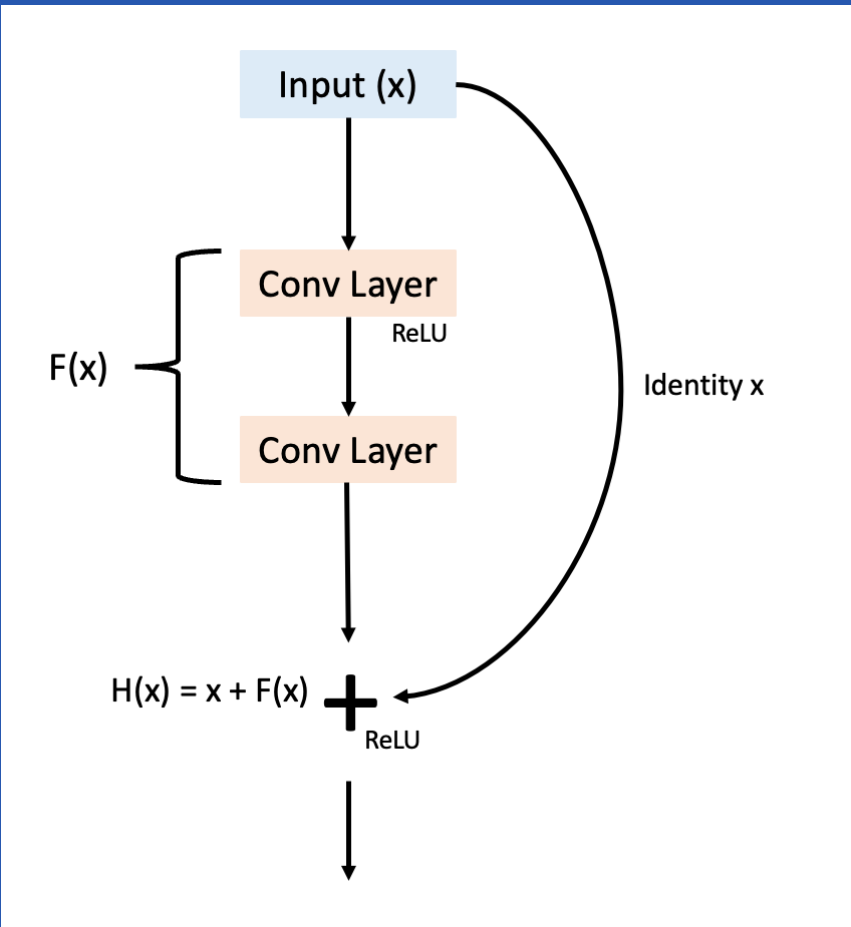
AI/ML for Lattice



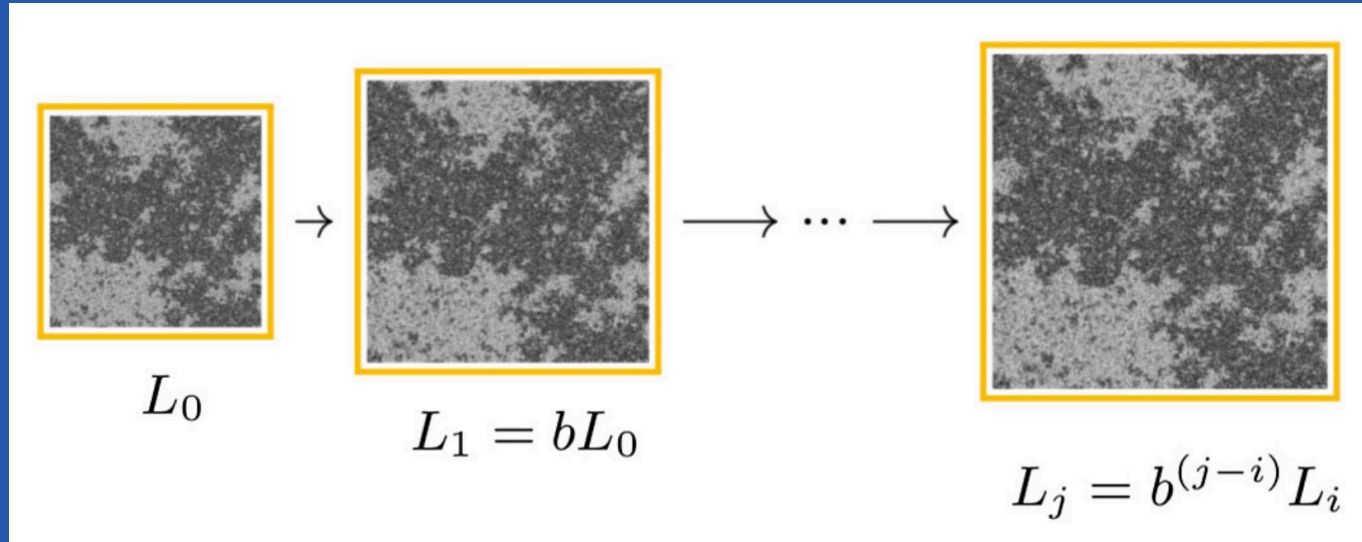
Cr. Xiao-Yong Jin



Attention



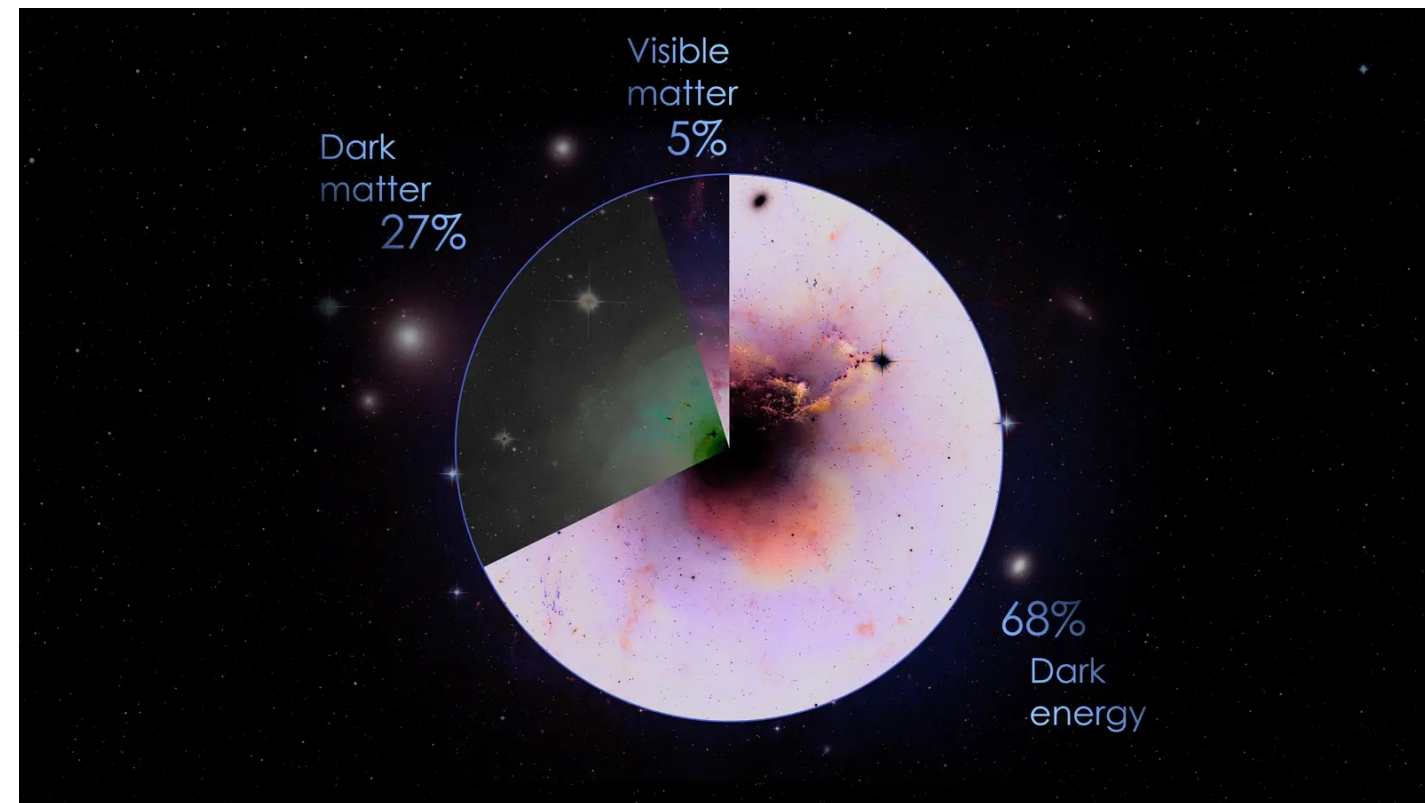
ResNet



Cr. D. Bachtis, et al., PRL 128 (2022) 2107.00466

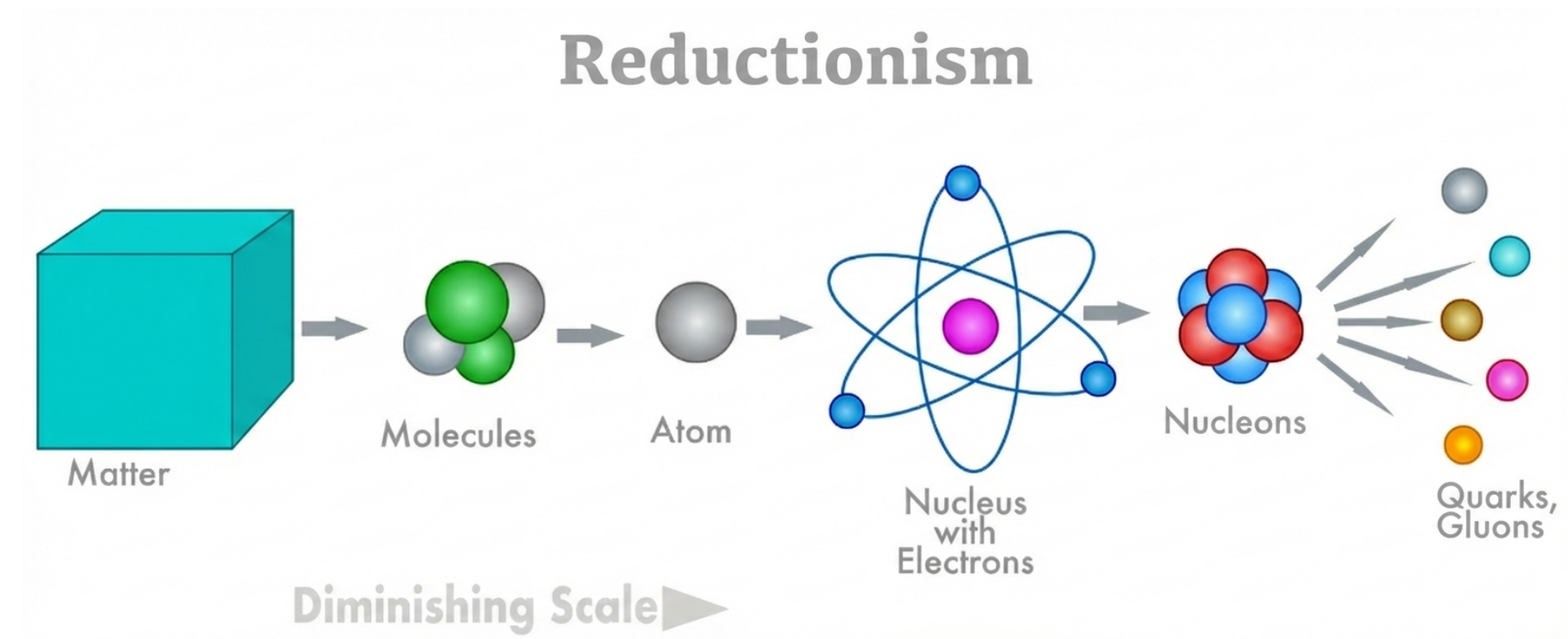
Visible Universe

- Only 5% of the universe is visible; *D. N. Spergel, Science 347.6226 (2015)*
- The visible universe can be understood as a hierarchy: molecules → atoms → nuclei → nucleons → ...;

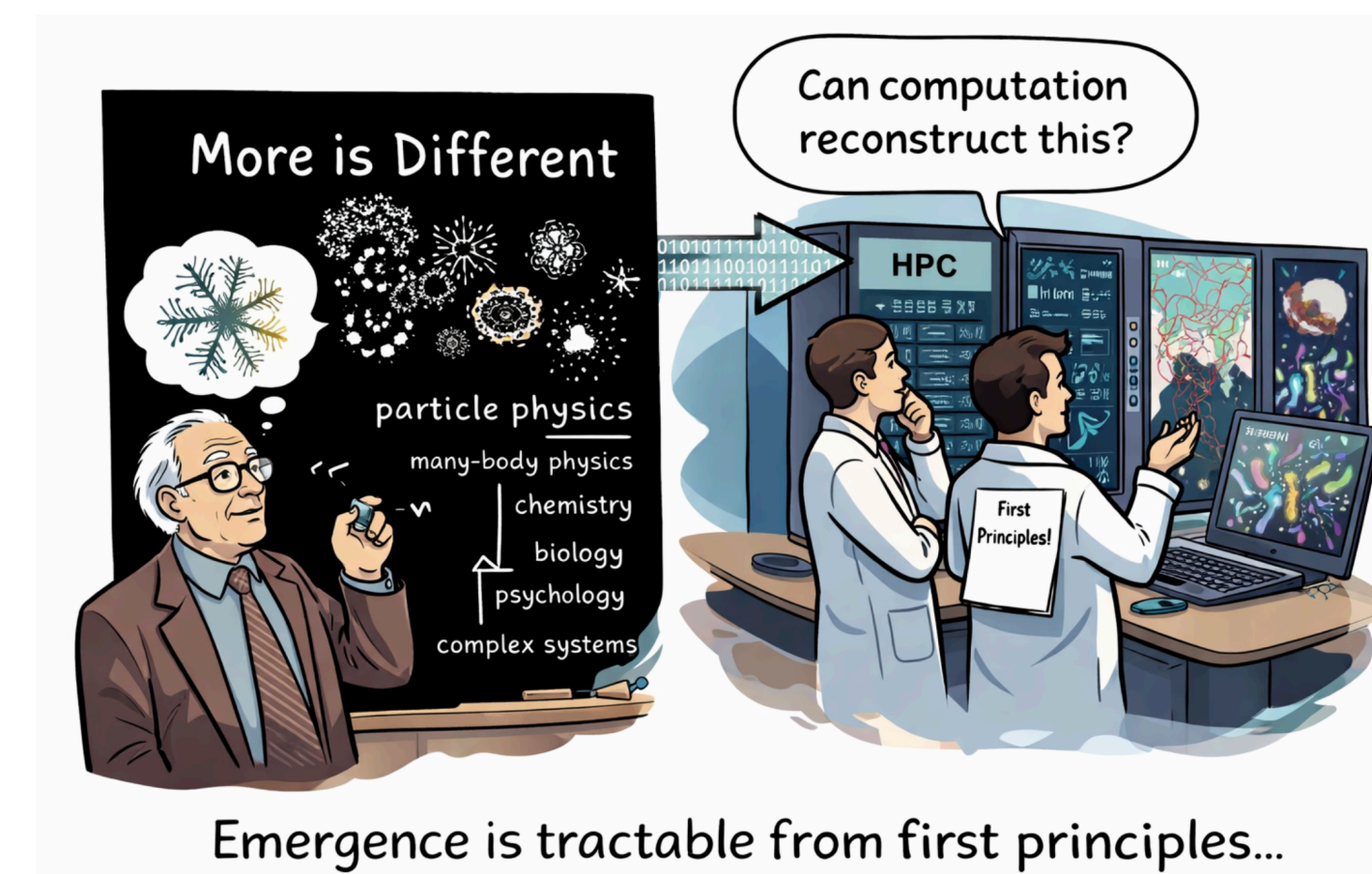


Cr. NASA's Goddard Space Flight Center

Reductionism motivates the search for fundamental degrees of freedom.



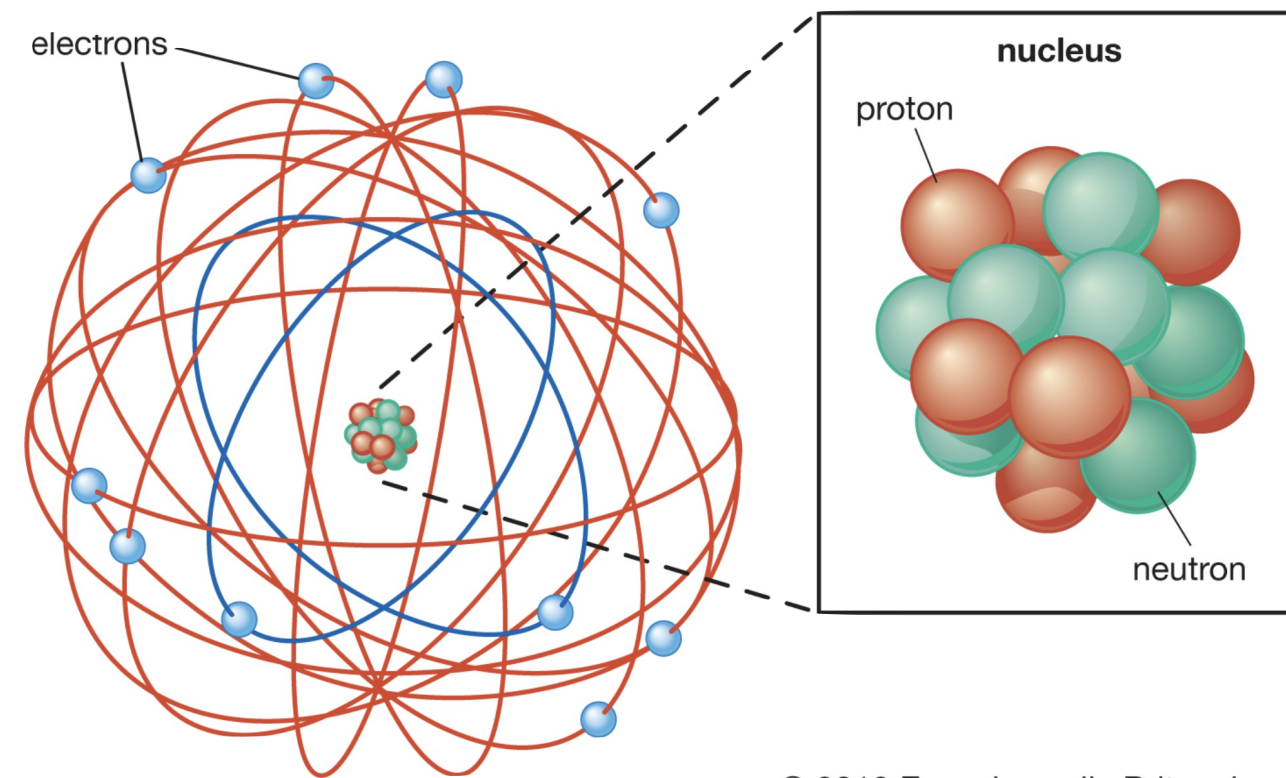
- More is different? *P. W. Anderson, Science 177.4047 (1972)*
- Emergence *After 50 years, things are changing...*
- Broken symmetry *Computation becomes more and more important in this era*
- Complexity (large number of d.o.f., strongly coupled system)



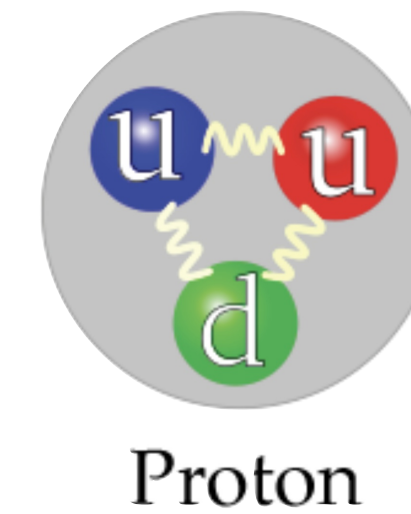
- **Modern computation makes macroscopic emergence increasingly tractable from first principles.**

Visible Universe

- The inner structure of hadrons are still sophisticated if we step closer;

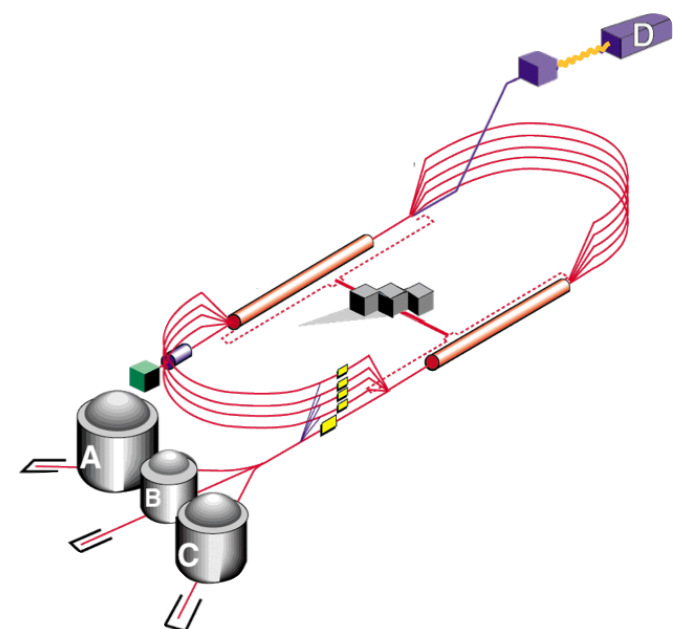


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- Many experiments have been designed to probe the internal structure of hadrons.

CEBAF(JLab)



Cr. Dave Gaskell

HERA



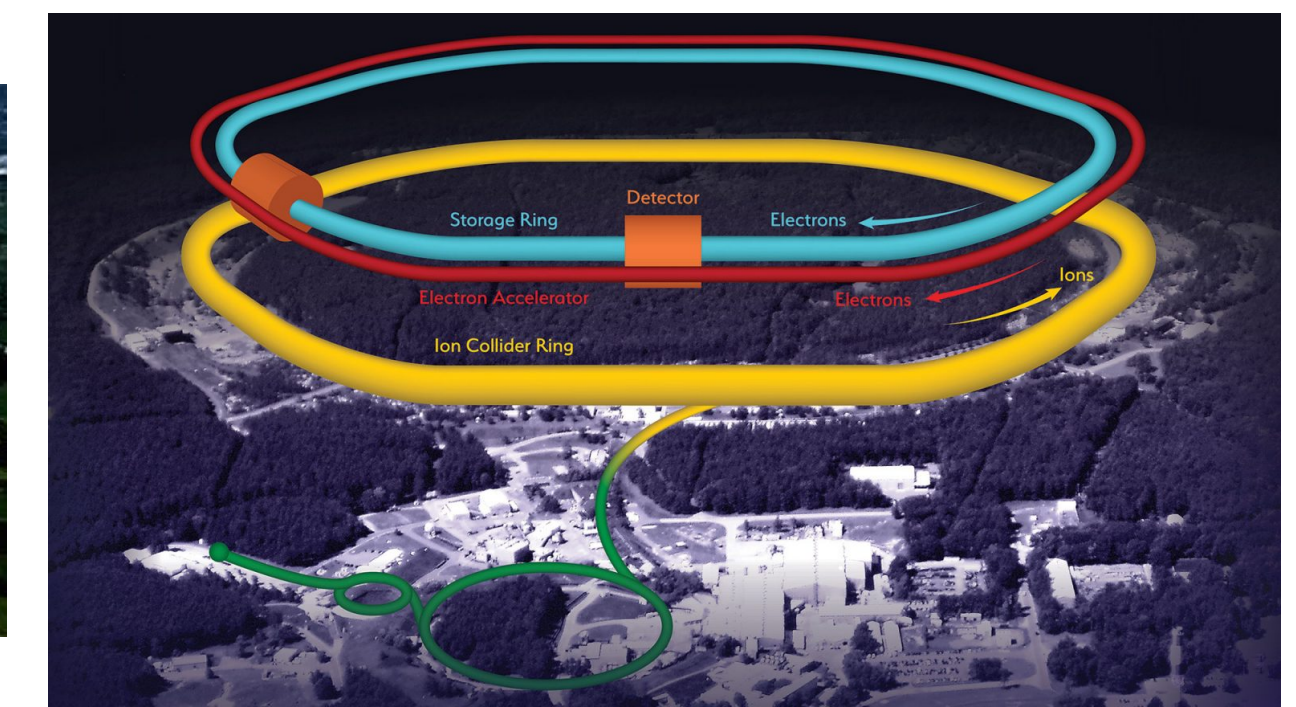
Cr. DESY

LHC



Cr. CERN

EIC



Cr. BNL

Parton Physics

- Our knowledge of hadrons is still limited:

- Spin, mass ...

Perturbation theory

Non-relativistic EFTs

- How to describe a strongly coupled relativistic moving bound state?

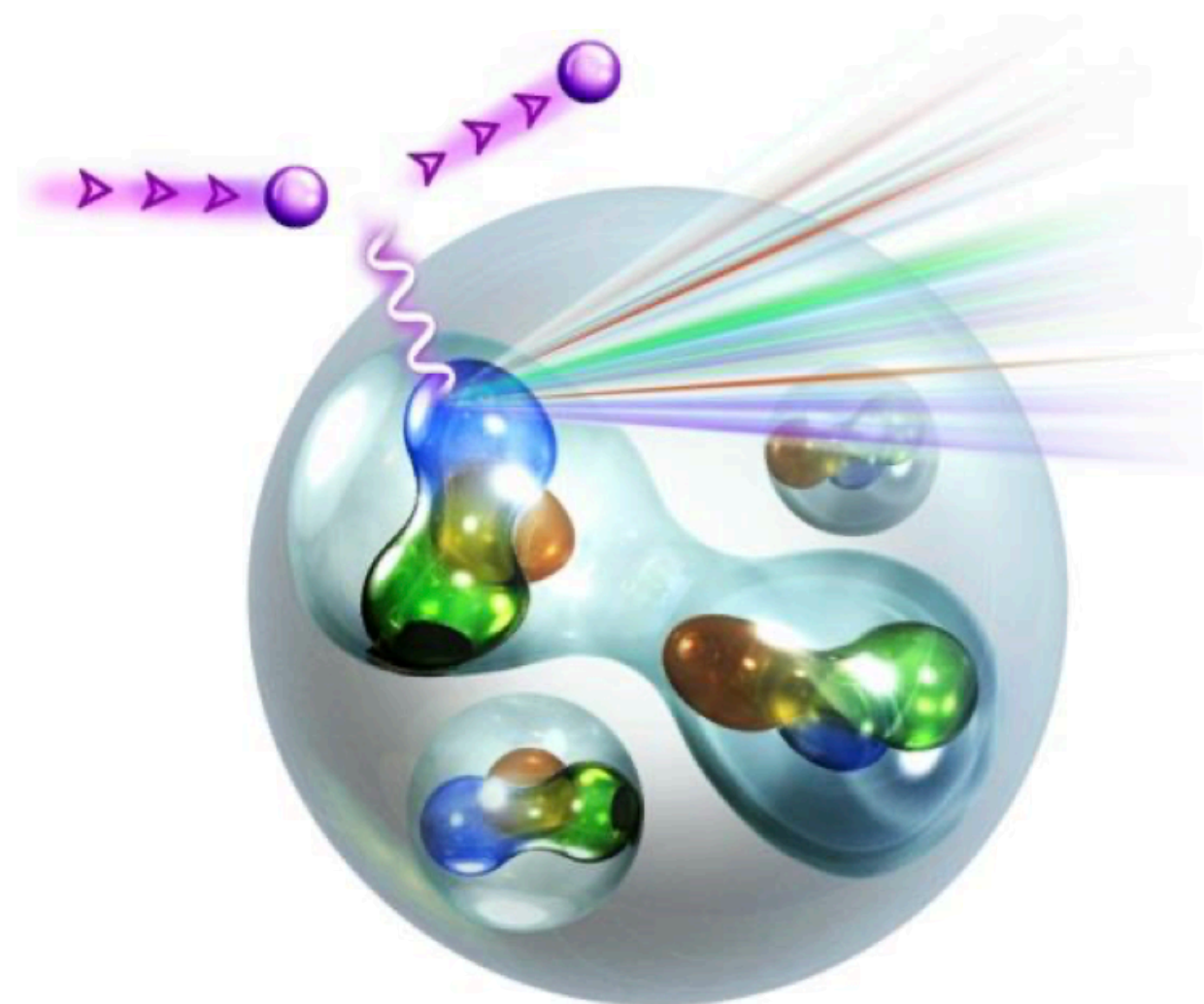
- The language from Feynman: Parton Model in the infinite momentum frame

R. P. Feynman, Conf. Proc. C 690905 (1969)

- Quarks and gluons (partons) are “frozen” in the transverse plane;

- During a high-energy collision, the struck parton appears like a free particle.

Bjorken Scaling → point-like structure



Cr. Dave Gaskell

The many faces of the proton

QCD bound state of **quarks** and **gluons**

Origin of mass?

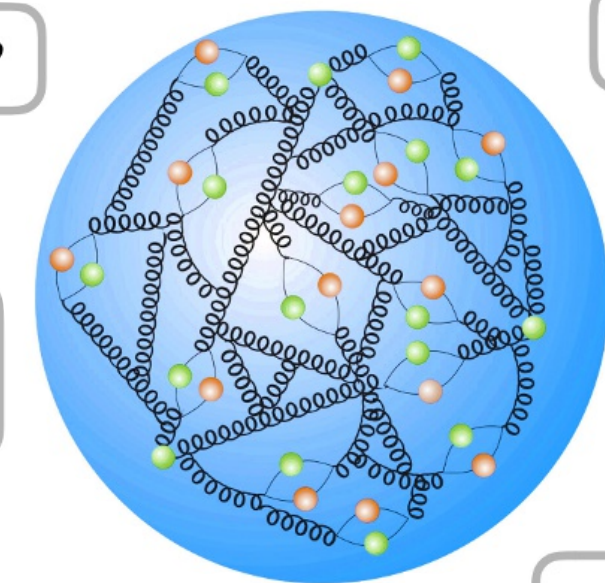
Origin of spin?

Gluon-dominated matter?

3D imaging?

Heavy quark content?

Nuclear modifications?



Cr. Juan Rojo

Collinear Factorization

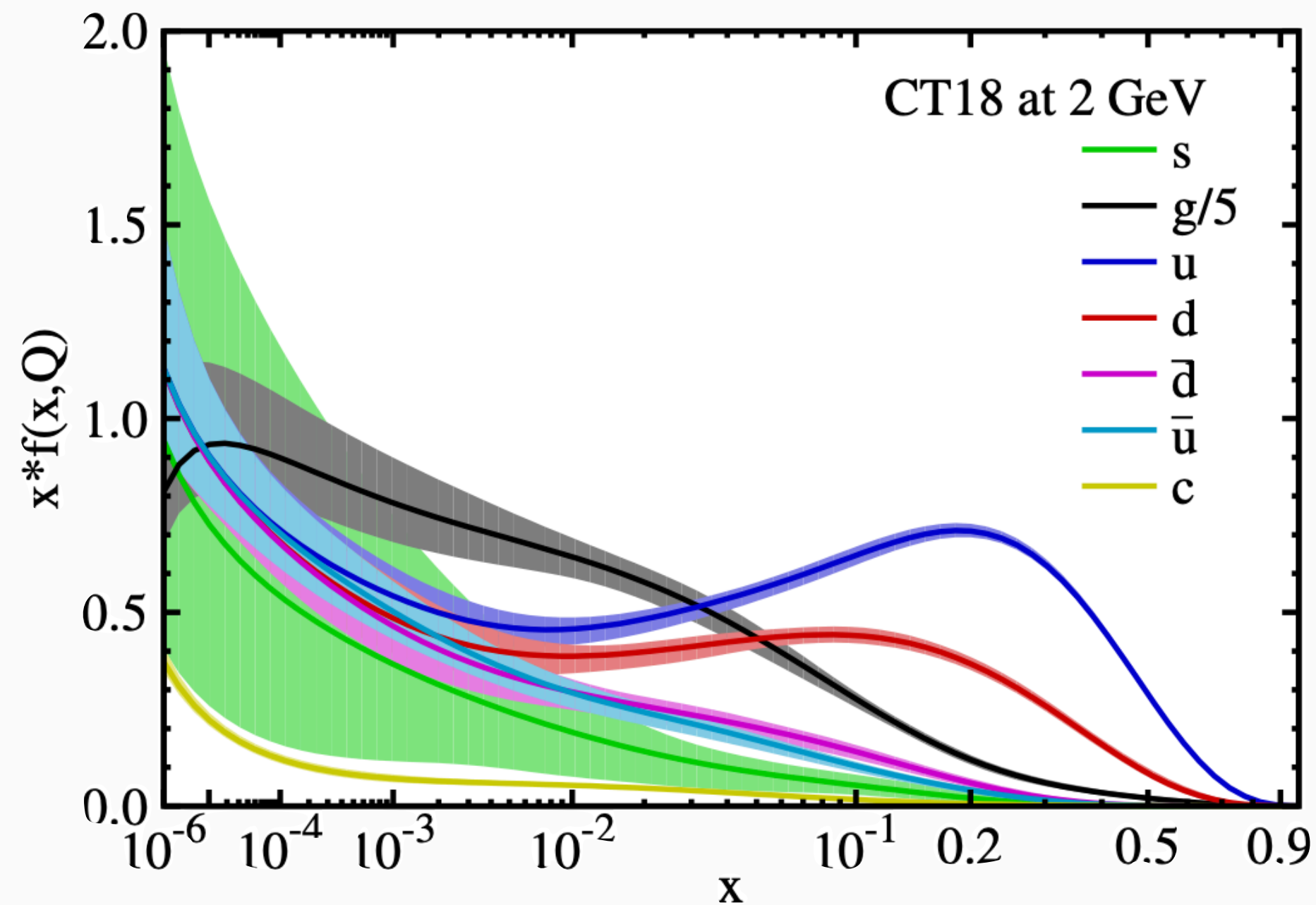
$$\sigma_{\text{DIS}} \propto \left| \begin{array}{c} l \\ l' \\ q \\ P \end{array} \right|^2 \approx \left| \begin{array}{c} k \approx xP \\ p \\ P \end{array} \right|^2 \otimes \left| \begin{array}{c} l \\ l' \\ q \\ xP \end{array} \right|^2$$

The diagram shows the collinear factorization of the deep inelastic scattering cross-section. It is expressed as the product of three terms: the squared amplitude of the lepton-quark interaction, the squared parton distribution function (PDF) for finding a quark with momentum $k \approx xP$ inside a proton with momentum P , and the squared amplitude of the perturbative scattering process between the lepton and the struck quark.

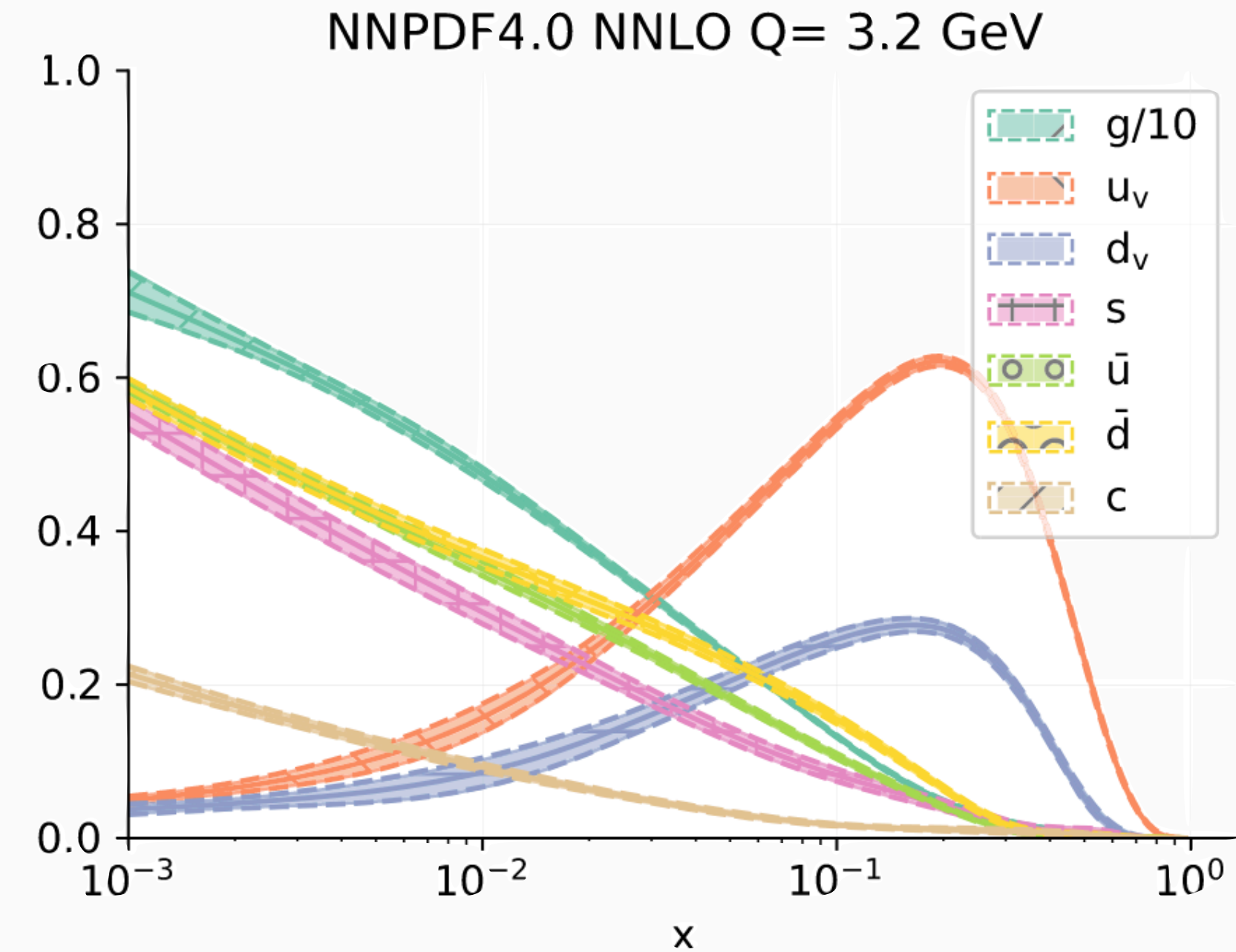
R. Boussarie, et al., "TMD Handbook", [arXiv:2304.03302 [hep-ph]]

Global Analysis of PDFs

- Since PDFs are universal in different high-energy processes, many efforts have been spent in extracting PDFs from experimental data.



T. J. Hou, et al., Phys. Rev. D 103 (2021)



R. D. Ball, et al. [NNPDF], Eur. Phys. J. C 82 (2022)

Parameterization form as prior of fit

$$f_i(x, Q_0) = a_0 x^{a_1-1} (1-x)^{a_2} P_i(y; a_3, a_4, \dots)$$

$$xf_k(x, Q_0; \theta) = A_k x^{1-\alpha_k} (1-x)^{\beta_k} \text{NN}_k(x; \theta)$$

Lattice Calculation of Partonic Physics

- **Collinear distributions: Pion/Kaon DA, Nucleon PDFs**
- **TMD distributions: Pion TMDs, Nucleon TMDs**
- **Main collaborators: Yong Zhao (ANL), Xiang Gao (BNL), Swagato Mukherjee (BNL),
Peter Petreczky (BNL), Rui Zhang (MIT), Dennis Bollweg (Flatiron), Xiangdong Ji
(UMD), Yushan Su (UMD), Wei Wang (SJTU), Yi-Bo Yang (ITP)**

Lattice QCD: First-principle Non-perturbative Method

○ Path integral formalism

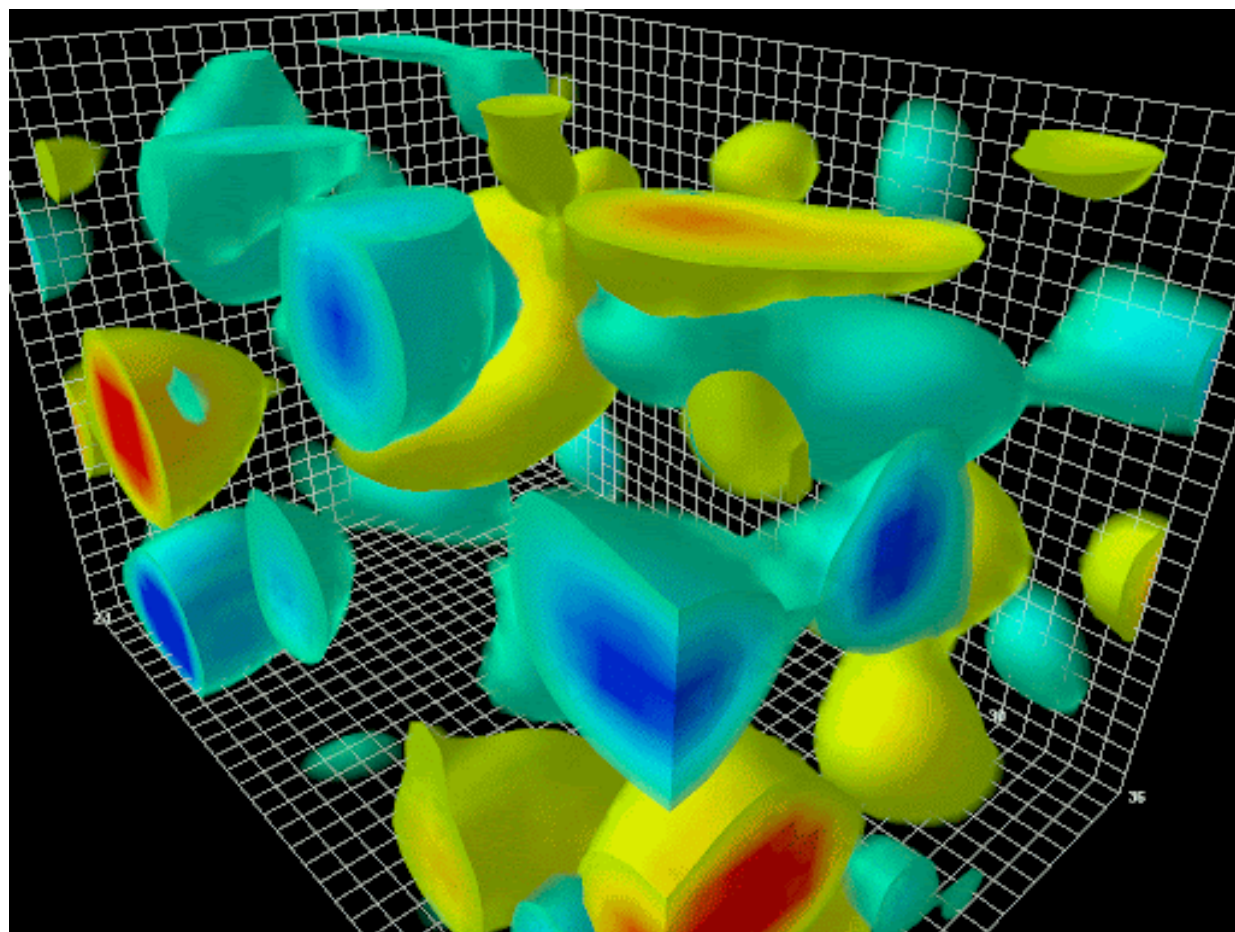
$$Z = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{iS_{\text{QCD}}[A,\psi,\bar{\psi}]} \xrightarrow[t \rightarrow -it_E]{\substack{\text{Wilson link} \\ U_\mu(x) = e^{igaA^\mu(x)}}} Z_E = \int \mathcal{D}U e^{-S_E^g[U]} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-\bar{\psi} M[U] \psi} = \int \mathcal{D}U \boxed{e^{-S_E^g[U]} \det M[U]}$$

Wick rotation

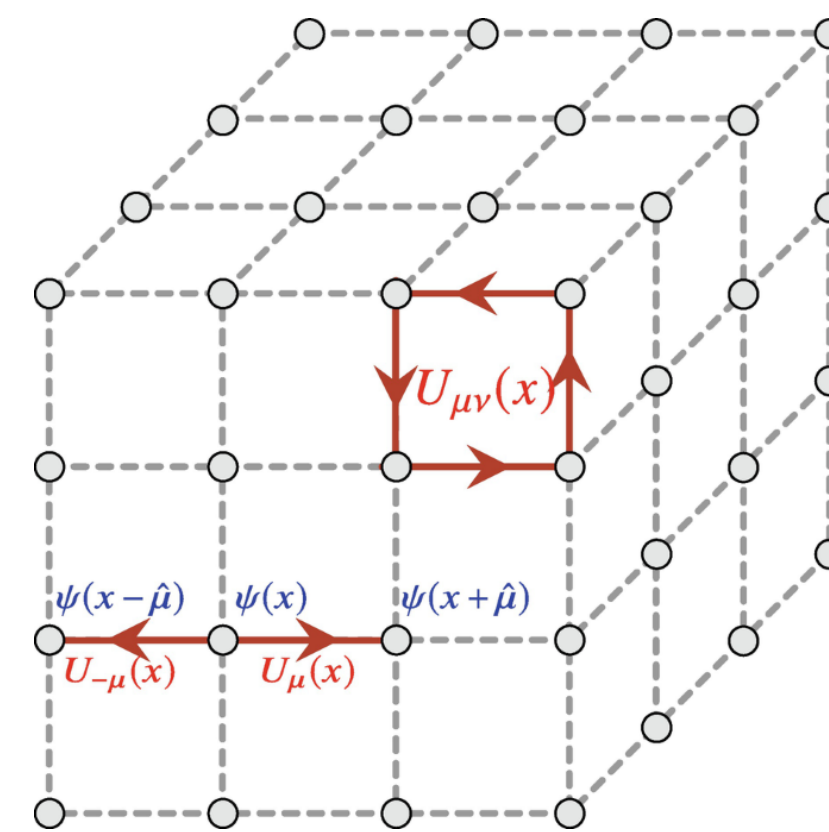
Sampling probability for configuration U

○ Monte Carlo sampling

$$\langle \hat{O} \rangle = \frac{1}{Z_E} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \hat{O} e^{-S_E^{\text{QCD}}[A,\psi,\bar{\psi}]} = \frac{1}{N} \sum_{i=1}^N O[U^{(i)}]$$



Cr. Derek Leinweber



Cr. Olaf Kaczmarek

Lattice QCD Calculation of PDFs

- As a first-principle non-perturbative method, Lattice QCD provides independent predictions of PDFs.
- Mellin Moments
 - Up to $\langle x^3 \rangle$ *C. Alexandrou, et al., Phys. Rev. D 92 (2015); G. S. Bali, et al., Phys. Rev. D 98 (2018); ...*
 - Smeared operators for higher moments *Z. Davoudi, M. J. Savage, Phys.Rev.D 86 (2012); ...*
 - Gradient Flow for higher moments *A. Shindler, Phys. Rev.D 110 (2024); A. Francis, et al., PoS LATTICE2024, 336 (2025); ...*
- **Large Momentum Effective Theory (LaMET) (quasi-PDF)** *X. Ji, Phys.Rev.Lett. 110 (2013); X. Ji, et al., Rev.Mod.Phys. 93 (2021); X. Gao, et al., Phys. Rev. Lett. 128 (2022); ...*
- Short Distance Expansion
 - Pseudo PDF / Ioffe-time distribution *A. V. Radyushkin, Phys. Rev. D 96 (2017); C. Alexandrou, et al., Phys. Rev. D 98 (2018); ...*
 - Current-current correlator *V. M. Braun, et al., Nucl. Phys. B 685 (2004); V. M. Braun, et al., Eur. Phys. J. C 55 (2008); R. S. Sufian, et al., Phys. Rev. D 102 (2020); ...*
- Operator Product Expansion (OPE)
 - Compton amplitude *A. J. Chambers, et al., Phys. Rev. Lett. 118 (2017); M. Gockeler, et al. [QCDSF], Phys. Rev. Lett. 92 (2004); ...*
 - Heavy-quark Operator Product Expansion (HOPE) *W. Detmold, and C. J. David Lin, Phys. Rev. D 73 (2006); W. Detmold, et al. [HOPE], Phys. Rev. D 105 (2022); ...*
- Hadronic Tensor *K. F. Liu, Phys. Rev. D 62 (2000); K. F. Liu, and S. J. Dong, Phys. Rev. Lett. 72 (1994); ...*

Large-Momentum Effective Theory(LaMET)

- PDF is defined from a light-cone correlator in a hadron, which is boost invariant.

Parton model (IMF)

$$f_{\Gamma}(x, \mu) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-i\lambda x} \frac{1}{2P^+} \left\langle P \left| \bar{\psi}(\xi^-) W(\xi^-, 0) \Gamma \psi(0) \right| P \right\rangle \longleftrightarrow \left\langle \left| \vec{P} \right| = \infty \left| O(t=0) \right| \left| \vec{P} \right| = \infty \right\rangle$$

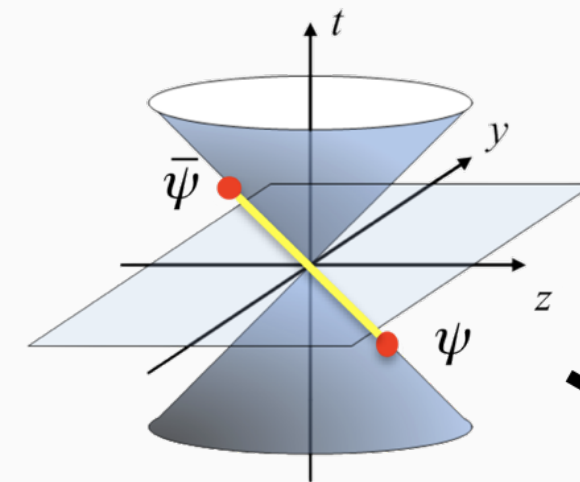
- Define a quasi distribution with large-momentum states and time-independent operators.

Different orders of limit, but pert.

$$\tilde{f}_{\Gamma}^0(y, P^z, \mu) = P^z \int \frac{dz}{2\pi} e^{iz(yP^z)} \frac{1}{2P^t} \langle P | \bar{\psi}_0(z) W(z, 0) \Gamma \psi_0(0) | P \rangle$$

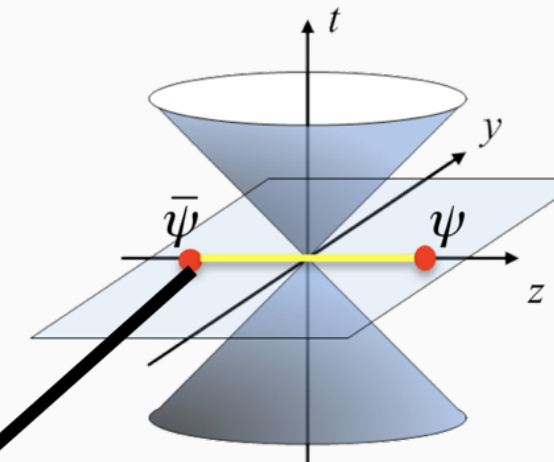
$$\Lambda_{\text{QCD}} \ll |\vec{P}| \ll \frac{\pi}{a}$$

Large-momentum expansion



X. Ji, PRL 110 (2013)
X. Ji, et al., RMP 93 (2021)
X. Ji, NPB 1007 (2024)

Lorentz boost & Matching



Light-cone distribution:

Cannot be directly calculated on the lattice
Critical point with infinite correlation length

Equal-time correlation (for quasi distribution):

Directly calculable on the lattice
Boost is dynamic, while projection is simple on lattice

- LaMET enables us to obtain the precision-controlled x -distribution of PDFs in $x \in [x_{\min}, x_{\max}]$.

Pert. matching kernel

Power corrections

$$f(x, \mu) = C\left(\frac{y}{x}, \frac{P^z}{\mu}\right) \otimes \tilde{f}\left(y, \frac{P^z}{\mu}\right) + O\left(\frac{\Lambda_{\text{QCD}}^2}{(2xP^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{(2(1-x)P^z)^2}\right)$$

Y. Su, et al., Nucl. Phys. B 991 (2023);

X. Ji, et al. JHEP 08 (2023);

R. Zhang, et al., Phys. Lett. B 844 (2023);

Y. Liu, et al., JHEP 2024 (2024);

X. Ji, et al., JHEP 03 (2025);

Distribution Amplitude of Pion / Kaon

- State-of-the-art calculation of pion / kaon DA in 2022:

J. Hua, M. Chu, JH, et al. [LPC], PRL 129 (2022), 2201.09173

- Three lattice spacing: 0.06 fm, 0.09 fm and 0.12 fm;

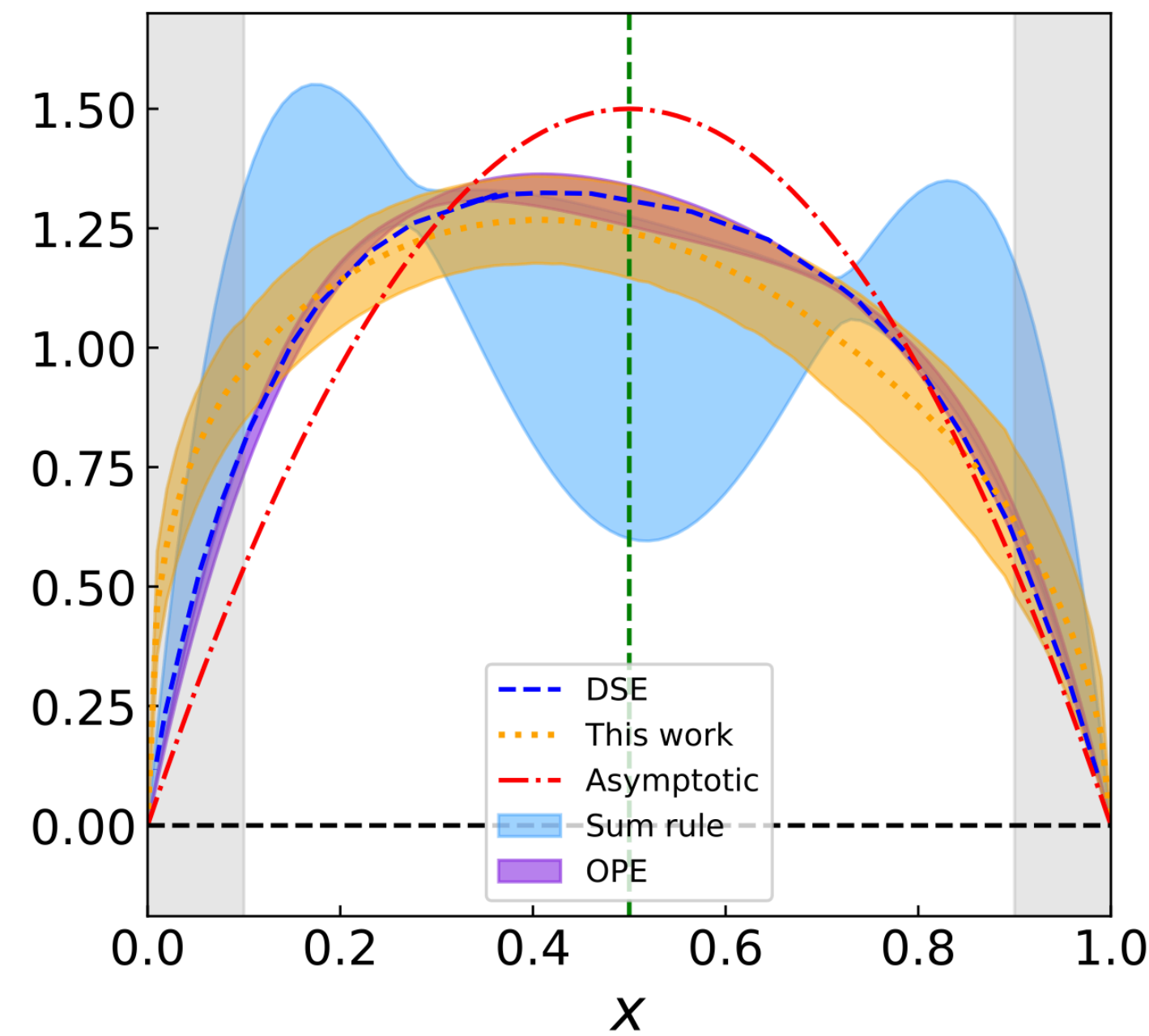
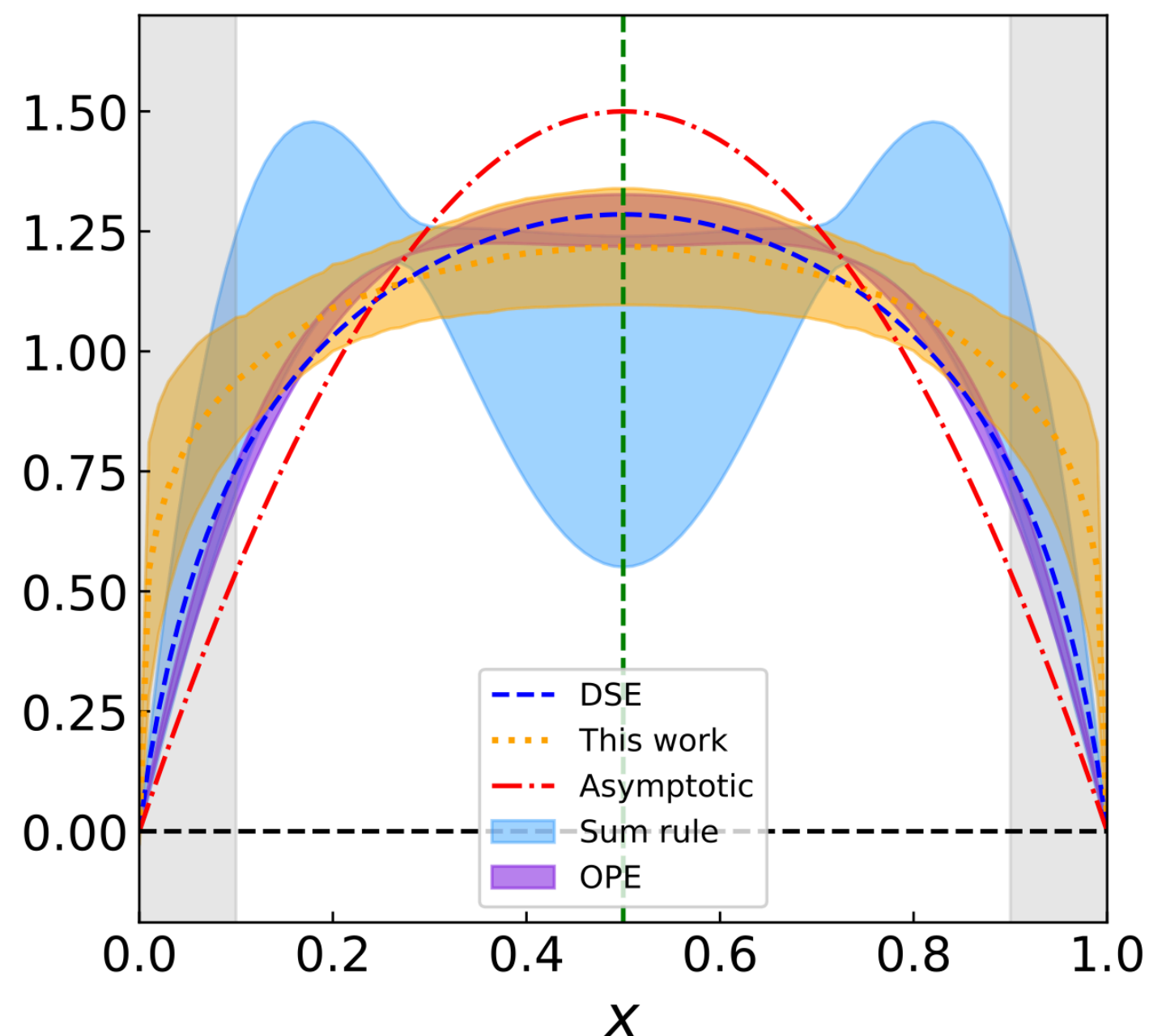
- Physical quark masses;

- Three meson momenta up to 2 GeV;

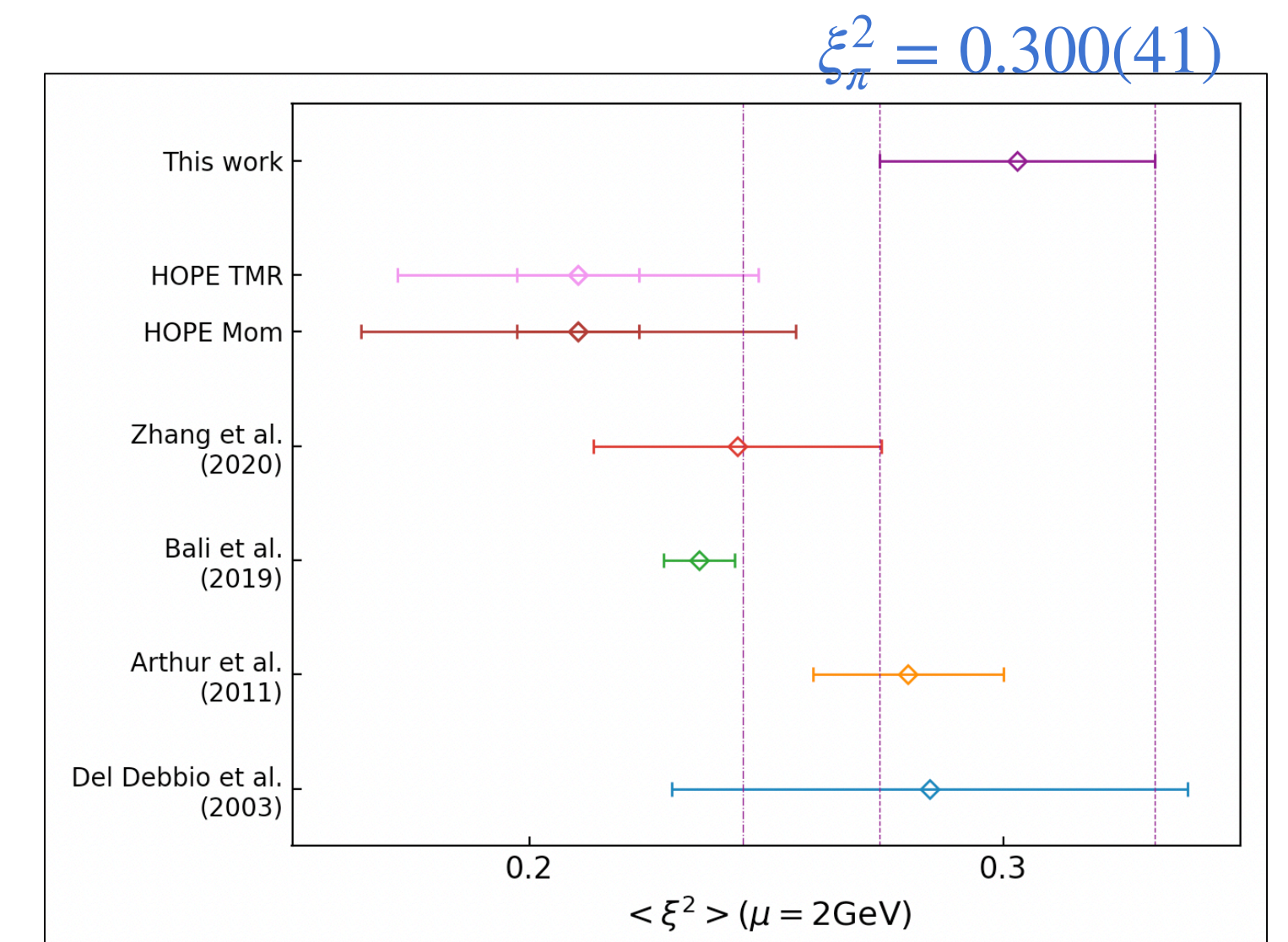
- Hybrid scheme with self-renormalization; *Y. Huo, Y. Su, et al. (LPC), NPB 969 (2021)*

$$\int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle 0 | \bar{\psi}_1(0) \not{n} \gamma_5 U(0, \xi^-) \psi_2(\xi^-) | M(P) \rangle = if_M(p \cdot n) \phi_M(x)$$

DA



We saw a larger second moment of pion DA



$$\xi_M^n = \int_0^1 dx (2x-1)^n \phi_M(x)$$

3D Imaging of Hadron

x is the momentum fraction in the longitudinal (hadron momentum) direction.

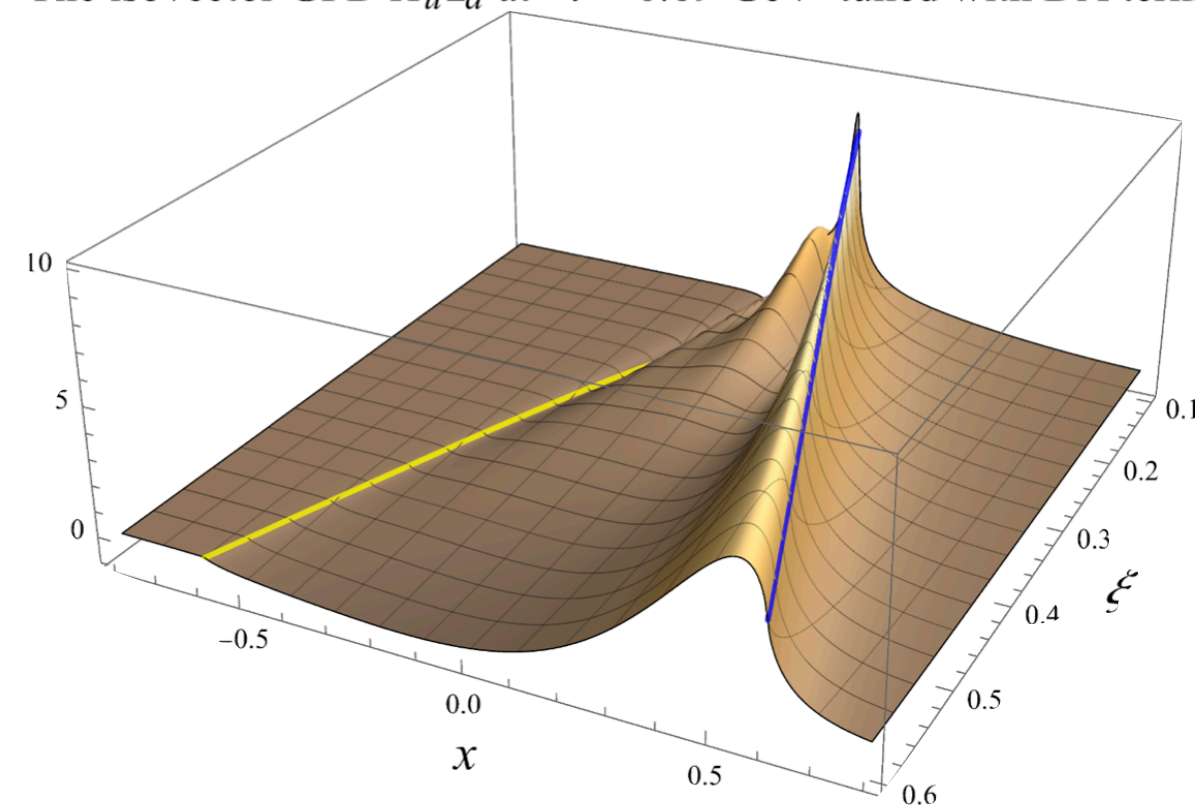
Wigner Distribution / GTMD

$$W(x, \vec{r}_\perp, \vec{k}_\perp)$$

$$\int d^2\vec{k}_\perp$$

Generalized Parton Distributions (GPDs)

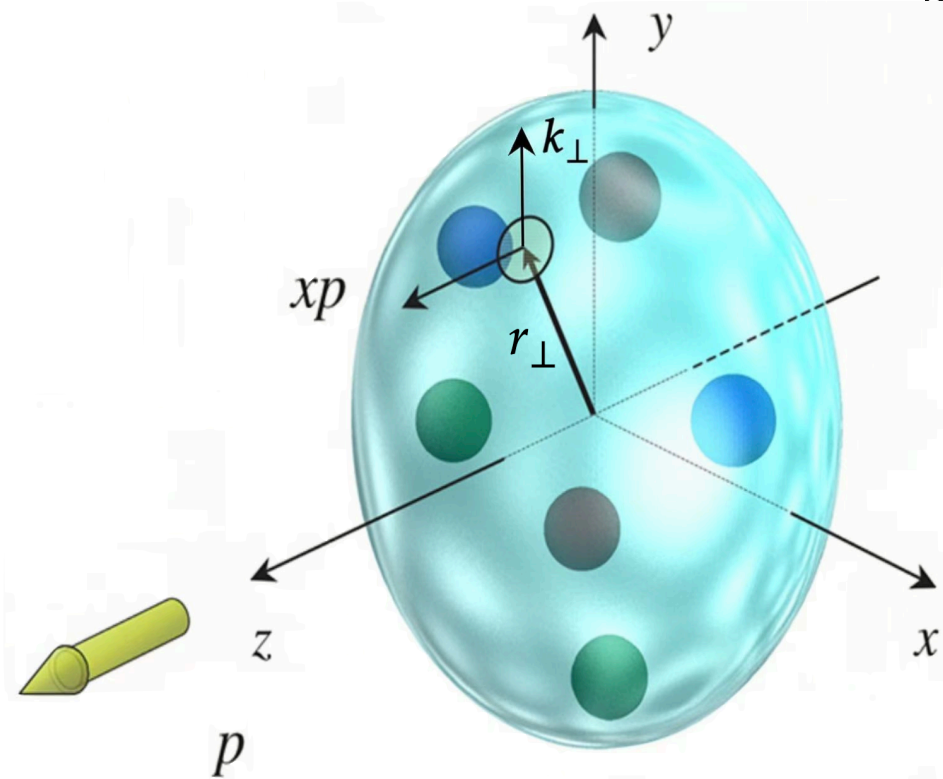
The isovector GPD H_{u-d} at $-t = 0.69 \text{ GeV}^2$ tuned with DA terms



Y. Guo, et al., JHEP 09 (2022)

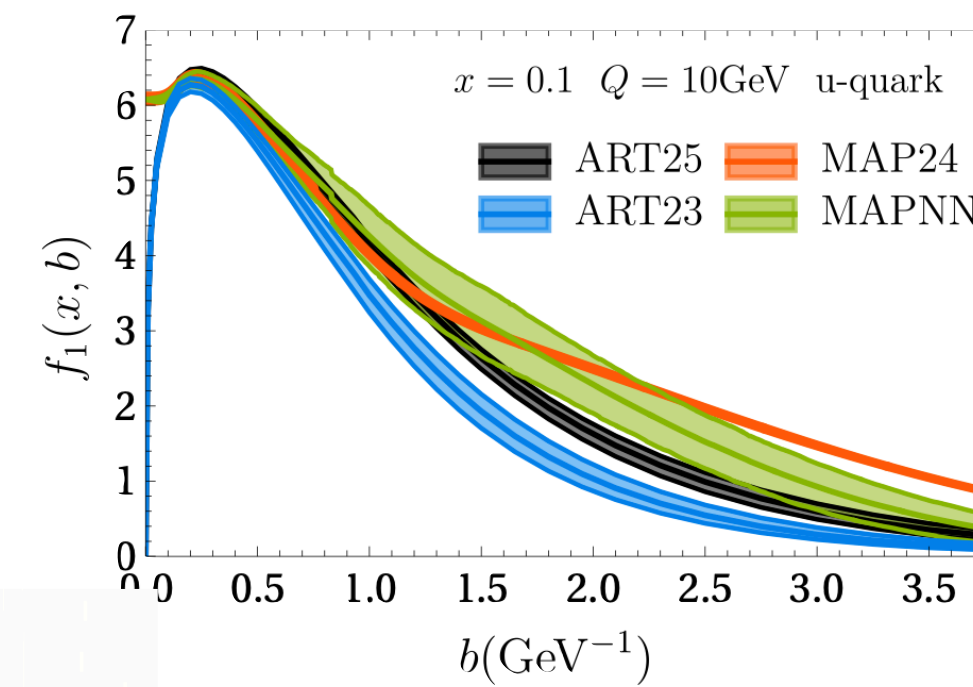
Latest results of GUMP 1.0 in *Y. Guo, et al., PRL 135 (2025)*

$$\int d^2\vec{r}_\perp$$

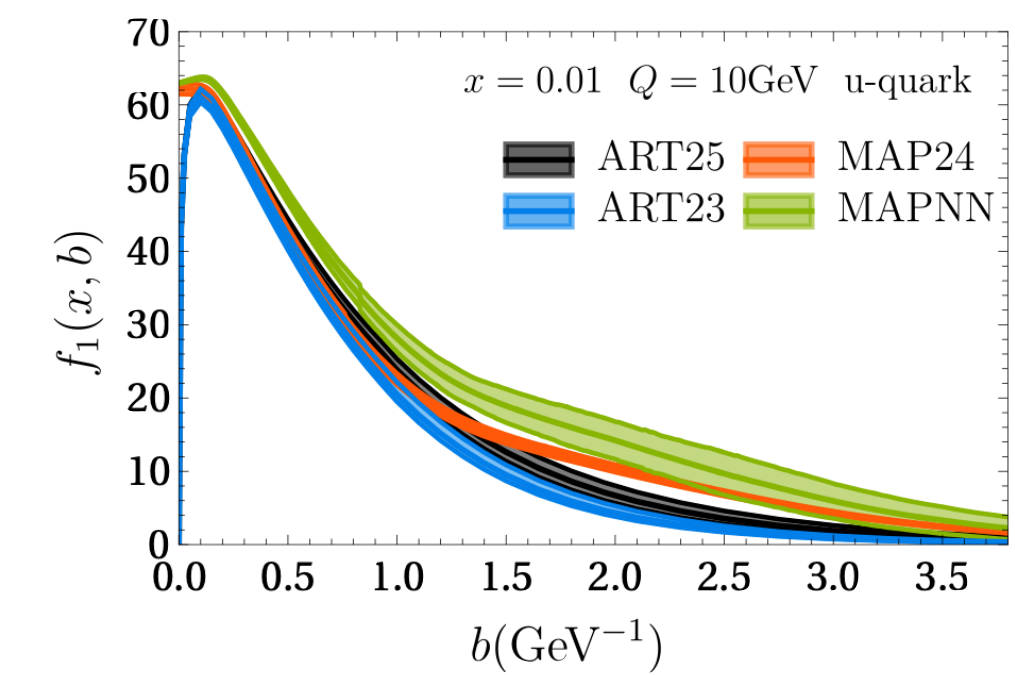


$$\int d^2\vec{r}_\perp$$

Transverse-Momentum-Dependent distributions (TMDs)



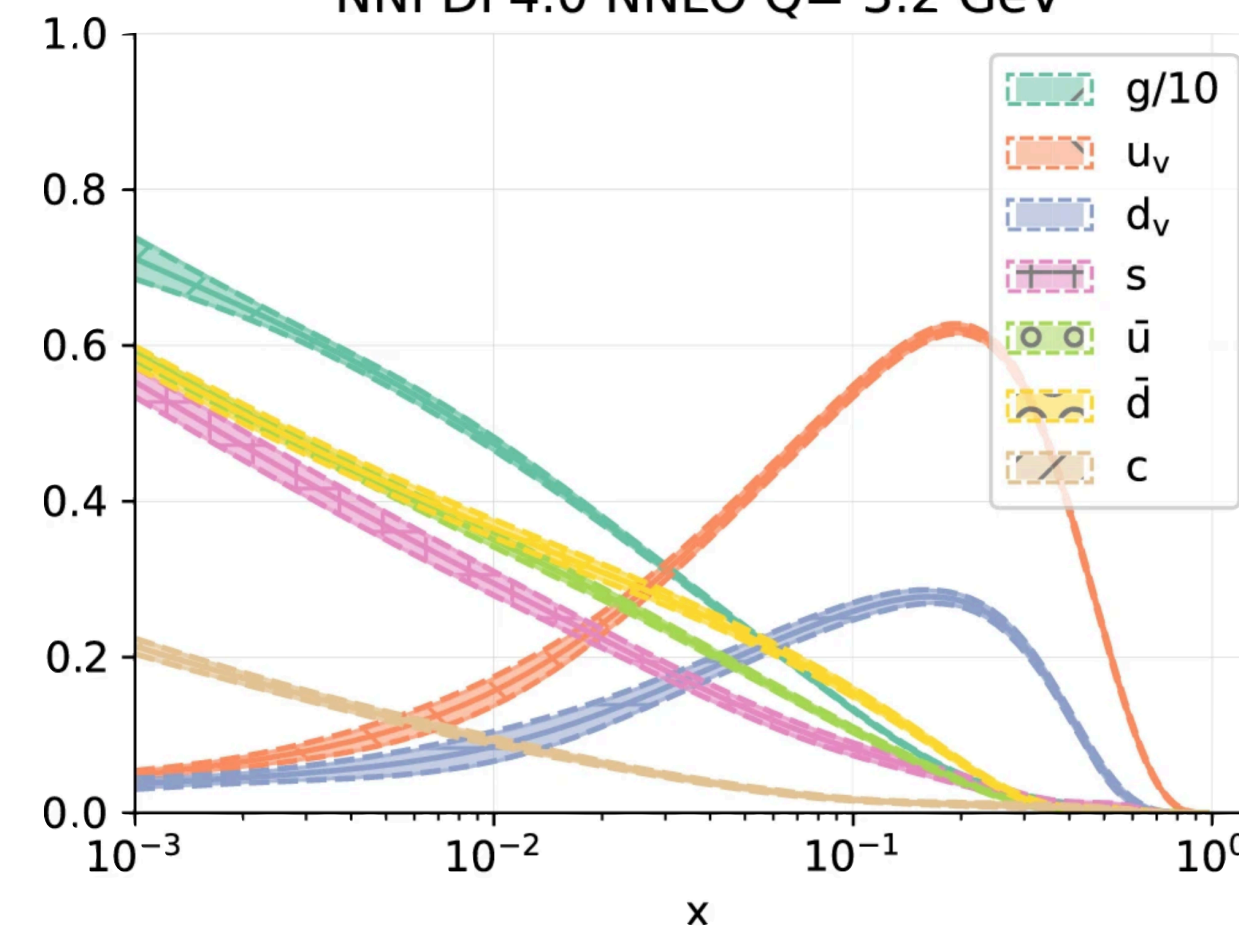
V. Moos, et al., JHEP 11 (2025)



$$\int d^2\vec{k}_\perp$$

Parton Distribution Functions (PDFs)

NNPDF4.0 NNLO $Q = 3.2 \text{ GeV}$



R. D. Ball, et al. [NNPDF], Eur. Phys. J. C 82 (2022)

TMDs in Experiments

- **TMDPDFs**: the distribution densities of finding a parton carrying a longitudinal momentum fraction x and transverse momentum $k_\perp$ in a hadron;
- TMD processes are important processes in high energy collisions, like Drell-Yan process on LHC and Semi-Inclusive DIS on EIC;

TMD Factorization

$$\sigma_{\text{DY}} \propto \left| \text{Diagram 1} \right|^2 \approx \left| \text{Diagram 2} \right|^2 \otimes \left| \text{Diagram 3} \right|^2 \otimes \left| \text{Diagram 4} \right|^2$$

But why we need TMD factorization?

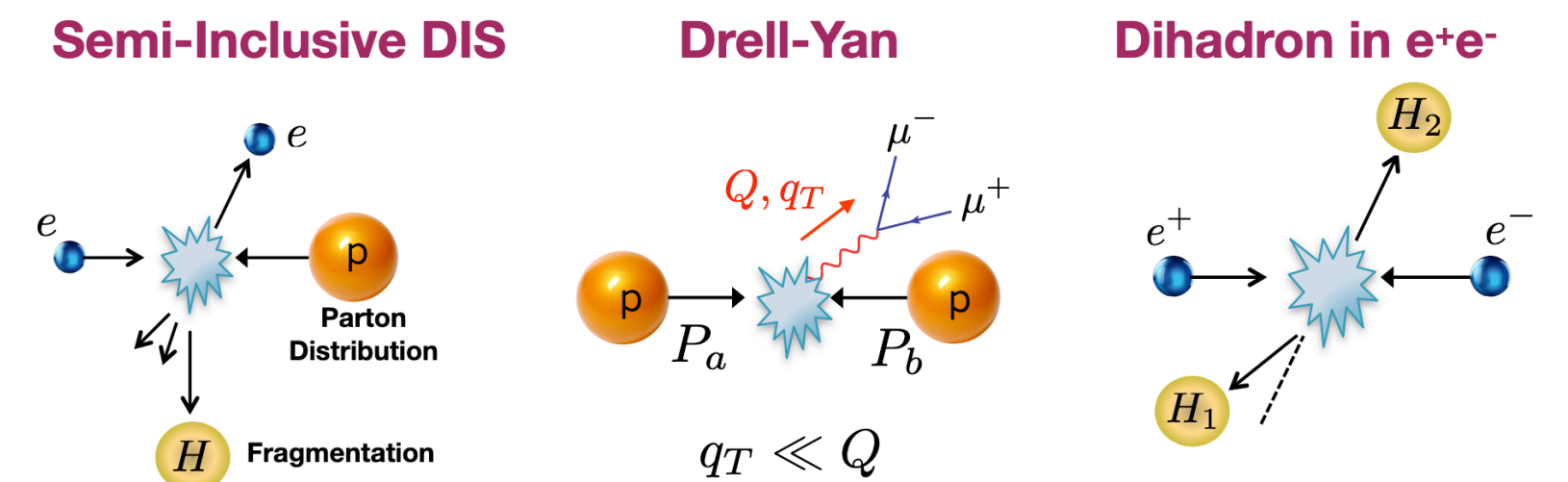
- In the Parton Model

When $q_T \sim Q$, collinear factorization is enough

$$\frac{d\sigma}{d^4q} = \sum_{i,j} \int_{x_a}^1 d\xi_a \int_{x_b}^1 d\xi_b \underbrace{f_{i/H_a}(\xi_a)}_{\text{PDF}} f_{j/H_b}(\xi_b) \frac{d\hat{\sigma}_{ij}(\xi_a, \xi_b)}{d^4q} \left[1 + O\left(\frac{\Lambda_{\text{QCD}}^2}{q_T^2}, \frac{\Lambda_{\text{QCD}}^2}{Q^2}\right) \right]$$

When $\Lambda_{\text{QCD}} \lesssim q_T \ll Q$, TMD factorization is needed

$$\frac{d\sigma}{d^4q} = \frac{1}{s} \sum_{i \in \text{flavors}} \hat{\sigma}_i^{\text{TMD}}(Q) \int d^2\mathbf{k}_T \underbrace{f_{i/H_a}(x_a, \mathbf{k}_T)}_{\text{TMD PDF}} f_{i/H_b}(x_b, \mathbf{q}_T - \mathbf{k}_T) \left[1 + O\left(\frac{q_T^2}{Q^2}, \frac{\Lambda_{\text{QCD}}^2}{Q^2}\right) \right]$$



Leading Quark TMDPDFs ○ → Nucleon Spin ● → Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{○}$ Unpolarized		$h_1^\perp = \text{○} - \text{○}$ Boer-Mulders
	L		$g_1 = \text{○} \rightarrow \text{○}$ Helicity	$h_{1L}^\perp = \text{○} \rightarrow \text{○}$ Worm-gear
	T	$f_{1T}^\perp = \text{○} - \text{○}$ Sivers	$g_{1T}^\perp = \text{○} - \text{○}$ Worm-gear	$h_1 = \text{○} - \text{○}$ Transversity $h_{1T}^\perp = \text{○} - \text{○}$ Pretzelosity

Soft Function

$$\sqrt{S_I(b_\perp; \mu)} \tilde{f}_\Gamma(x, b_\perp, P^z; \mu) = f(x, b_\perp; \mu, \zeta) H_f(x, P^z; \mu) \exp \left[\frac{1}{2} \ln \frac{(2xP^z)^2}{\zeta} \gamma^{\overline{\text{MS}}}(b_\perp; \mu) \right] + \text{Power corrections}$$

After regularization, the rapidity evolution is controlled by Collins-Soper scale: $\zeta = 2(xP^+)^2 e^{-2y_n}$

- The soft gluon radiation will lead to the existence of **soft functions**;
- Due to the factorization of the soft mode, the soft function contains the well-known rapidity divergence;

$$\underbrace{\int_{q_T}^Q \frac{dk}{k}}_{\text{full}} = \lim_{\tau \rightarrow 0} \left[\underbrace{\int_0^Q \frac{dk}{k} R_c(k, \tau)}_{\text{collinear}} + \underbrace{\int_{q_T}^\infty \frac{dk}{k} R_s(k, \tau)}_{\text{soft}} \right] = \ln \frac{Q}{q_T}$$

- There are two parts:

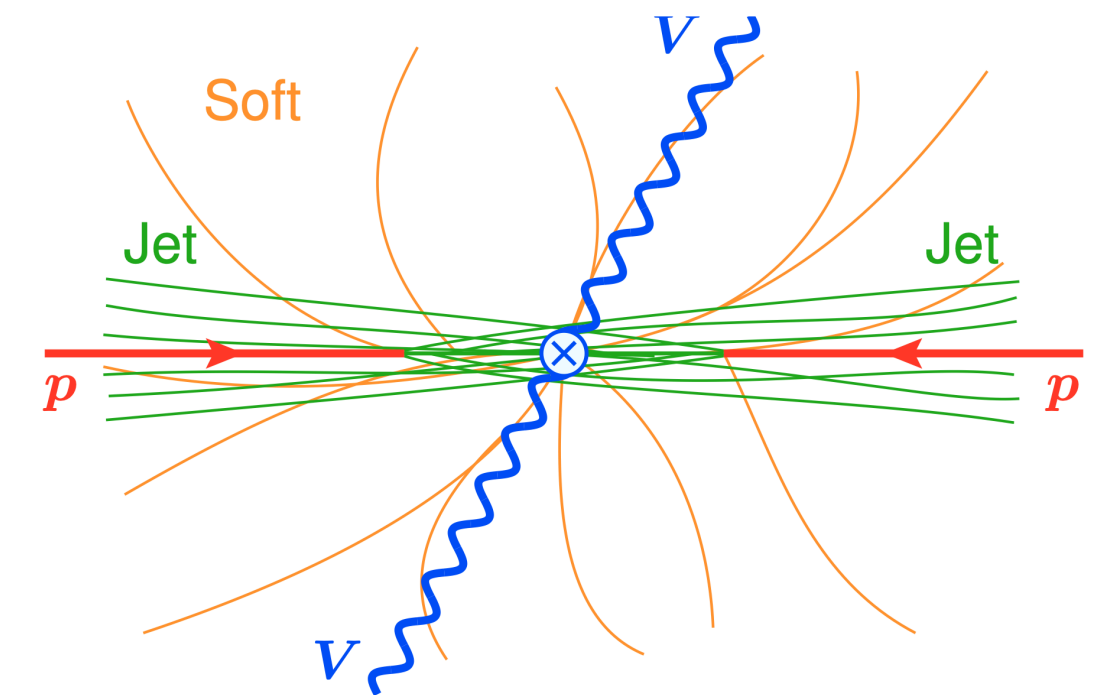
- Rapidity evolution kernel: CS kernel $\gamma^{\overline{\text{MS}}}(b_\perp; \mu)$ $\gamma^{\overline{\text{MS}}}(b_\perp; \mu) = 2 \frac{d}{d \ln \zeta} \ln f(x, b_\perp; \mu, \zeta)$

- Rapidity independent part: intrinsic soft function $S_I(b_\perp; \mu)$

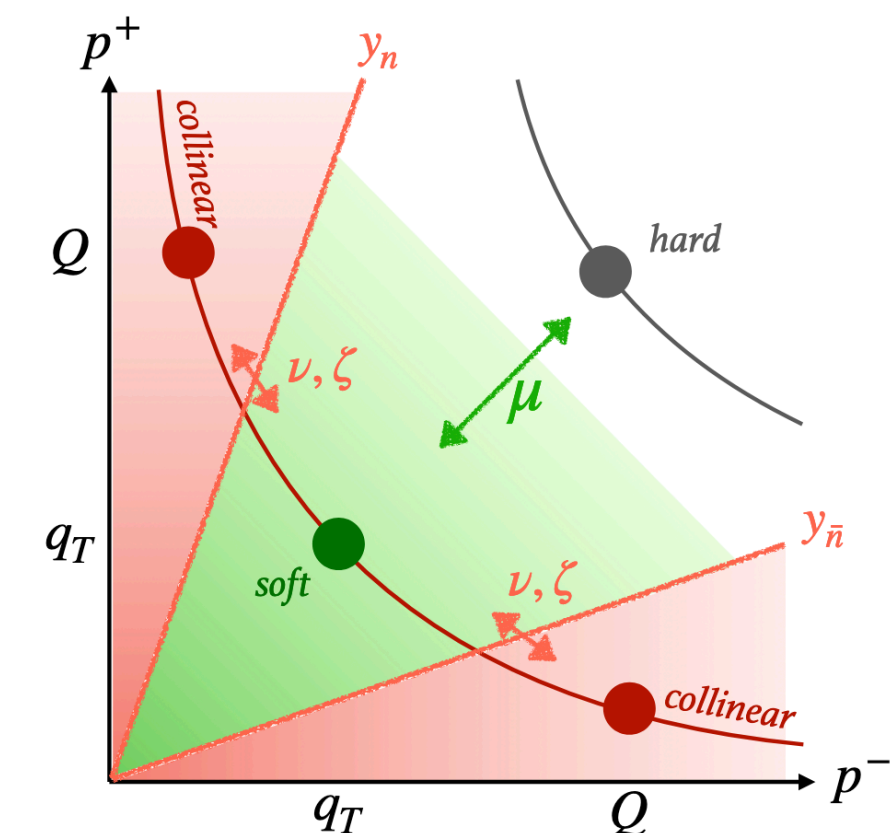
- CS kernel can be extracted from the rapidity evolution of TMDs

$$\gamma^{\overline{\text{MS}}}(b_\perp, P_1, P_2; \mu) = \frac{1}{\ln(P_2/P_1)} \ln \frac{H_f(x, \bar{x}, P_1; \mu) \tilde{f}_{\gamma^t}(x, b_\perp, P_2; \mu)}{H_f(x, \bar{x}, P_2; \mu) \tilde{f}_{\gamma^t}(x, b_\perp, P_1; \mu)}$$

$H_f(x, P^z; \mu) = |C_{\text{TMD}}(xP^z; \mu)|^2$ is the TMD hard kernel for matching.



M. Ebert, PhD Thesis (2017)



R. Boussarie, et al., 2304.03302 (2023)

Soft Function

- The intrinsic soft function cannot be directly calculated on lattice because of two light-like Wilson lines in different directions

X. Ji, Y. Liu, Y. Liu, Nucl.Phys.B 955 (2020)

- Fortunately, it can be extracted from the meson form factor

$$F(b_{\perp}, P_1, P_2, \Gamma, \Gamma') \equiv -4N_c \frac{\langle P_2 | \bar{q}(b_{\perp}) \Gamma q(b_{\perp}) \bar{q}(0) \Gamma' q(0) | P_1 \rangle}{f_{\pi}^2(P_1 \cdot P_2)}$$

- The form factor satisfies the factorization formula

$$F(b_{\perp}, P^z) = \int dx_1 dx_2 H_F(x_1, x_2, P^z; \mu) \phi^{\dagger}(x_1, b_{\perp}, y_n; \mu, \zeta_1, \bar{\zeta}_1) \phi(x_2, b_{\perp}, -y_n; \mu, \zeta_2, \bar{\zeta}_2)$$

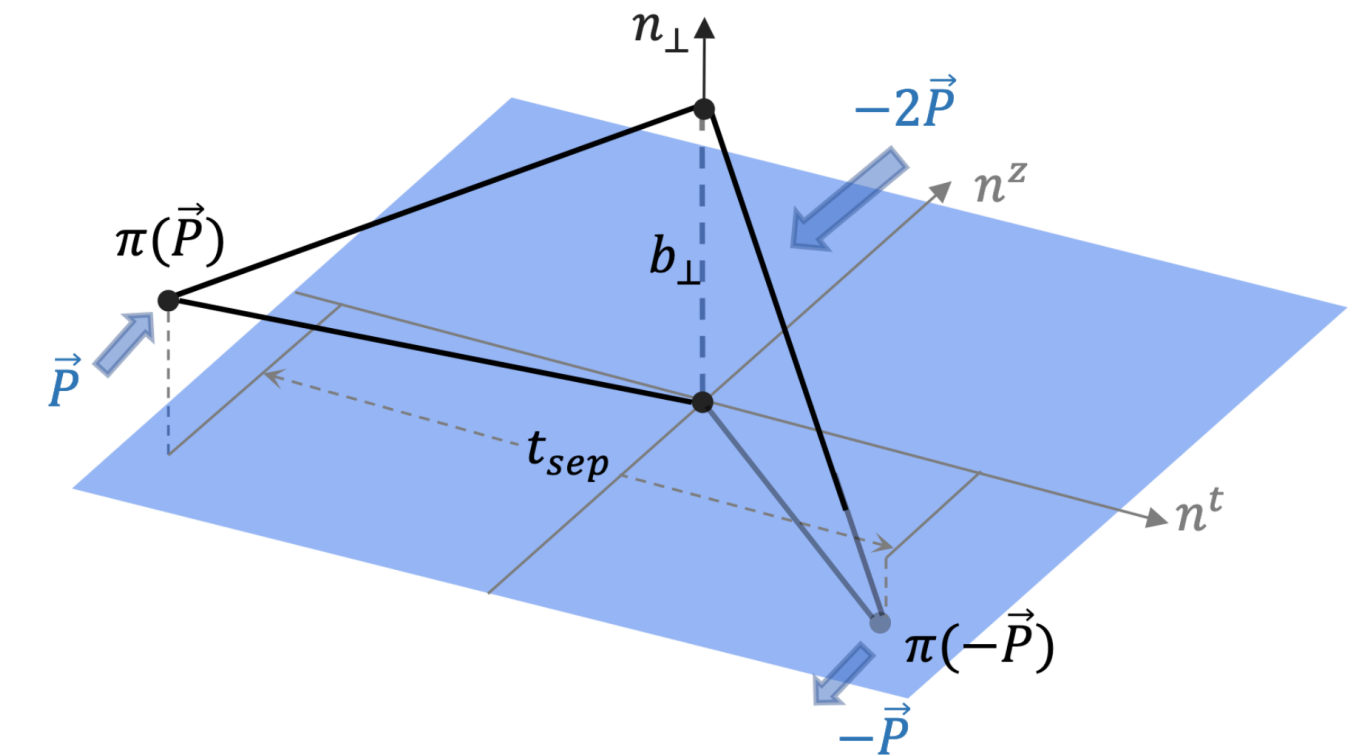
$$\phi(x, b_{\perp}, \dots) = \int_{-\infty}^{\infty} \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \langle 0 | \bar{\psi}(b^{\mu}) W_{\square}(b^{\mu}, 0) \gamma^+ \gamma^5 \psi(0) | P \rangle \text{ is TMD wave function.}$$

$$H_F(x_1, x_2, P^z; \mu) = C_{\text{Sud}}(x_1, x_2, P^z; \mu) \cdot C_{\text{Sud}}(\bar{x}_1, \bar{x}_2, P^z; \mu), \text{ where } C_{\text{Sud}} \text{ is the Sudakov kernel.}$$

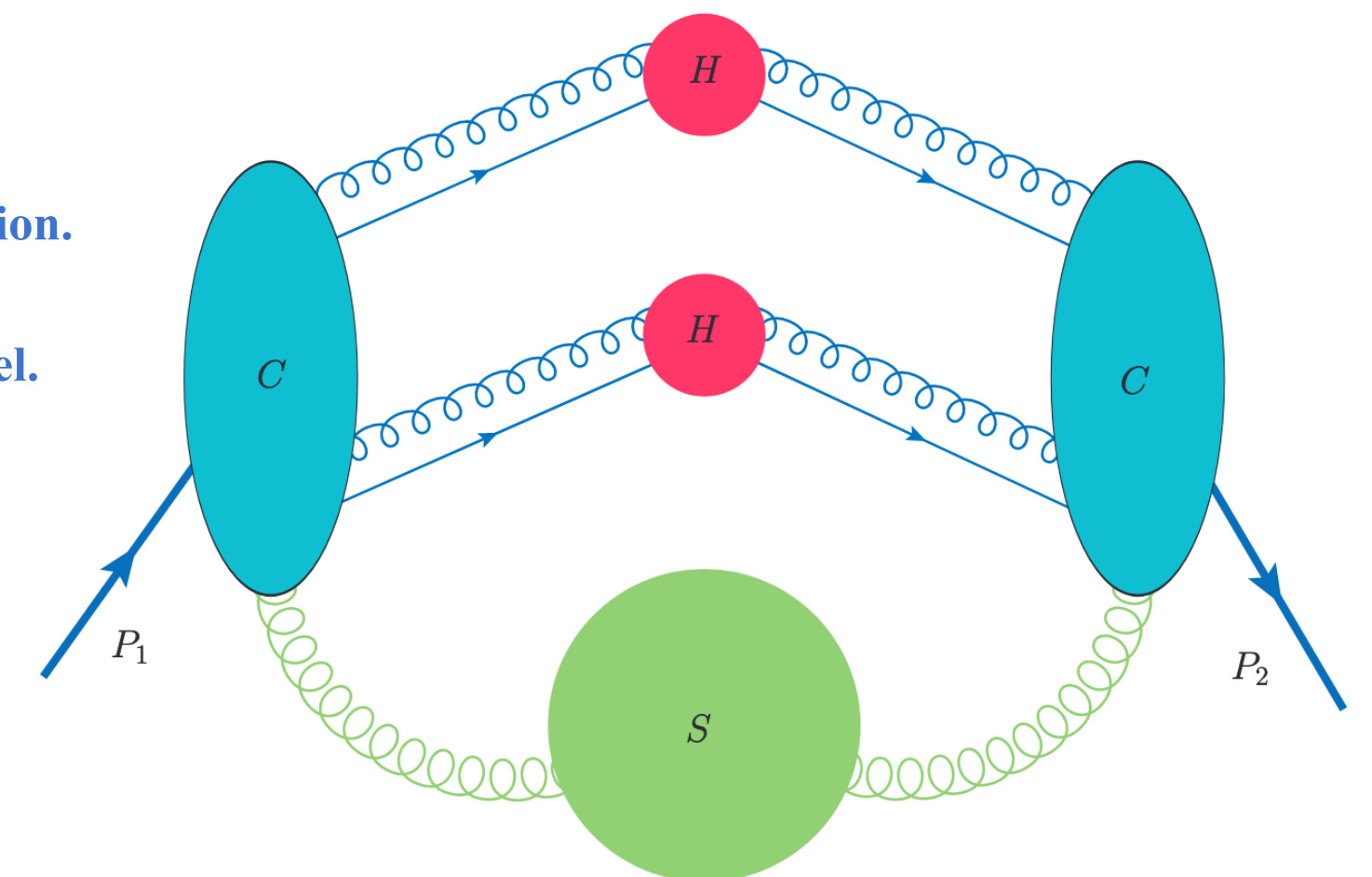
- Therefore, the intrinsic soft function can be extracted via

$$S_I(b_{\perp}; \mu) = \frac{F(b_{\perp}, P^z)}{\int dx_1 dx_2 H_F(x_1, x_2, P^z; \mu) \tilde{\Phi}^{\dagger}(x_1) \tilde{\Phi}(x_2)} \text{ with } \tilde{\Phi}(x) \equiv \frac{\tilde{\phi}_{\Gamma}(x, b_{\perp}, P^z; \mu)}{H_{\phi}(x, \bar{x}, P^z; \mu)}$$

$\tilde{\phi}_{\Gamma}(x, b_{\perp}, P^z; \mu)$ is quasi-TMD wave function.

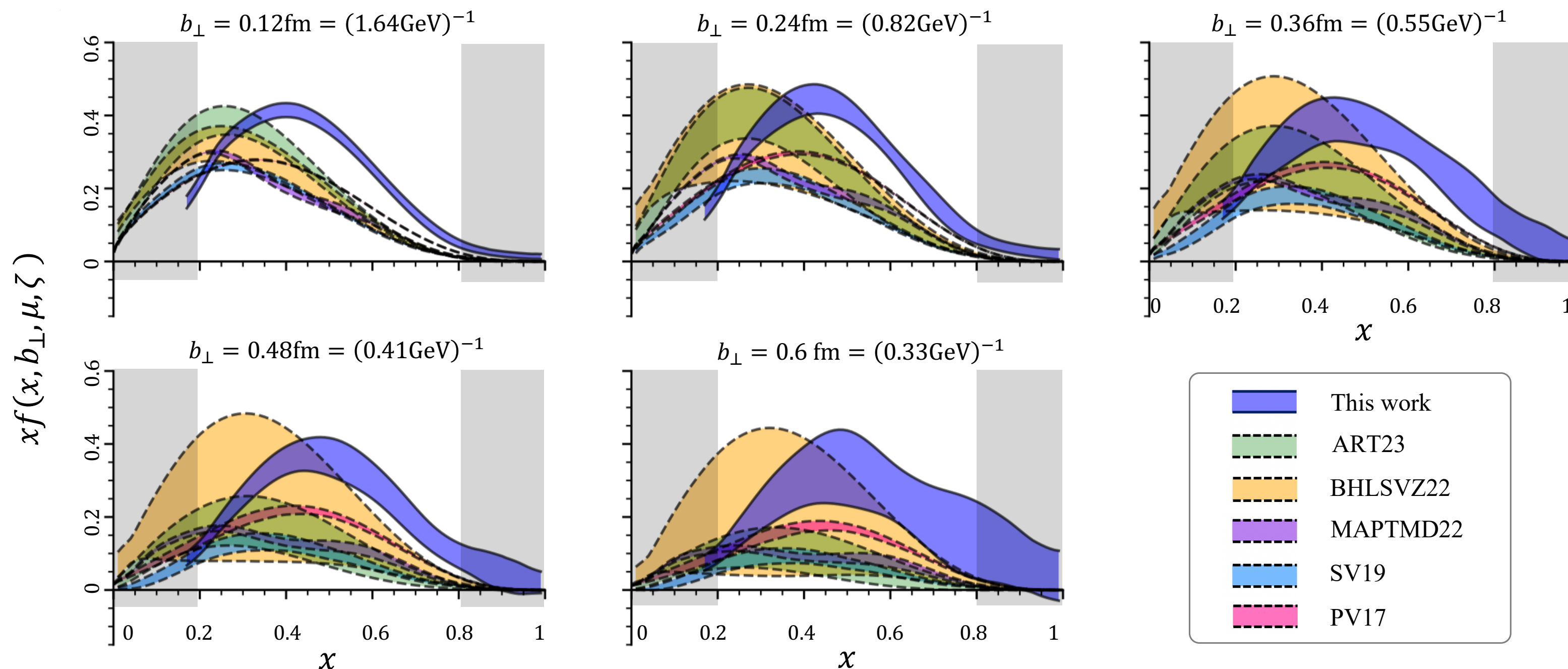


Q. A. Zhang, et al., Phys. Rev. Lett. 125 (2020)



Z. F. Deng, et al., JHEP 09 (2022)

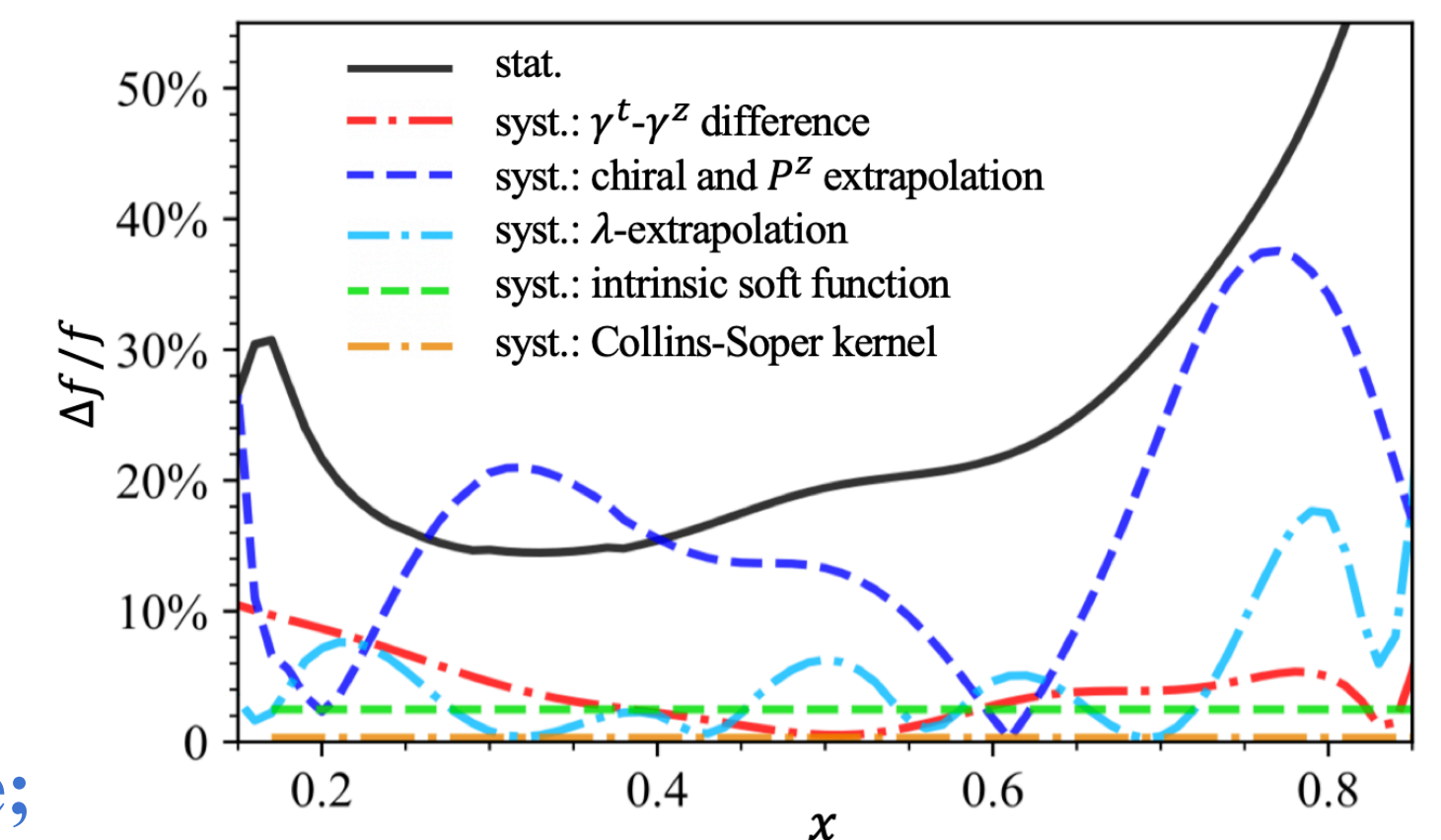
Unpolarized Nucleon TMDPDFs



JH, et al. (LPC), PRD 109 (2024)

- ◆ $a = 0.12$ fm
- ◆ $P_{\text{max}}^z = 2.58$ GeV
- ◆ Physical limit of m_{π}^{val}
- ◆ N3LL matching
(Next-to-next-to-next-to-leading logarithmic)
- ◆ NLO soft function
(Next-to-leading order)

- First lattice QCD calculation of x-dependent nucleon TMDPDFs;
- Provide first-principles predictions in comparison with global analysis;
- Systematics are estimated and included in the final results;
- Constrained by the limited computing resources, it is hard to implement on a finer lattice;
- Due to the signal-to-noise ratio (SNR), it is hard to probe the large $b_{\perp}$ region.



Coulomb Gauge Method

- Define a quasi distribution in CG without Wilson line: *X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)*
Y. Zhao, PRL 133 (2024)

$$\tilde{f}_{\text{CG}}^0(x, b_{\perp}, P^z, \mu) = P^z \int \frac{dz}{2\pi} e^{iz(xP^z)} \frac{1}{2P^t} \langle P | \bar{\psi}_0(z, b_{\perp}) \Gamma \psi_0(0) | P \rangle \Big|_{\vec{\nabla} \cdot \vec{A} = 0}$$

- Why choose CG?

X. Ji, Y. S. Liu, Y. Liu, J. H. Zhang and Y. Zhao, RMP 93 (2021)

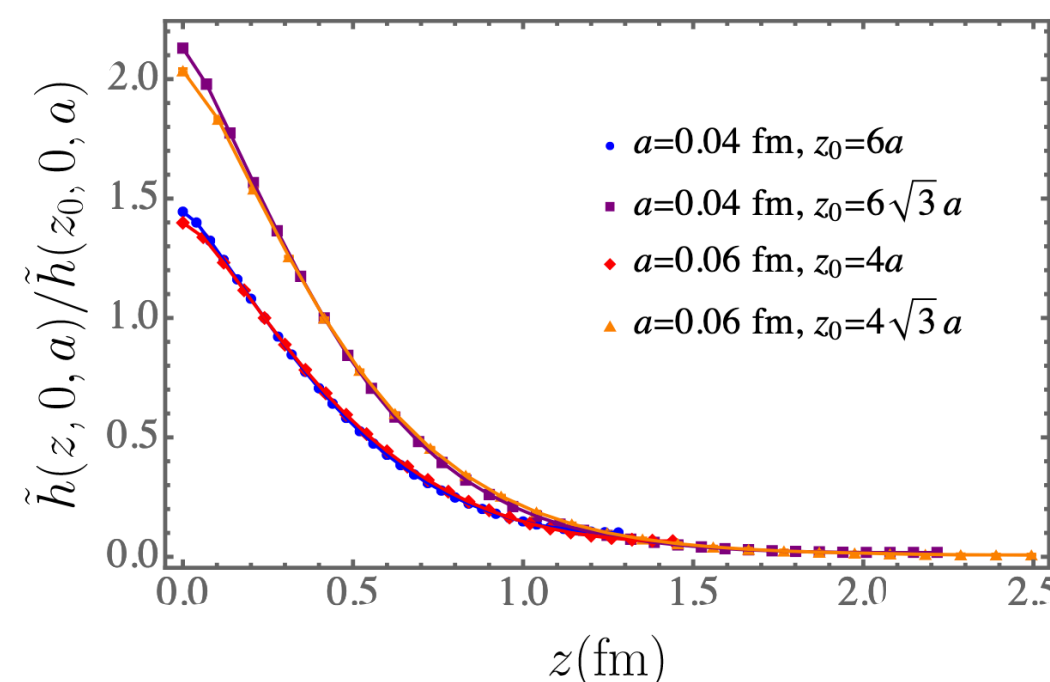
- $\vec{\nabla} \cdot \vec{A} = 0$ becomes $A^+ = 0$ in the infinite boost, so the quasi distribution in CG belongs to the universality class in LaMET;

$$\psi_C(x) = e^{-ie \frac{1}{\nabla^2} \vec{\nabla} \cdot \vec{A}} \psi_0(x) \xrightarrow{P^z \rightarrow \infty} e^{-ie \frac{1}{(\nabla^+)^2} \nabla^+ A^+} \psi_0(x) = e^{-ie \frac{1}{\nabla^+ \pm 0} A^+} \psi_0(x) = e^{-ie \int_{\pm\infty}^x dy^- A^+(y^-, x_{\perp})} \psi_0(x)$$

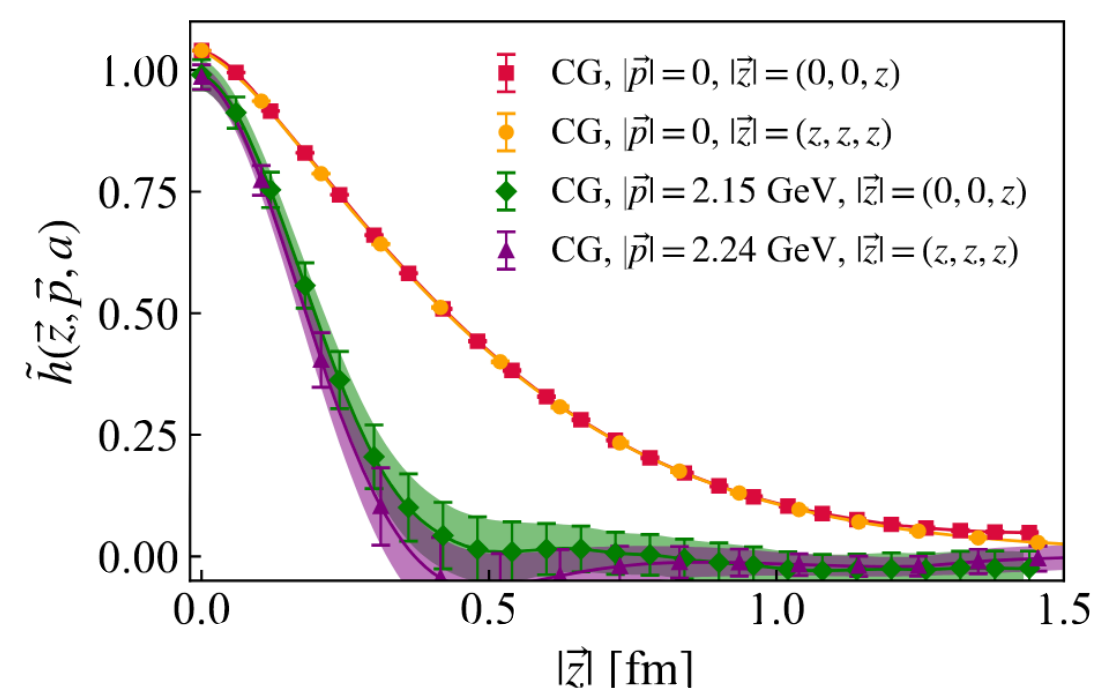
Wilson line on light-cone

- No self-energy of the Wilson link, improving the SNR significantly;
- Simplified renormalization $\bar{\psi}_0(z, b_{\perp}) \Gamma \psi_0(0) = Z_{\psi}(a) [\bar{\psi}(z, b_{\perp}) \Gamma \psi(0)]$;
- Larger off-axis momenta (3D rotational symmetry).

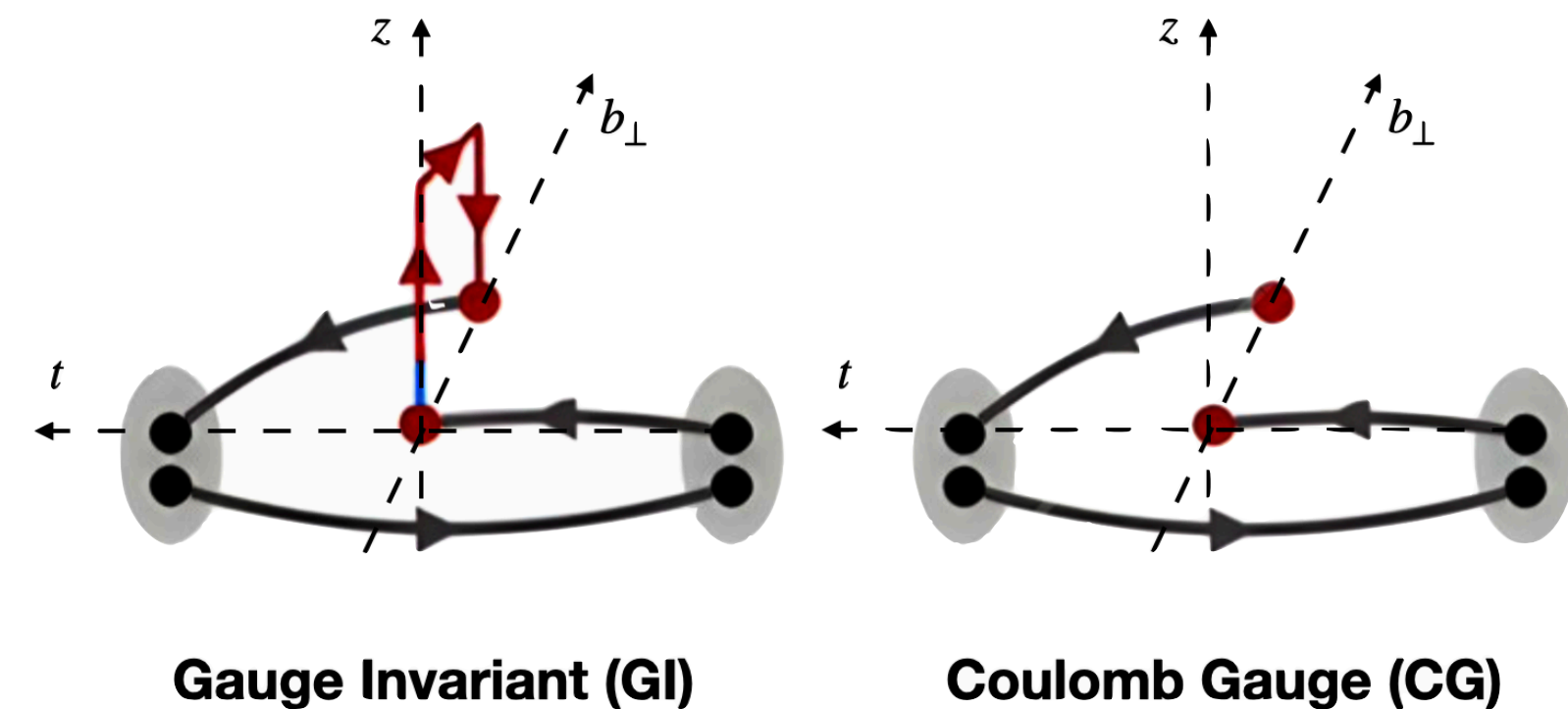
No linear divergence



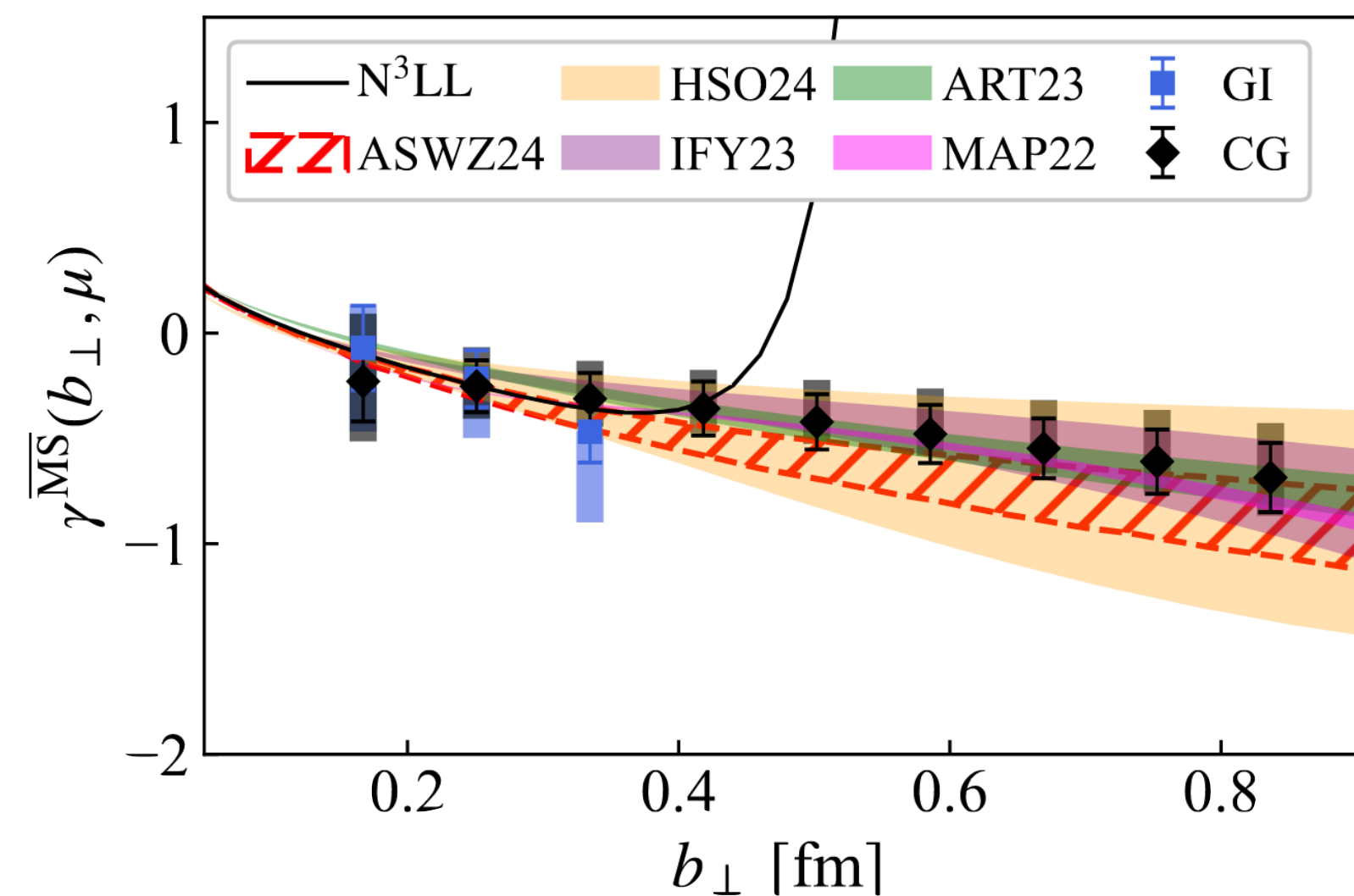
3D rotational symmetry



X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)



Collins-Soper kernel in CG v.s. GI



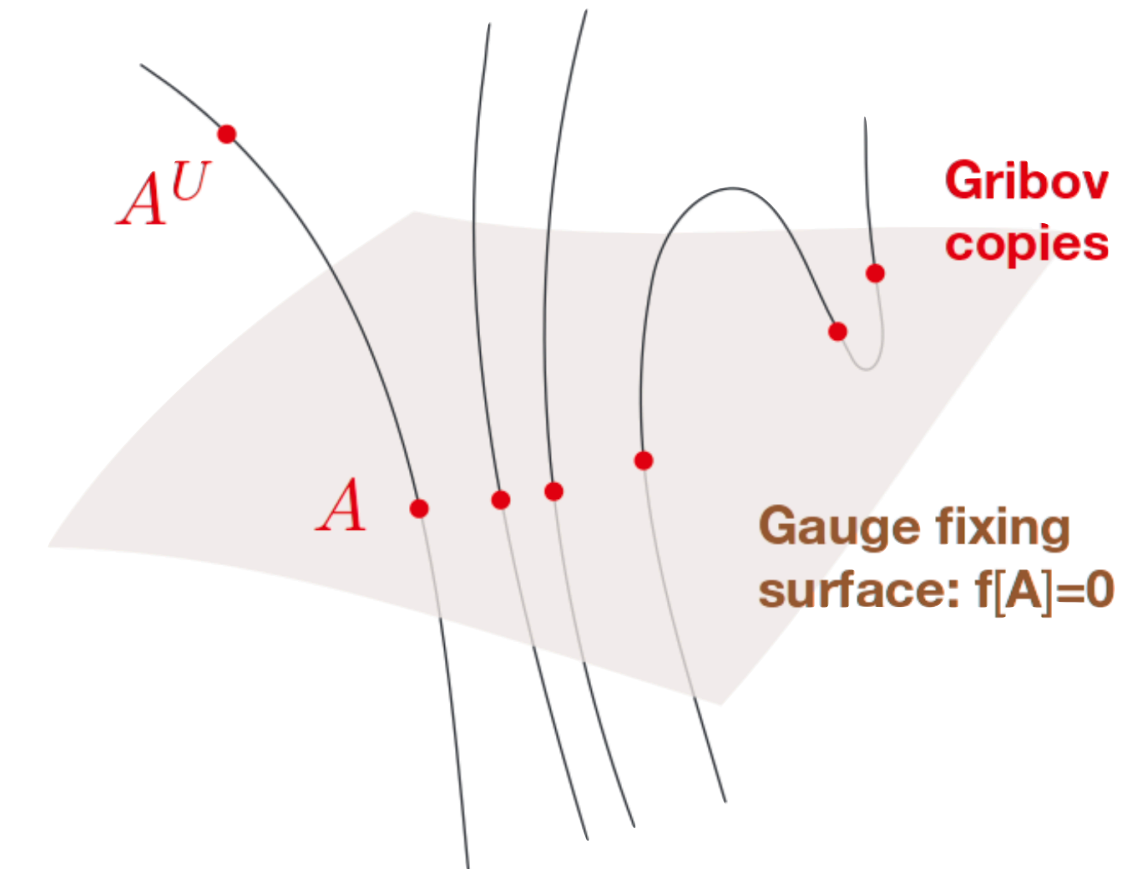
D. Bollweg, et al., Phys.Lett.B 852 (2024)

Gribov Copies

- Gauge fixing is equivalent to finding the stationary point of a functional, such as $F_{\text{CG}}[A, \Omega] \equiv \frac{1}{2} \sum_{\mu=1}^3 \int d^4x A_{\Omega\mu}^a(x) A_{\Omega}^{\mu a}(x)$

$$\delta F_{\text{CG}}[A, \Omega] = - \sum_{\mu=1}^3 \int d^4x (D_{\mu ab}^{\Omega} \theta_b) A_{\Omega}^{\mu a} = - \sum_{\mu=1}^3 \int d^4x (\partial_{\mu} \theta_a - g f^{cab} A_{\Omega\mu}^c \theta_b) A_{\Omega}^{\mu a} = \sum_{\mu=1}^3 \int d^4x \theta_a (\partial_{\mu} A_{\Omega}^{\mu a})$$

- The gauge fixing condition may have many solutions in non-perturbative case, such as Lattice QCD;
- There are two kinds of effects from Gribov copies



Cr. Diego Fiorentini

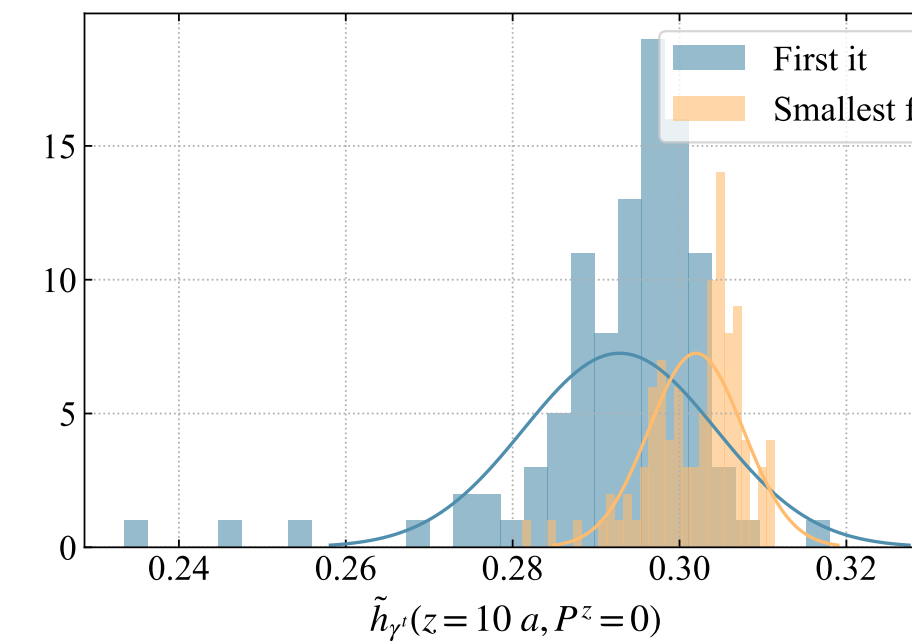
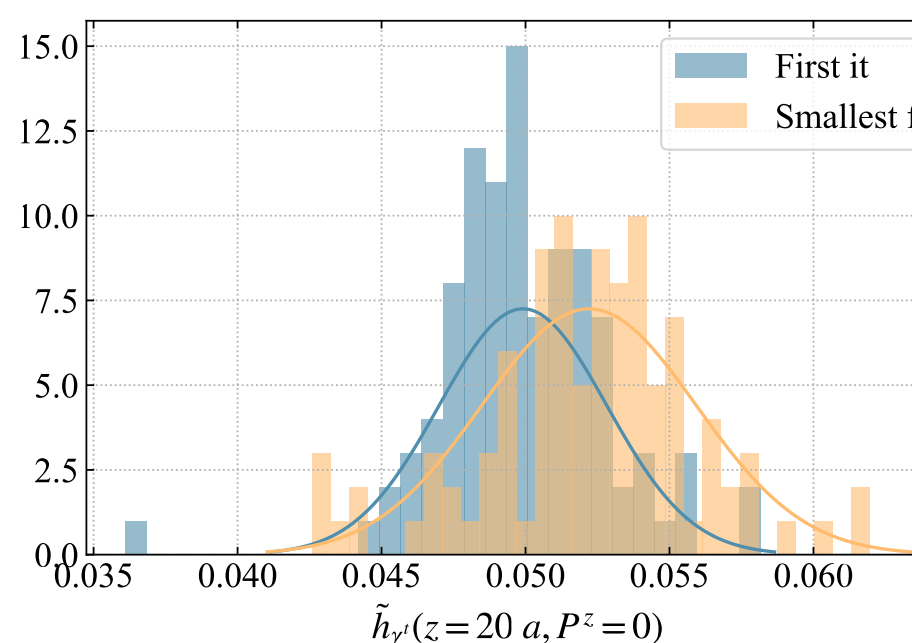
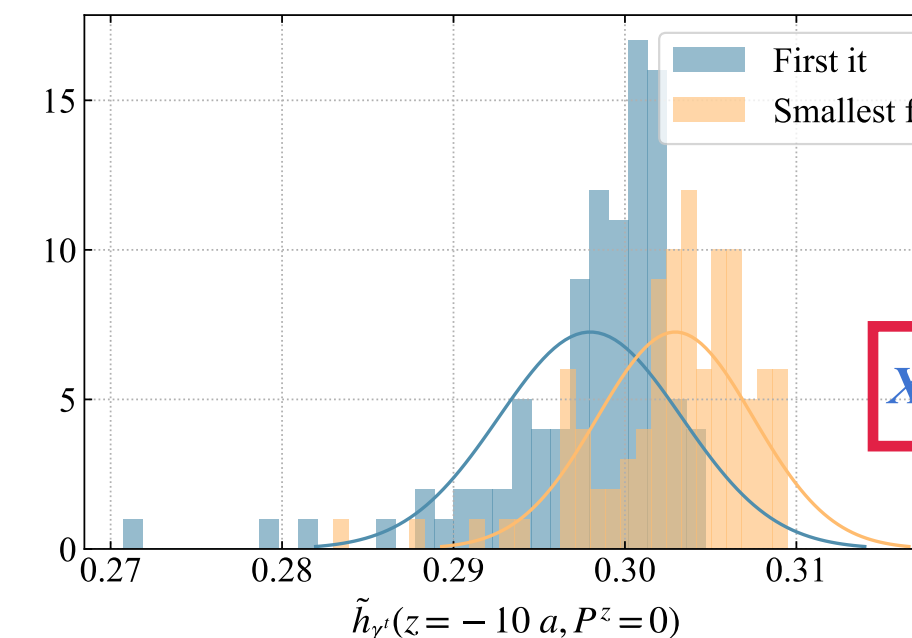
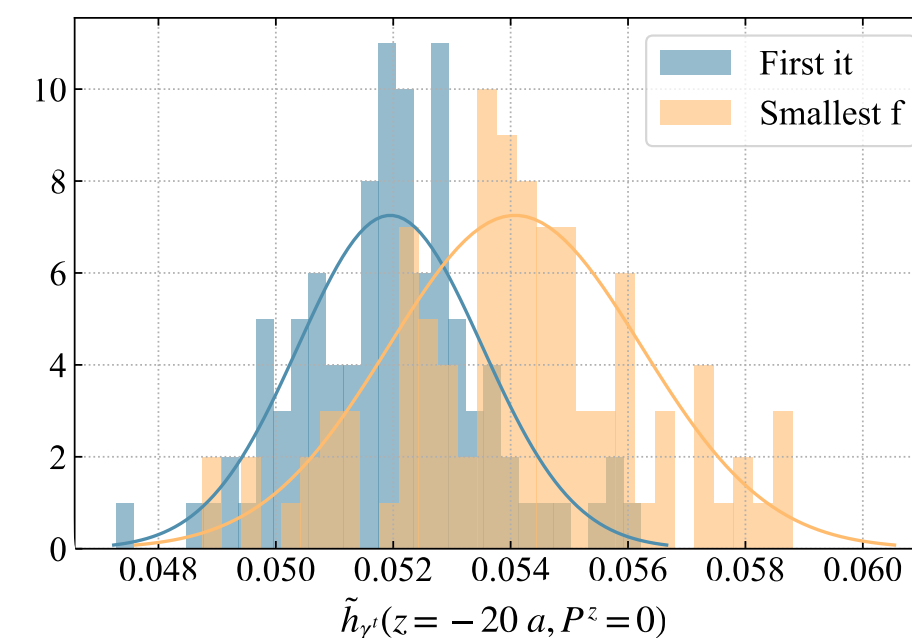
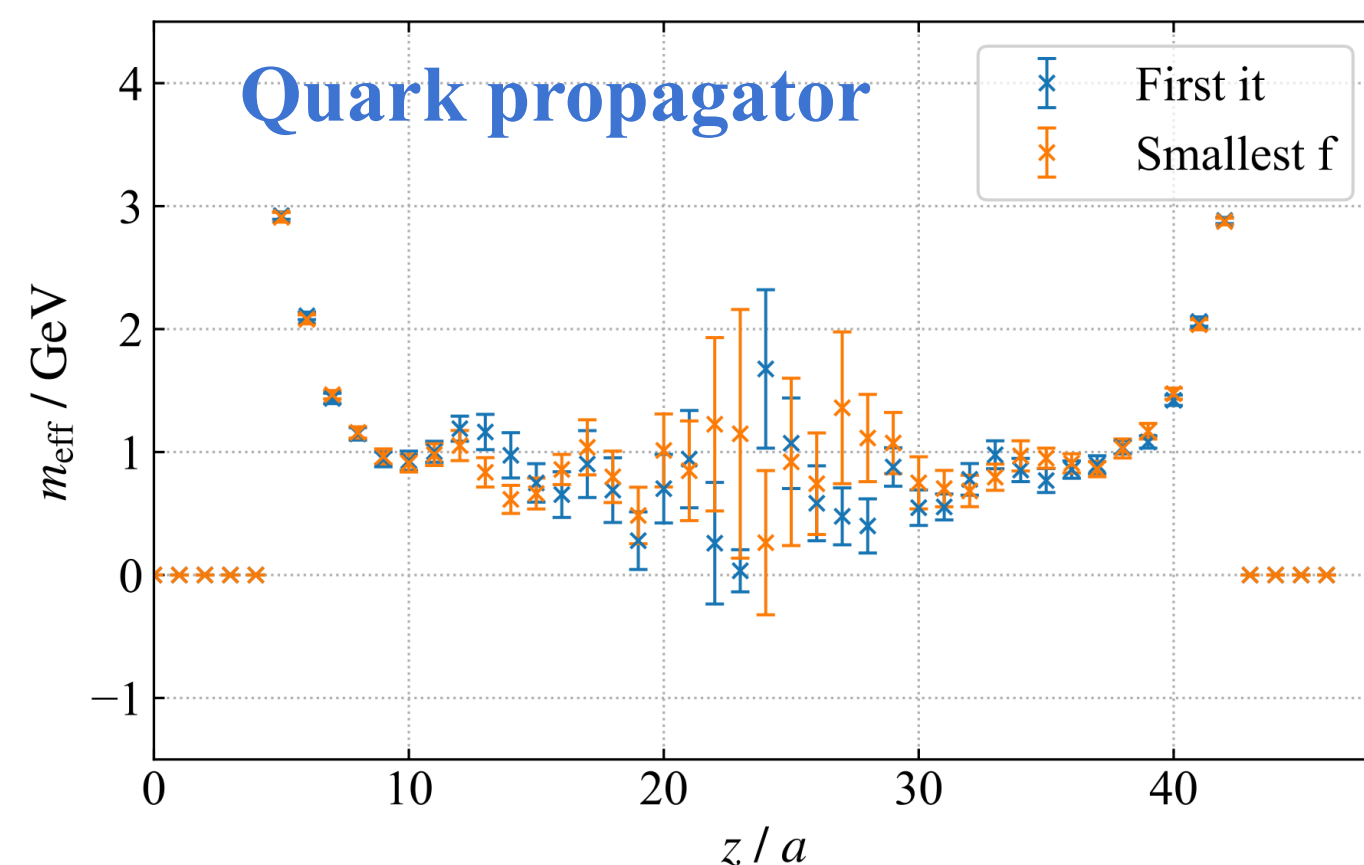
- Gribov noise: not separable from the statistical uncertainty

Quasi-PDF

- Measurement distortion

No significant distortion in quark correlators

(With $\delta F/F < 10^{-8}$)



X. Gao, JH, R. Zhang and Y. Zhao, CPL 41 (2024)

Pion TMDs from Coulomb Gauge Method

- 2+1 flavor HISQ ensemble by HotQCD with volume $L_s \times L_t = 48^3 \times 64$;

Dispersion relation: $E^2 = m_0^2 + c_1 P^2 + c_2 a^2 P^4$

- Lattice spacing is $a = 0.06$ fm;

D. Bollweg, X. Gao, JH, S. Mukherjee and Y. Zhao, PRD 112 (2025)

- Pion mass of sea quark: $m_\pi^{\text{sea}} = 160$ MeV;

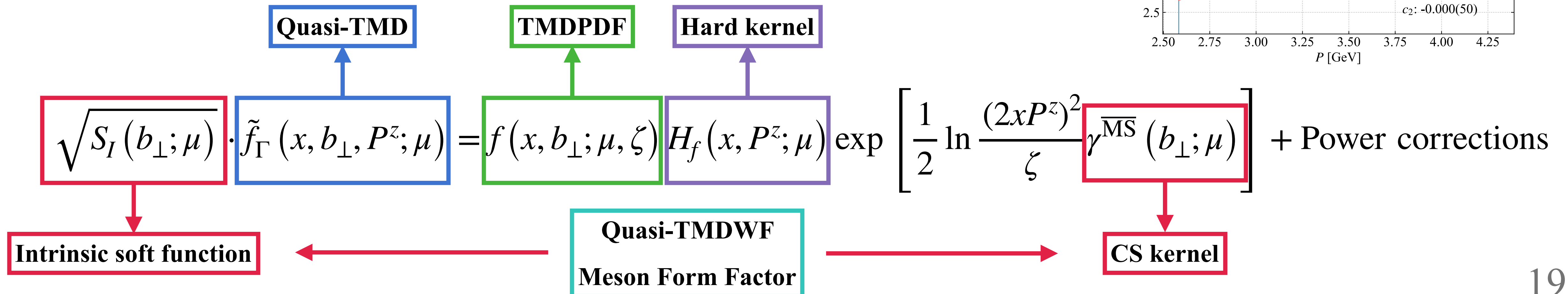
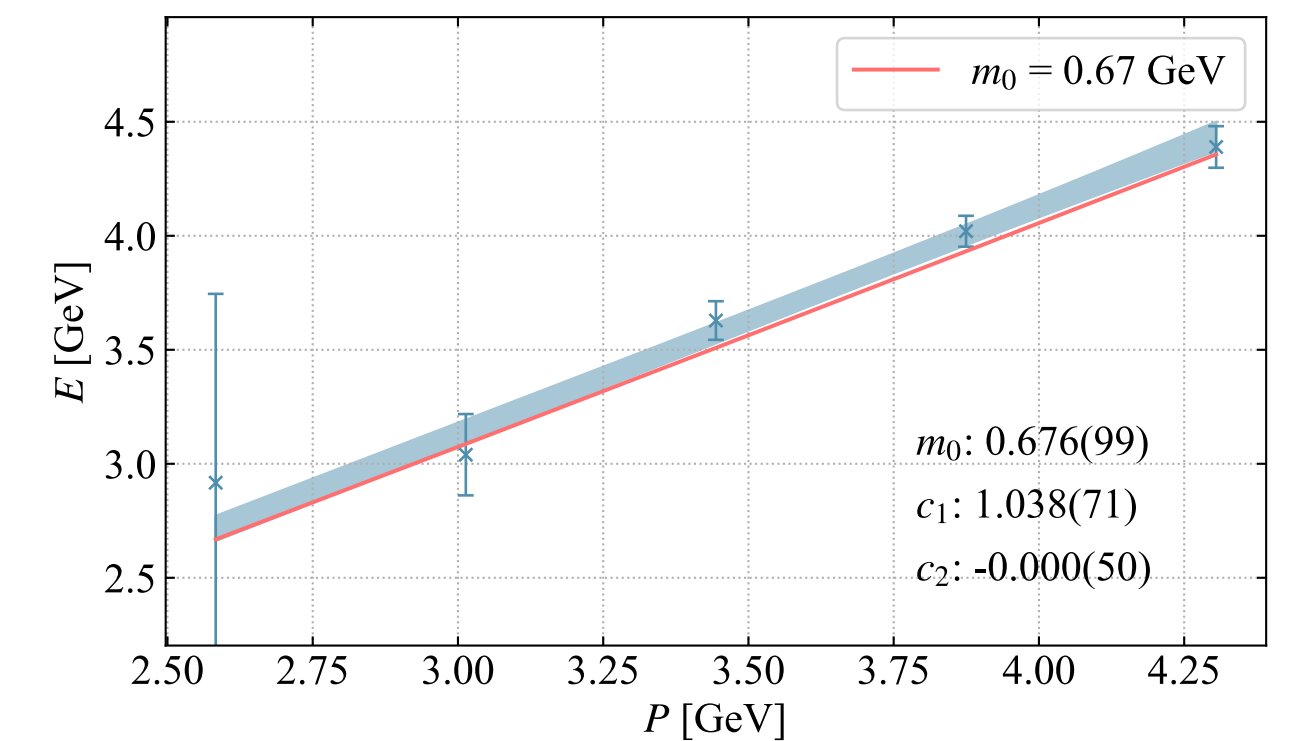
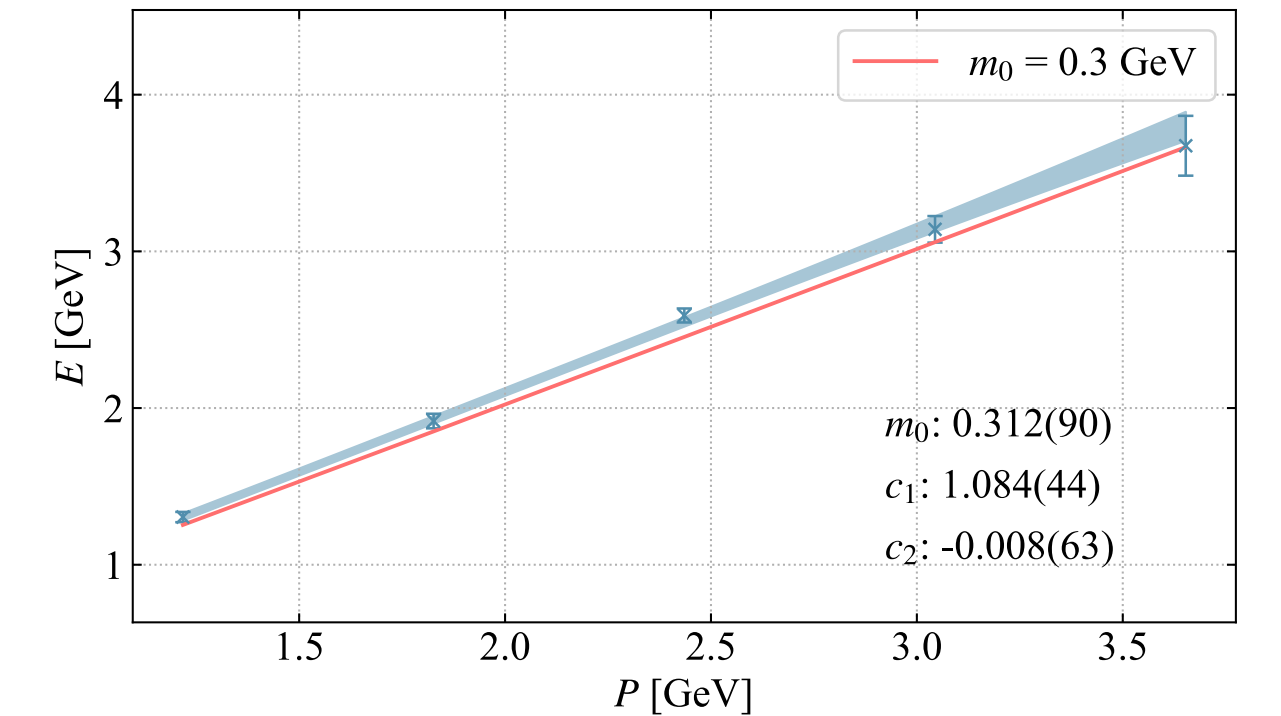
- Pion mass of valence quark for quasi-TMD: $m_\pi^{\text{val}} = 300$ MeV;

- Off-axis ($\vec{n} = (1,1,0)$) hadron momenta for quasi-TMD: 1.83 GeV, 2.43 GeV and 3.04 GeV;

- Pion mass of valence quark for qTMDWF and meson form factor: $m_\pi^{\text{val}} = 670$ MeV;

- On-axis hadron momenta for qTMDWF: 3.44 GeV, 3.87 GeV and 4.30 GeV;

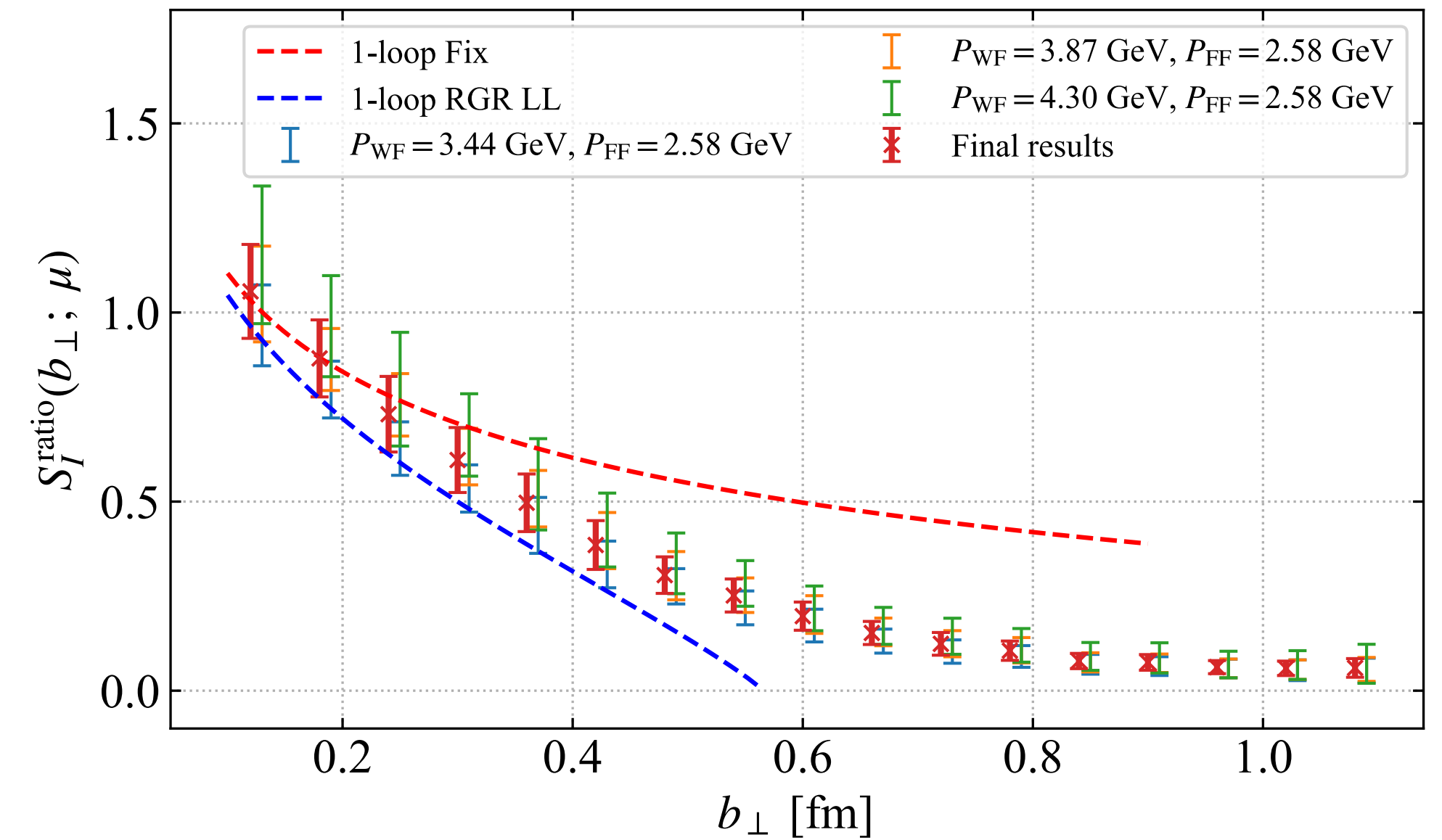
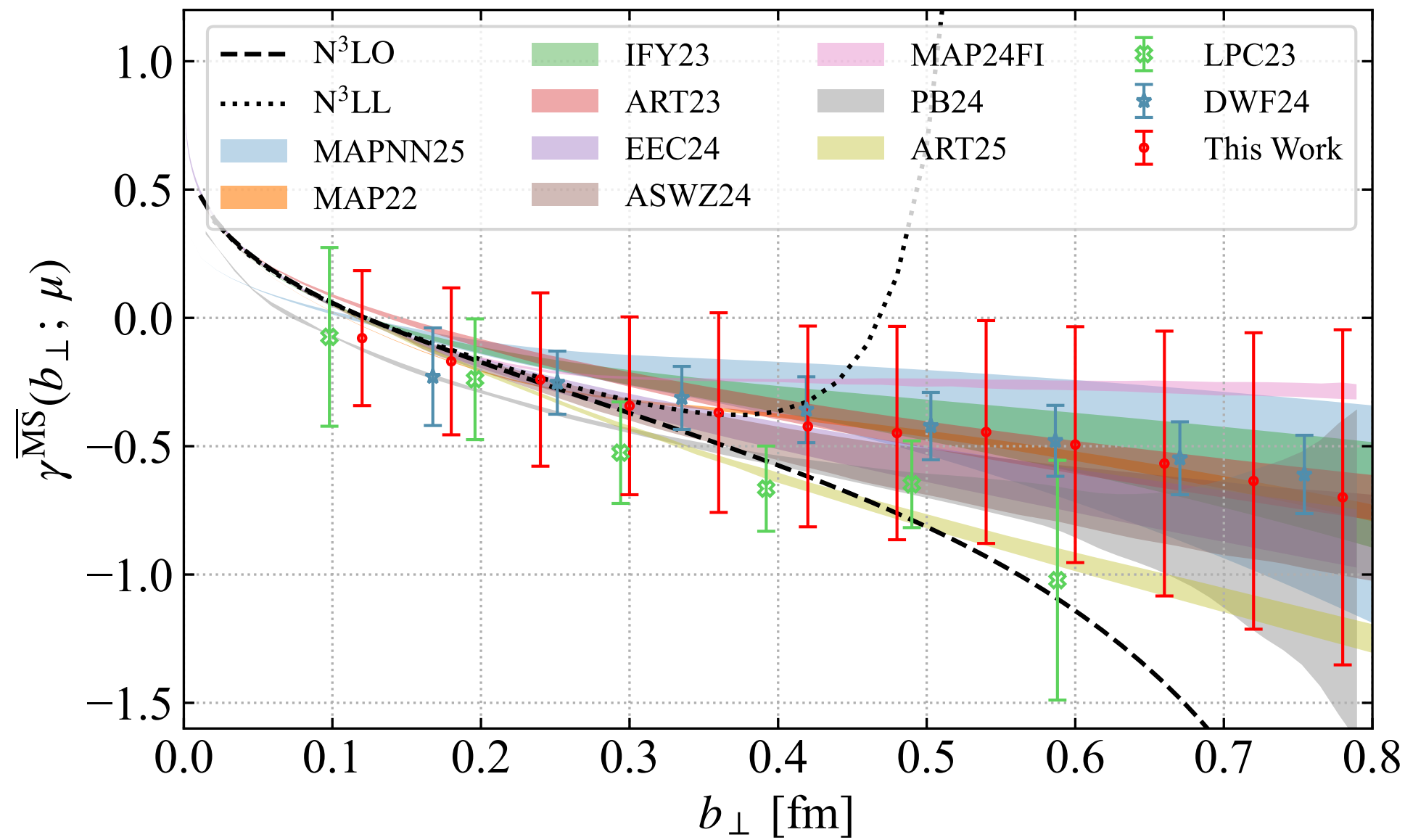
- On-axis hadron momenta for meson form factor: 2.58 GeV.



CS Kernel & Intrinsic Soft Function

$$\gamma^{\overline{\text{MS}}}(b_{\perp}, P_1, P_2; \mu) = \frac{1}{\ln(P_2/P_1)} \ln \frac{H_{\phi}(x, \bar{x}, P_1; \mu) \tilde{\phi}_{\gamma^z \gamma^5}(x, b_{\perp}, P_2; \mu)}{H_{\phi}(x, \bar{x}, P_2; \mu) \tilde{\phi}_{\gamma^z \gamma^5}(x, b_{\perp}, P_1; \mu)}$$

$$S_I(b_{\perp}; \mu) = \frac{F(b_{\perp}, P^z)}{\int dx_1 dx_2 H_F(x_1, x_2, P^z; \mu) \tilde{\Phi}^{\dagger}(x_1) \tilde{\Phi}(x_2)} \text{ with } \tilde{\Phi}(x) \equiv \frac{\tilde{\phi}_{\Gamma}(x, b_{\perp}, P^z; \mu)}{H_{\phi}(x, \bar{x}, P^z; \mu)}$$



Systematic uncertainty is estimated by the spread of the mean values between these three momentum pairs.

- There is a notable tension among recent results in phenomenology (MAP24FI & ART25);
- The large uncertainty is mainly caused by the **small Lorentz boost factor** at such a heavy pion mass ($m_{\pi}^{\text{val}} = 670 \text{ MeV}$);
- Our lattice results are consistent with the perturbation theory in the small $b_{\perp}$ regime;
- Thanks to the absence of linear divergence, our final results of the intrinsic soft function can go beyond $b_{\perp} \sim 1 \text{ fm}$.

Pion TMDs from Coulomb Gauge Method

Pion momentum larger than **3 GeV** on the lattice with

$a = 0.06$ fm;

Thanks to the absence of linear divergence in Coulomb

gauge method, we get pion TMDPDF up to **$b_{\perp} > 1$ fm**;

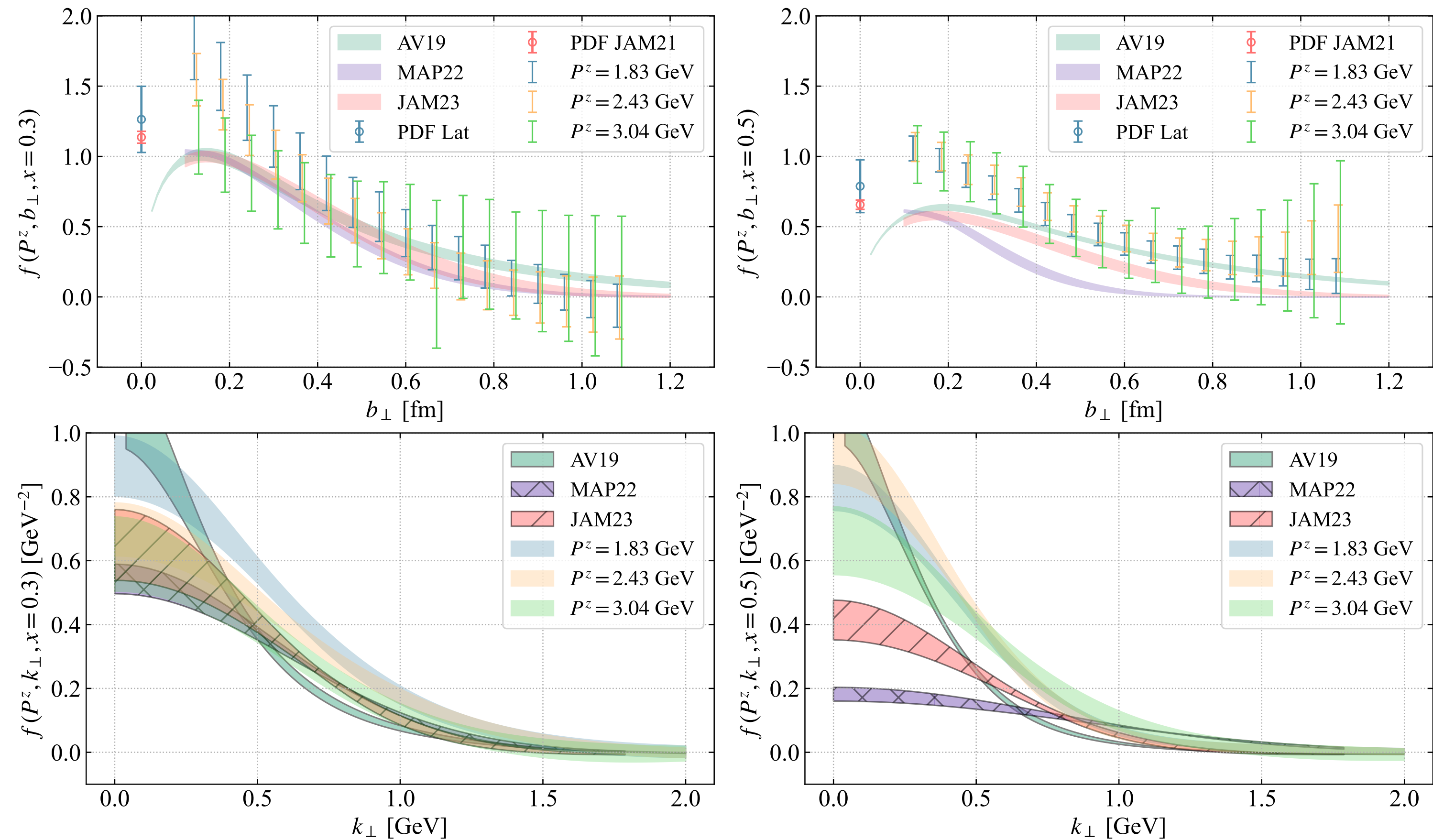
Different phenomenological collaborations agree well

at relative small x , where we saw consistency;

Lattice provides predictions at relative large x , where

phenomenological results exhibit discrepancies.

D. Bollweg, X. Gao, JH, S. Mukherjee and Y. Zhao, PRD 112 (2025)

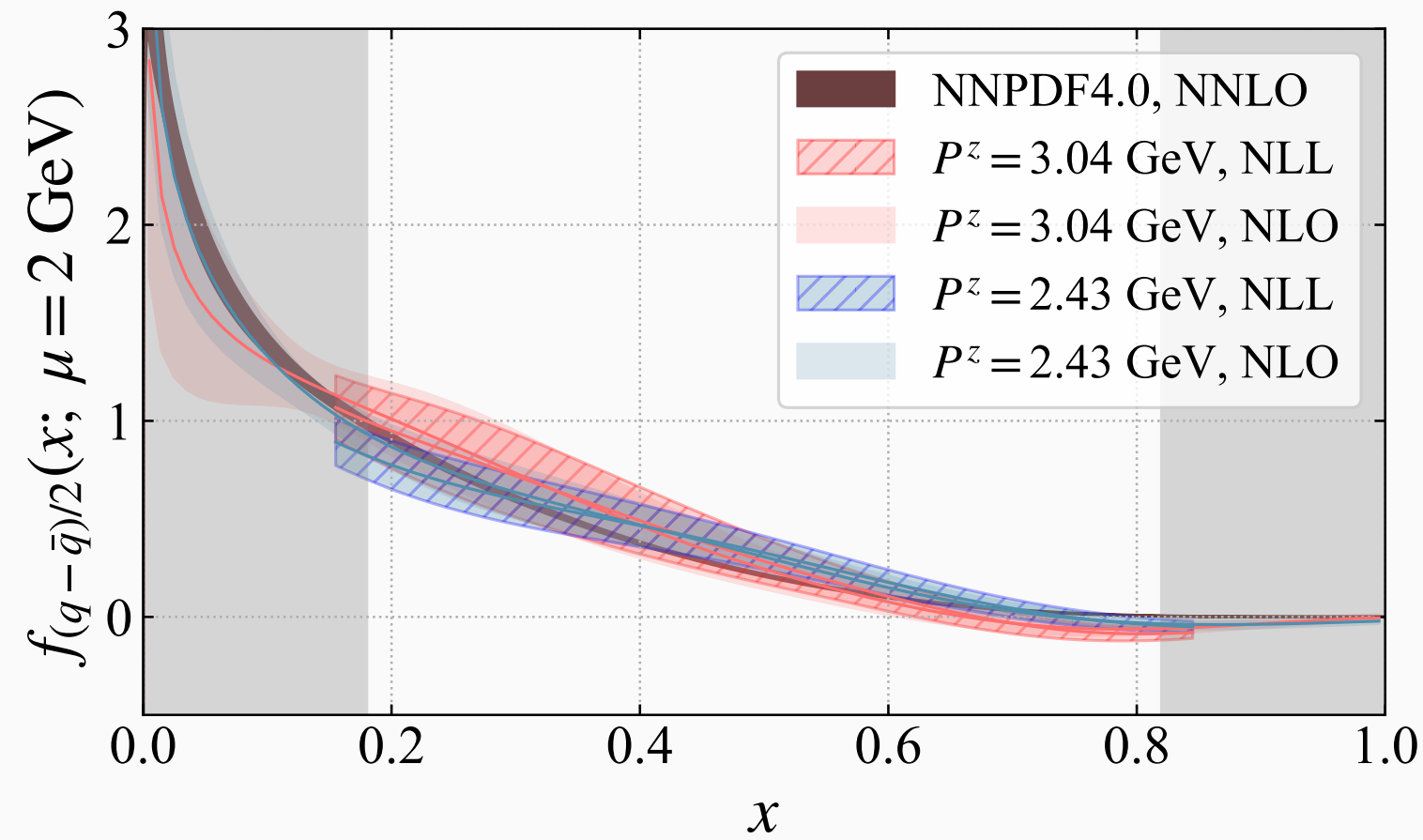


A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), PRD 107 (2023); P. C. Barry, et al. (JAM) PRD 108 (2023)

Nucleon PDFs from Coulomb Gauge Method

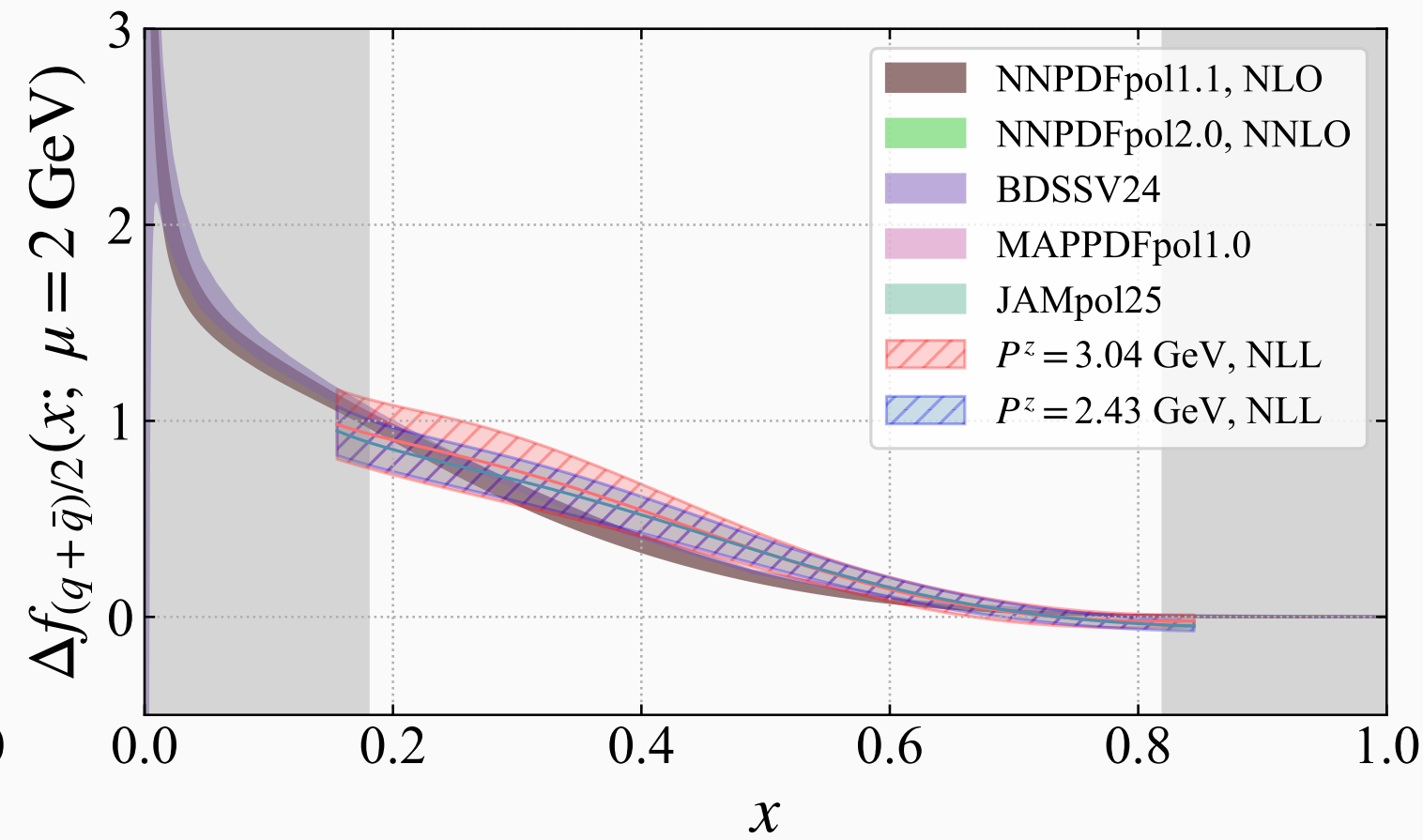
X. Gao, JH, et al., 2602.11283

Unpolarized PDF $(q - \bar{q})/2$



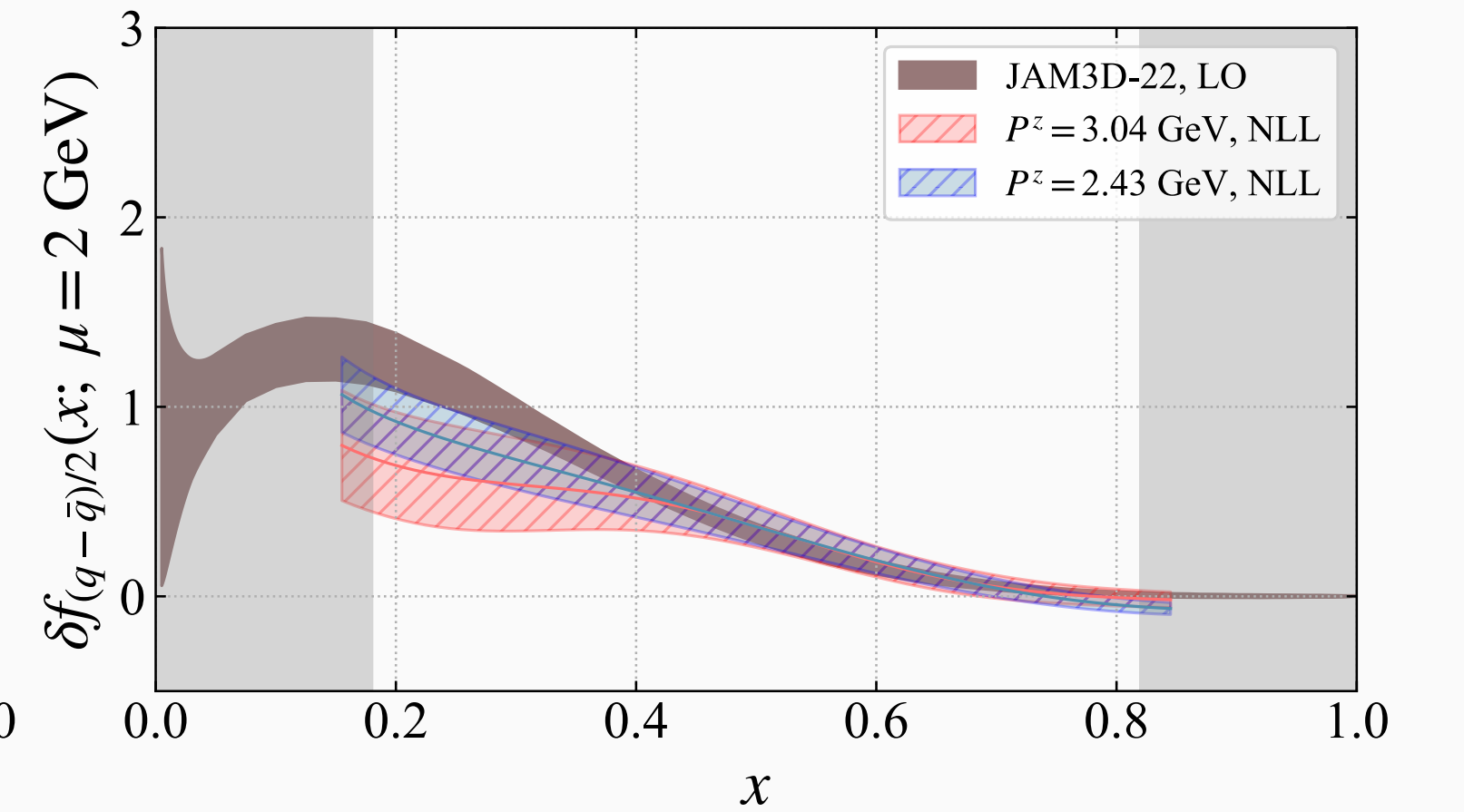
NNPDF4.0: R. D. Ball, et al. [NNPDF], EPJC 82 (2022)

Helicity PDF $(q + \bar{q})/2$



NNPDFpol1.1: E. R. Nocera et al. [NNPDF], NPB 887 (2014)
 NNPDFpol2.0: J. Cruz-Martinez et al. [NNPDF], JHEP 07 (2025)
 BDSSV24: I. Borsa et al., PRL 133 (2024)
 MAPPDFpol1.0: V. Bertone et al. [MAP], PLB 865 (2025)
 JAMpol25: C. Cocuzza et al. [JAM], 2506.13616

Transversity PDF $(q - \bar{q})/2$



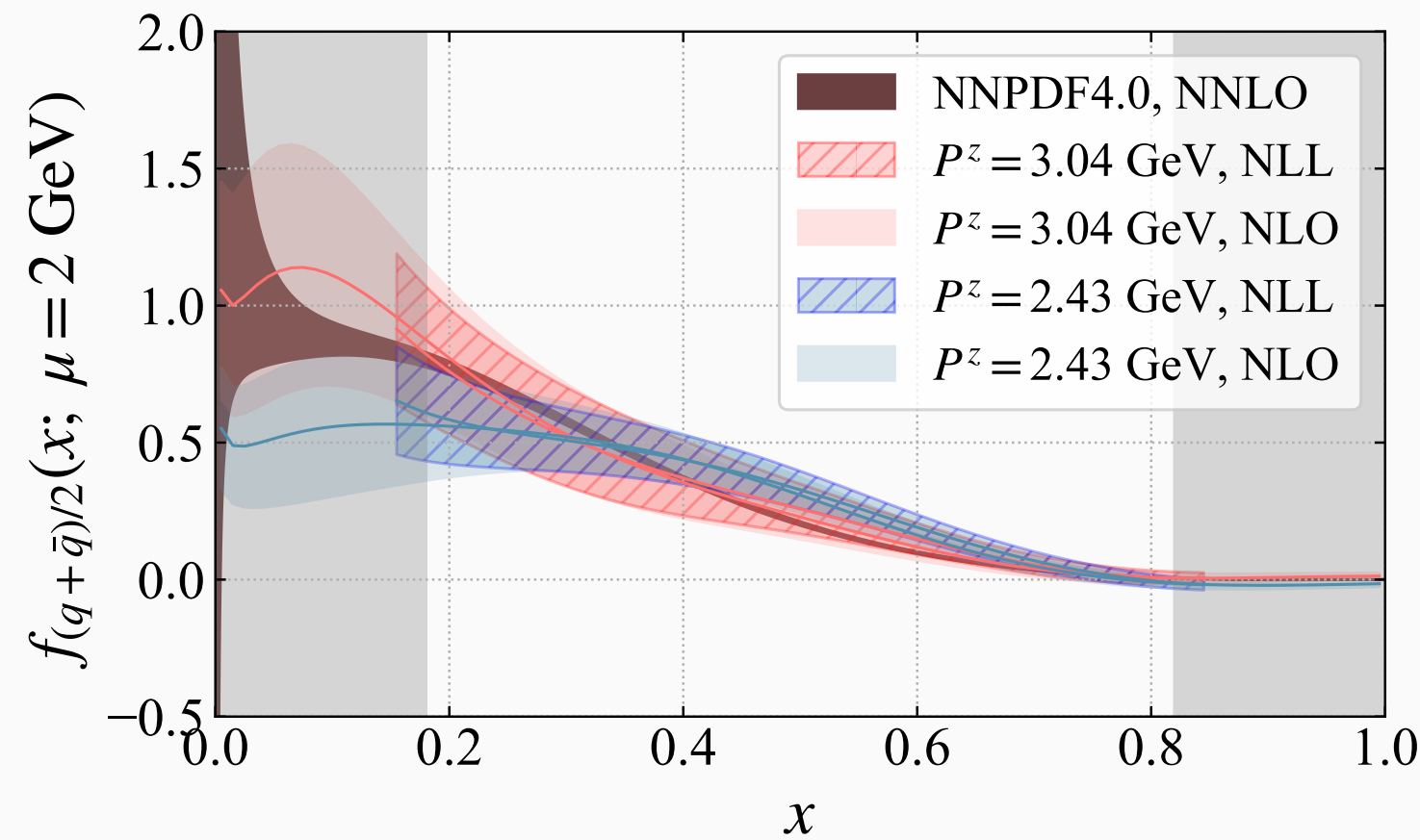
JAM3D-22: L. Gamberg, et al. [JAM], PRD 106 (2022)

- This work presents the **first lattice calculation** of the nucleon PDFs for all three polarizations using the **CG method**;
- The results of **real part contribution** in all three PDFs show encouraging agreement with existing results at **moderate x region**;
- The **NLO** and **NLL with RGR** results show an aligned behavior at **moderate x**, where LaMET can make reliable prediction;

Nucleon PDFs from Coulomb Gauge Method

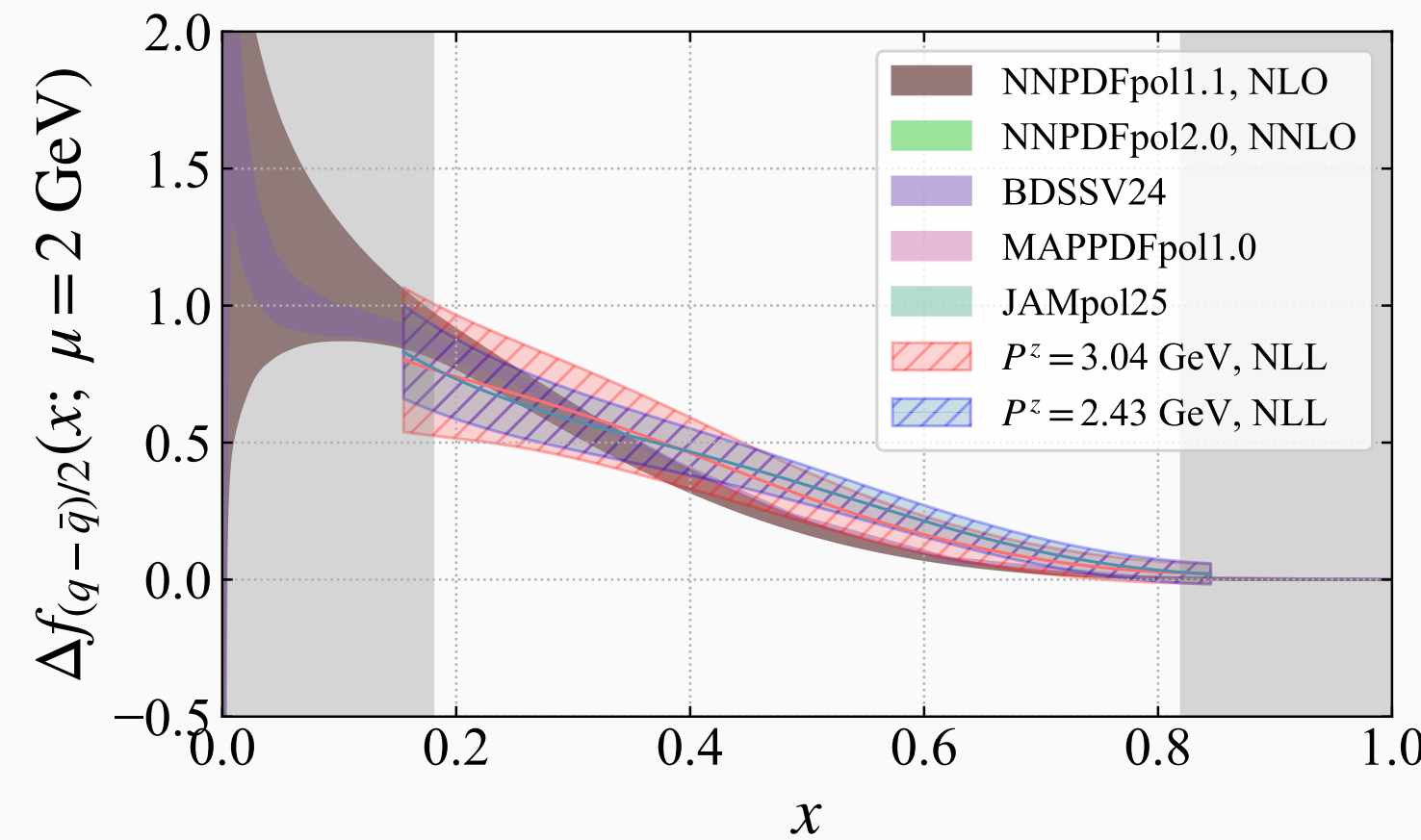
X. Gao, JH, et al., 2602.11283

Unpolarized PDF $(q + \bar{q})/2$



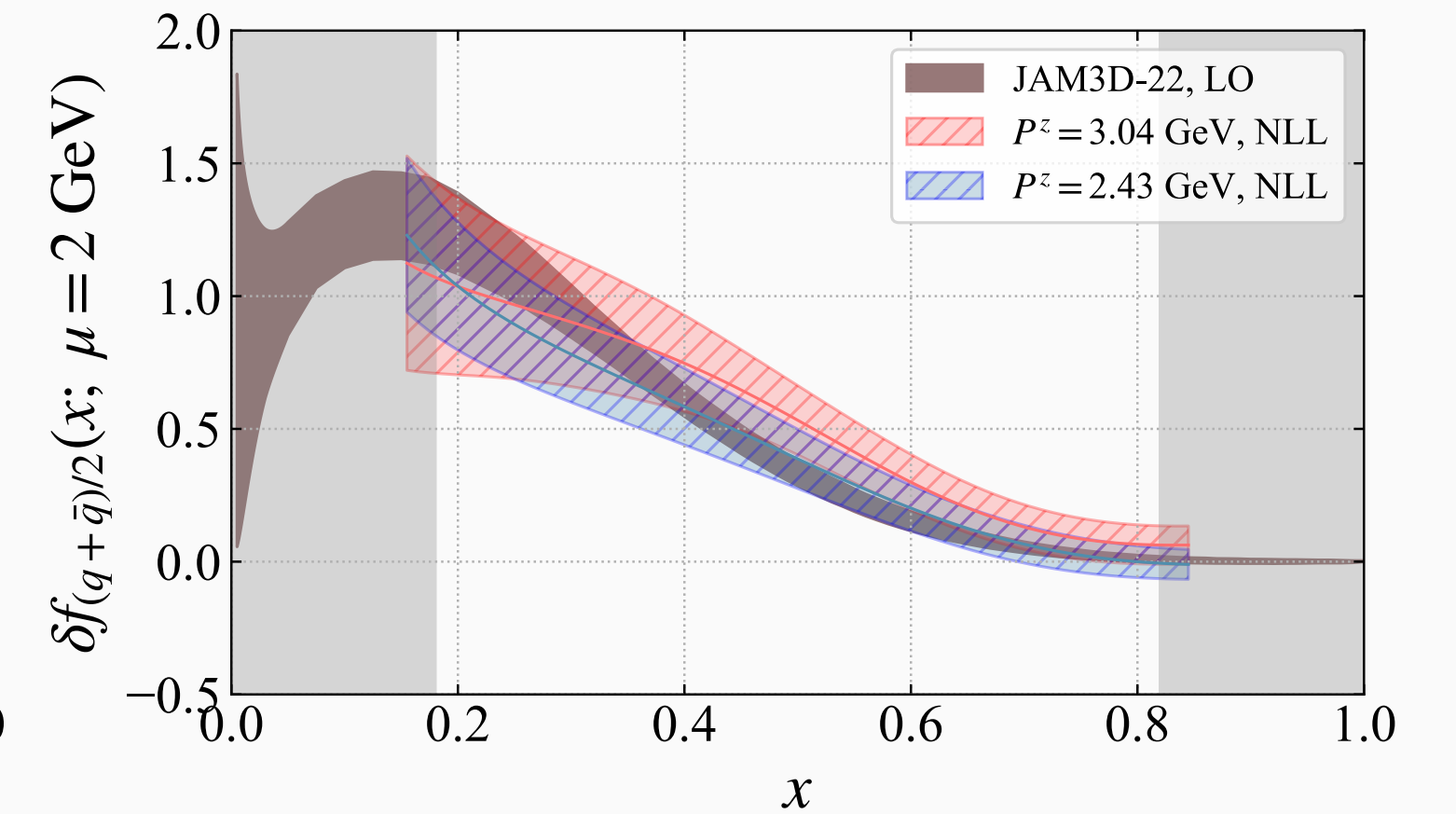
NNPDF4.0: R. D. Ball, et al. [NNPDF], EPJC 82 (2022)

Helicity PDF $(q - \bar{q})/2$



NNPDFpol1.1: E. R. Nocera et al. [NNPDF], NPB 887 (2014)
 NNPDFpol2.0: J. Cruz-Martinez et al. [NNPDF], JHEP 07 (2025)
 BDSSV24: I. Borsa et al., PRL 133 (2024)
 MAPPDFpol1.0: V. Bertone et al. [MAP], PLB 865 (2025)
 JAMpol25: C. Cocuzza et al. [JAM], 2506.13616

Transversity PDF $(q + \bar{q})/2$



JAM3D-22: L. Gamberg, et al. [JAM], PRD 106 (2022)

- For the imaginary part, the **discrepancy between two different momenta** is larger than that for the real part, possibly caused by excited-state contamination;
- In GI cases, it is also found that imaginary part of matrix elements is more sensitive to excited-state contamination.

C. Alexandrou, et al., Phys.Rev.D 103 (2021)

Ground State Fit

- Ratio of three-point and two-point correlators

$$R(t_{\text{sep}}, \tau) = \frac{C_{3\text{pt}}(t_{\text{sep}}, \tau)}{C_{2\text{pt}}(t_{\text{sep}})} = \frac{\sum_{n,m} z_n O_{nm} z_m^\dagger \cdot e^{-E_n(t_{\text{sep}} - \tau)} e^{-E_m \tau}}{\sum_n z_n z_n^\dagger \cdot (e^{-E_n t_{\text{sep}}} + e^{-E_n(L_t - t_{\text{sep}})})} \xrightarrow{t_{\text{sep}}, \tau, (L_t - t_{\text{sep}}) \rightarrow \infty} O_{00}$$

- Feynman-Hellmann (FH) correlators

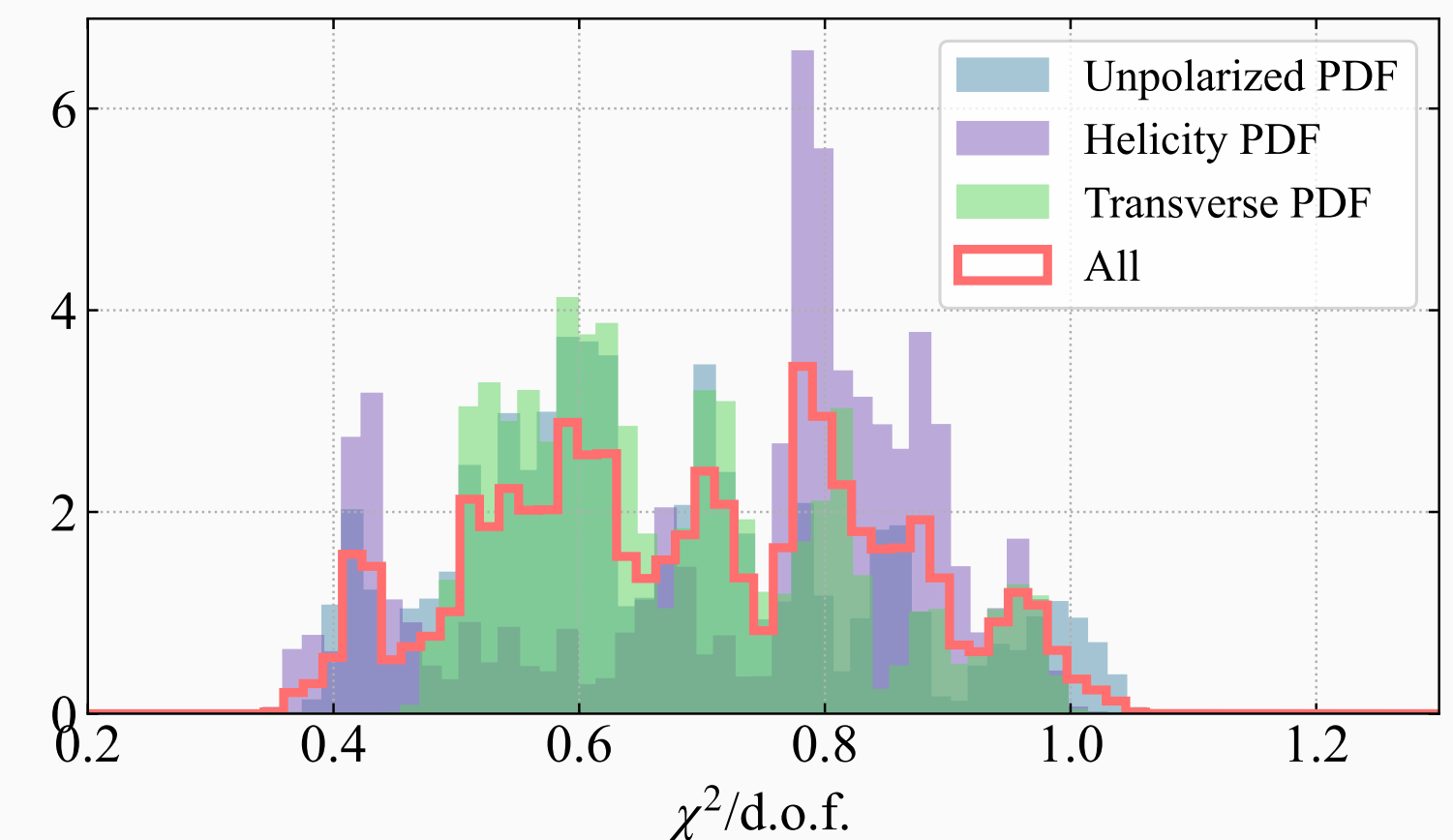
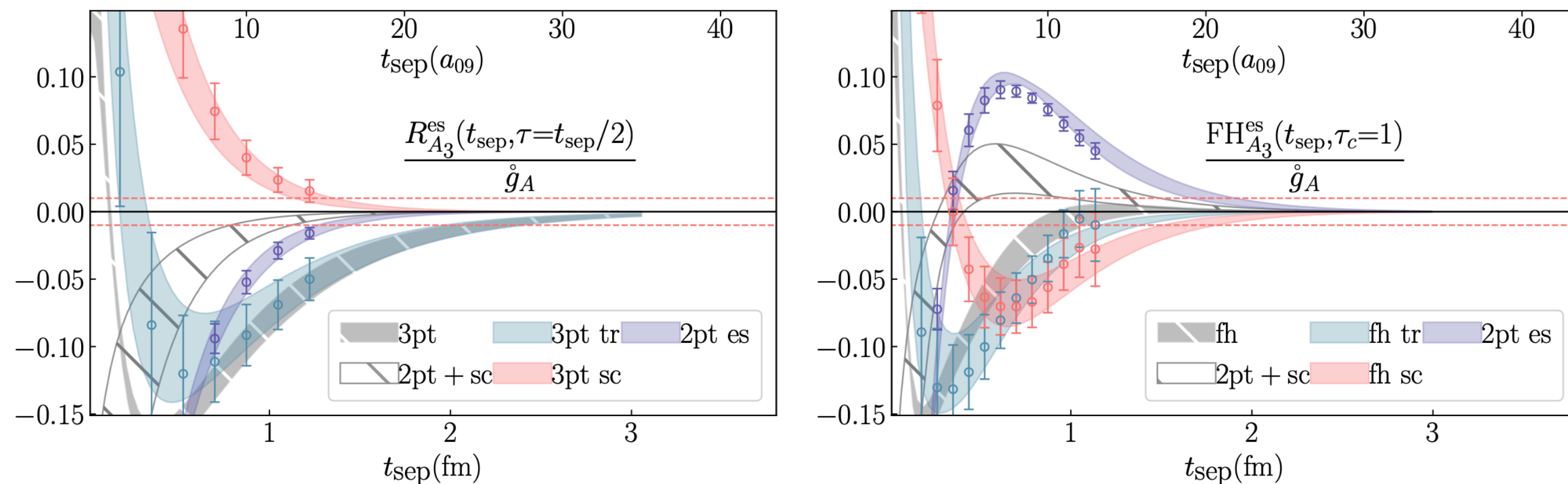
C. Bouchard, et al., PRD 96 (2017)

$$\text{FH}(t_{\text{sep}}, \tau_{\text{cut}}, dt) \equiv \frac{\sum_{t=\tau_{\text{cut}}}^{t=t_{\text{sep}}+dt-\tau_{\text{cut}}} R(t_{\text{sep}} + dt, t) - \sum_{t=\tau_{\text{cut}}}^{t=t_{\text{sep}}-\tau_{\text{cut}}} R(t_{\text{sep}}, t)}{dt} \xrightarrow{t_{\text{sep}}, \tau, (L_t - t_{\text{sep}}) \rightarrow \infty} O_{00}$$

JH, et al. (CalLat), PRC 105 (2022)

Cancellation of excited-state contamination in calculation of g_A

$\chi^2/\text{d.o.f.} \sim 1$ for all joint fits



AI/ML for Lattice

- **Neural Field Transformations for Hybrid Monte Carlo (NTHMC)**
- **AmSc Data: Organize the Lattice Dataset for AI Agents**
- **CritPt: a Frontier Physics Research Benchmark**
- **Main collaborators: Yong Zhao (ANL), James Osborn (ANL), Xiao-Yong Jin (ANL),
Peter Petreczky (BNL), Xiang Gao (BNL), Min-Yang Tian (UIUC), Wei-Yang Liu (SBU)**

Neural Field Transformations for Hybrid Monte Carlo (NTHMC)

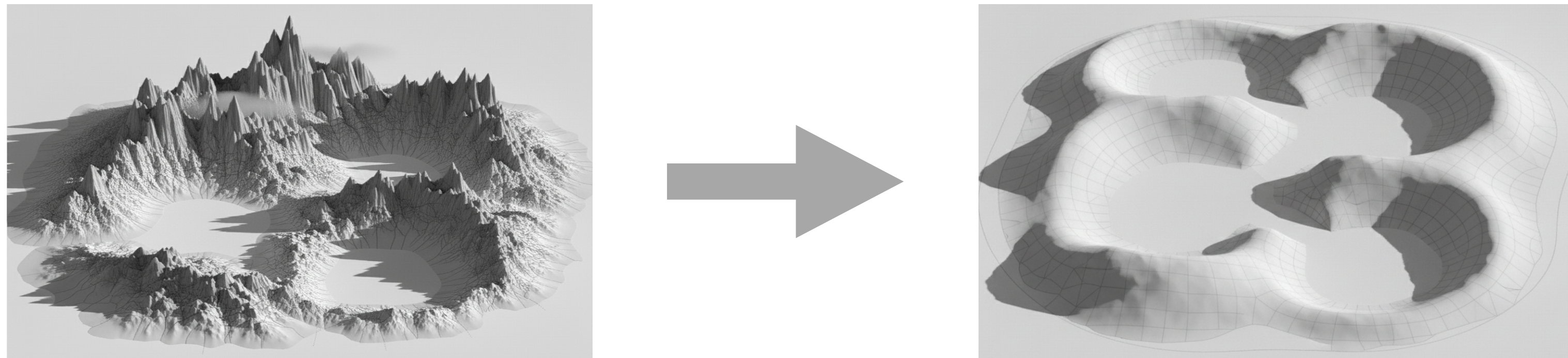
- **Motivation:** generate lattice configurations, critical slowing down, topology freezing;

JH, X.-Y. Jin, J. Osborn, Y. Zhao, NeurIPS 2025 ML4PS, 2511.02018

- **Method:** parametrize a field transformation to trivialize the action landscape using neural network (NTHMC);

$$\mathcal{Z} = \int D[U] e^{-S[U]}$$

$$\mathcal{F} : \tilde{U} \rightarrow U, \text{ then } S_{\text{FT}}(\tilde{U}) = S(\mathcal{F}(\tilde{U})) - \log \left| \det \mathcal{F}^* (\tilde{U}) \right|$$



- **Numerical results:** 2D U(1) gauge theory, improve the NTHMC efficiency via an optimized design, and check the scaling behavior on smaller lattice spacings and larger volumes;

Requirements of the Field Transformation

- **Gauge covariance:** suppose we do the field transformation $\mathcal{F} : \tilde{U}_{x,\mu} \rightarrow U_{x,\mu} = e^{\Pi_{x,\mu}} \tilde{U}_{x,\mu}$;

Gauge transform: $\tilde{U}_{x,\mu} \rightarrow \tilde{U}'_{x,\mu} = G(x) \tilde{U}_{x,\mu} G^\dagger(x + \mu)$ $\mathcal{F} \circ g[\tilde{U}] = g \circ \mathcal{F}[\tilde{U}]$, so $S_{\text{FT}}(\tilde{U})$ is gauge-invariant

$$U_{x,\mu} \rightarrow U'_{x,\mu} = G(x) U_{x,\mu} G^\dagger(x + \mu) = G(x) e^{\Pi_{x,\mu}} \tilde{U}_{x,\mu} G^\dagger(x + \mu), \text{ covariance requires } U'_{x,\mu} = e^{\Pi'_{x,\mu}} \tilde{U}'_{x,\mu} = e^{\Pi'_{x,\mu}} G(x) \tilde{U}_{x,\mu} G^\dagger(x + \mu)$$

So we need $G(x) e^{\Pi_{x,\mu}} G^\dagger(x) = e^{\Pi'_{x,\mu}}$, i.e. $\Pi_{x,\mu}$ should transform in adjoint representation.

In 2D U(1), we just need $\Pi_{x,\mu}$ to be gauge invariant.

- **Trackable Jacobian:** separate the lattice into disjoint subsets and update them sequentially;

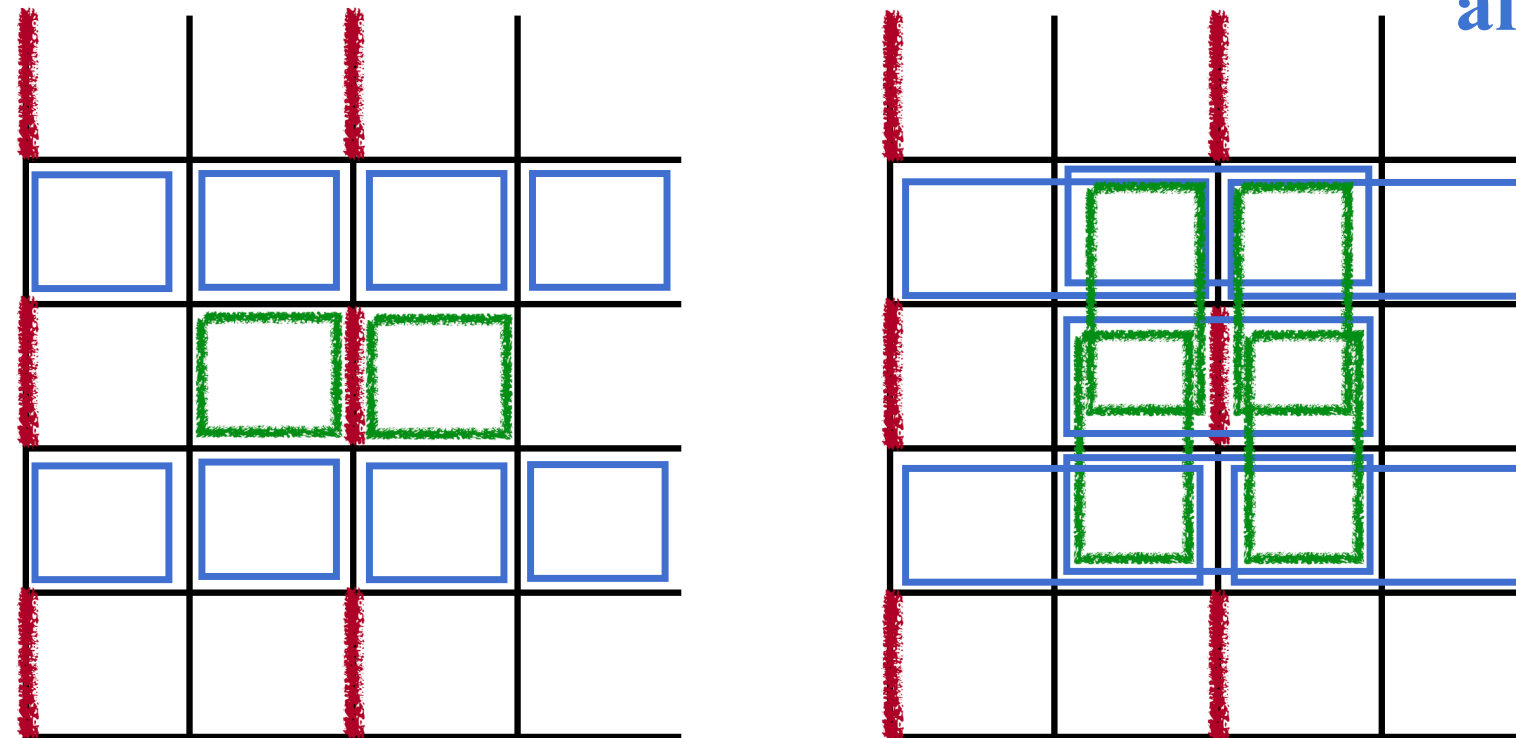
$$\begin{pmatrix} a & & & & \\ & b & & & \\ * & * & 1 & & \\ * & * & & 1 & \\ * & * & & & 1 \\ * & * & & & & 1 \end{pmatrix}$$

- **Local:** use convolutional neural network (CNN) with local variables.

Field Transformation

- $\mathcal{F} : \tilde{U}_{x,\mu} \mapsto U_{x,\mu} = e^{\Pi_{x,\mu}} \tilde{U}_{x,\mu}$, where $\Pi_{x,\mu} = \sum_l \epsilon_{x,\mu,l} W_{x,\mu,l}$;
- $W_{x,\mu,l}$ are group derivatives of gauge-invariant Wilson loops and coefficients $\epsilon_{x,\mu,l} = \phi[N_l(X, Y, \dots)]$ are parameterized using CNN;

Group derivative of a Wilson loop is in Lie algebra and transform in adjoint representation



- In the figure above, red lines mark one of the eight link subsets updated sequentially, green lines indicate Wilson loops $W_{x,\mu,l}$ and blue lines show the gauge-invariant features $(X, Y, \dots)$ used as CNN inputs.
- For 2D U(1), we have

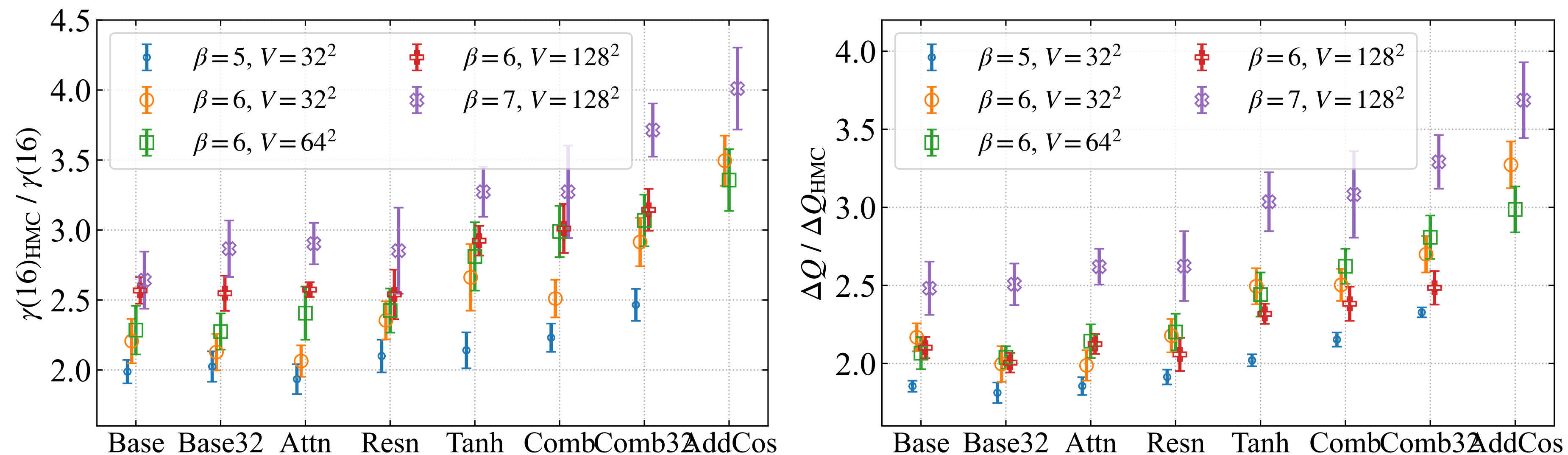
$$\mathcal{F} : \tilde{U}_{x,\mu} \mapsto U_{x,\mu} = e^{\sum_l \epsilon_{x,\mu,l} W_{x,\mu,l}} \tilde{U}_{x,\mu} = e^{\sum_p i\epsilon_{x,\mu,p} \sin(\theta_p) + \sum_r i\epsilon_{x,\mu,r} \sin(\theta_r)} \tilde{U}_{x,\mu}$$

$$\mathcal{F}_*(\tilde{U})_{x,\mu,x',\mu'} = \delta_{x,x'} \delta_{\mu,\mu'} \left[1 + \sum_p \epsilon_{x,\mu,p} \cos(\theta_p) + \sum_r \epsilon_{x,\mu,r} \cos(\theta_r) \right].$$

NTHMC Results

JH, X.-Y. Jin, J. Osborn, Y. Zhao, *NeurIPS 2025 ML4PS*, 2511.02018

Comparison of NTHMC with various network architectures

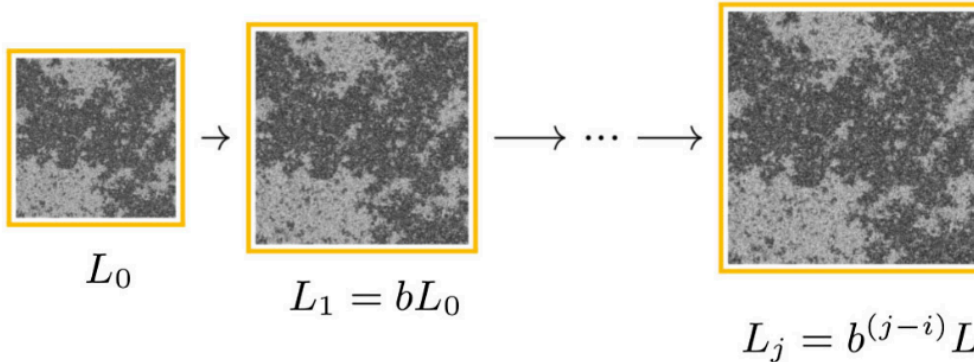


- NTHMC using Base model gives **150%** improvement on the sampling efficiency over the regular HMC;
- An optimized design of NTHMC gives an extra **65%** improvement on the sampling efficiency over the Base model;
- As the coupling increases, all NTHMC models exhibit a higher sampling efficiency compared to standard HMC, indicating that our method can be effectively extended to larger volumes and finer lattices;
- Interestingly, wider receptive field does not hinder transferability.

Future Work

- **Nucleon TMDPDFs using CG method: domain-wall fermion, physical quark mass;**
- **NTHMC in 4D SU(3);**
- **Inverse RG in Lattice gauge theory;**
- **Optimized profile for interpolating operators on lattice;**
- **Foundation model for lattice gauge theory;**
- **.....**

Inverse Renormalization Group Transformation

- Renormalization group (RG) is a semi-group, but the inverse transformation can be approximated numerically in discretized theories;
- Illustration of the inverse renormalization group (IRG):
 

D. Bachtis, et al., PRL 128 (2022)
- In lattice gauge theory, IRG can be used to upsample fine lattices from a coarse lattice, so that to avoid “Critical slowing down”;
- In 2D U(1) gauge theory, we have some preliminary tests:

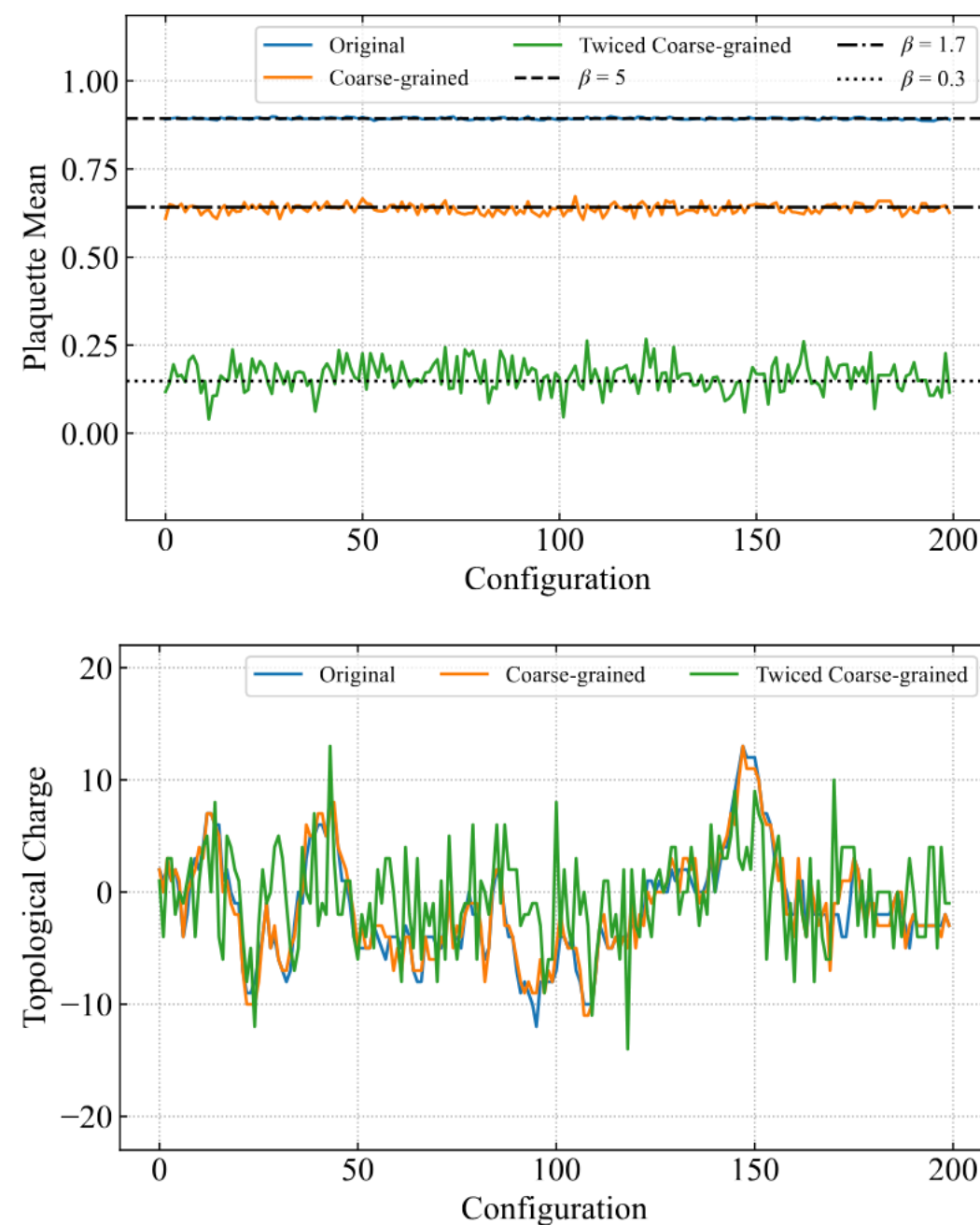


FIG. 4. RG comparison with one path

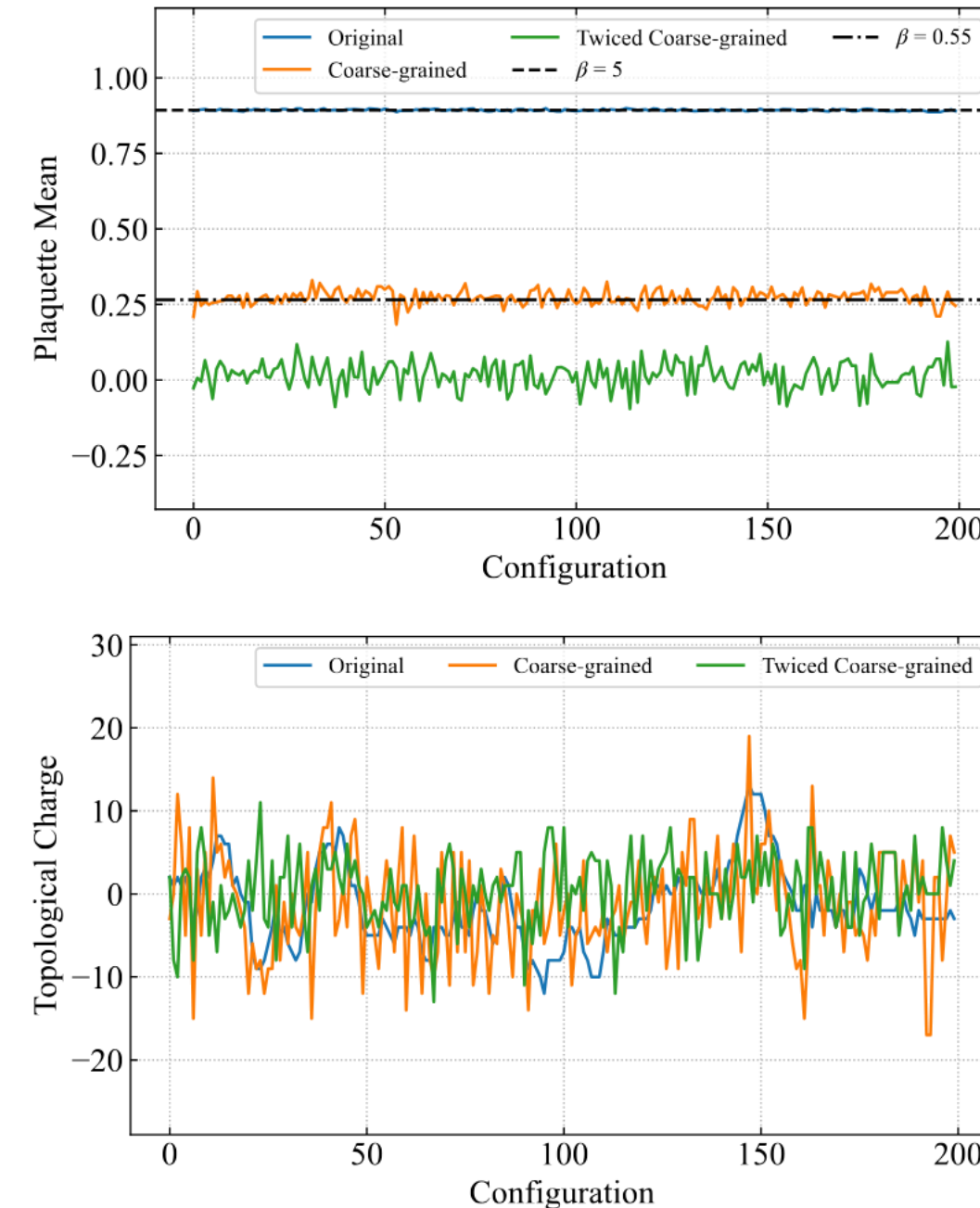


FIG. 5. RG comparison with two paths

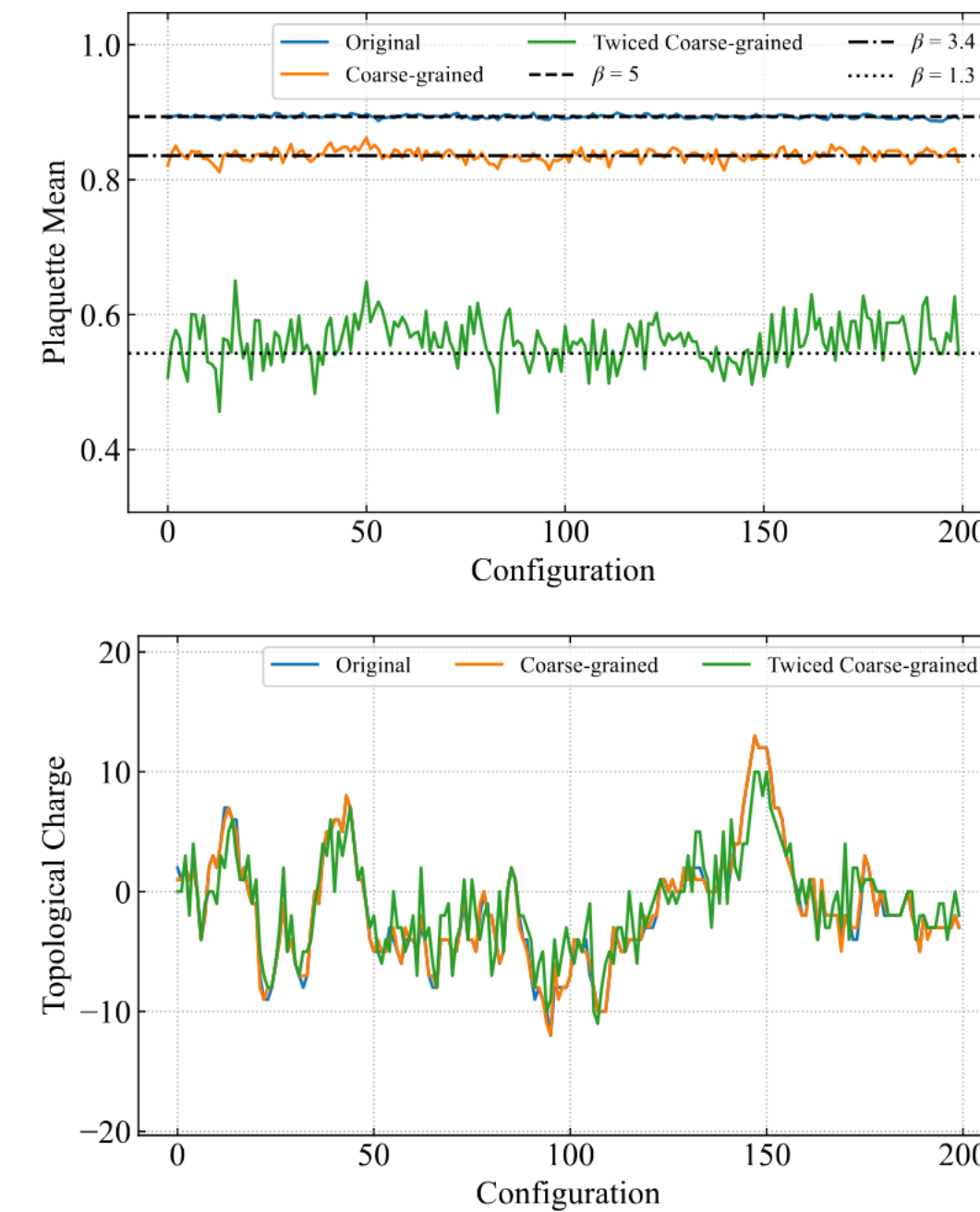
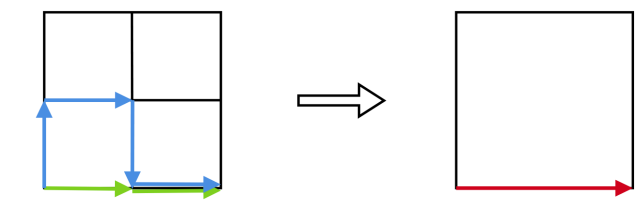
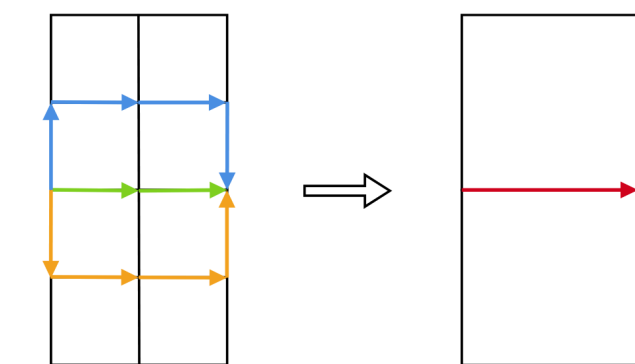


FIG. 6. RG comparison with three paths

Two paths



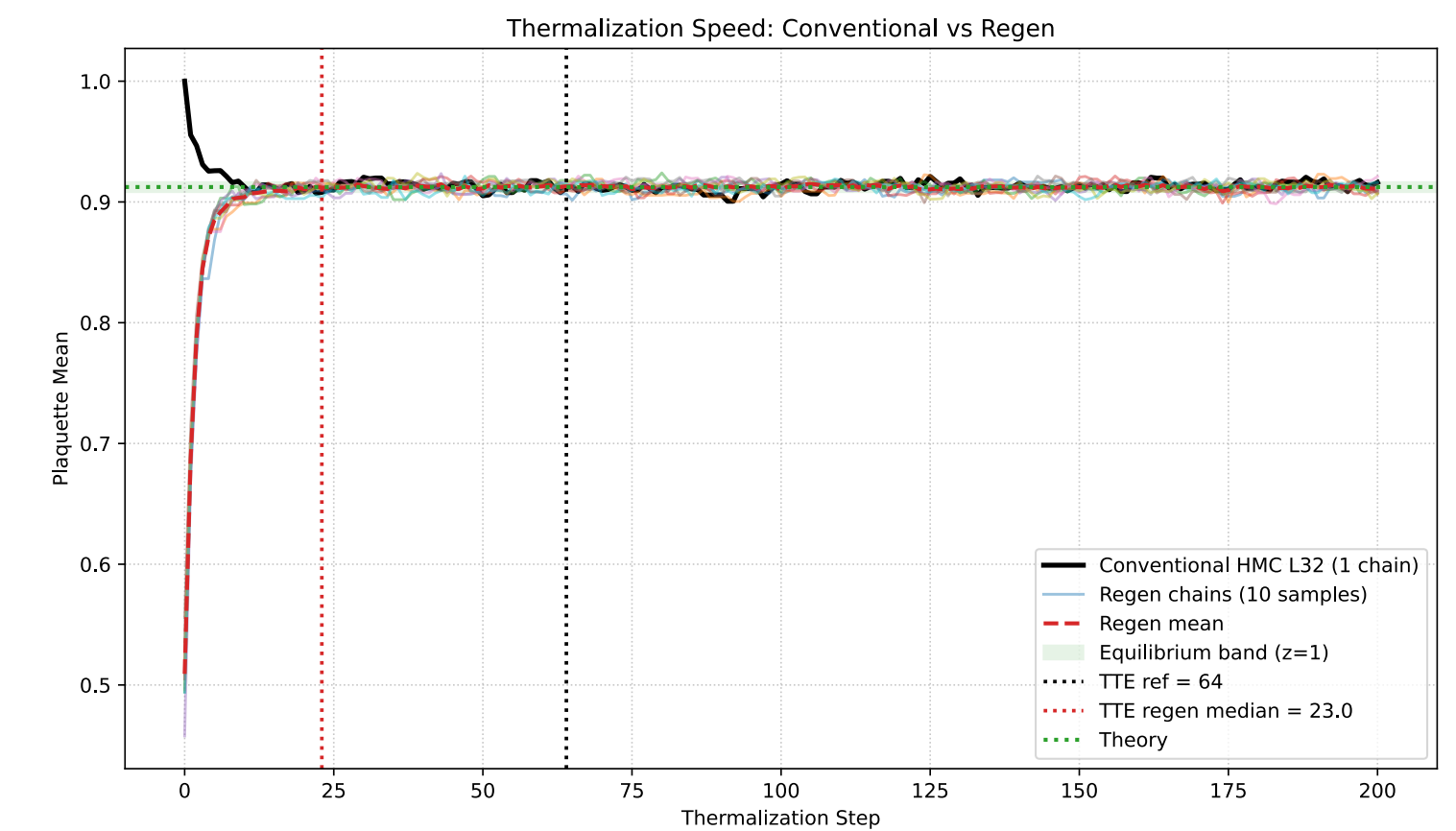
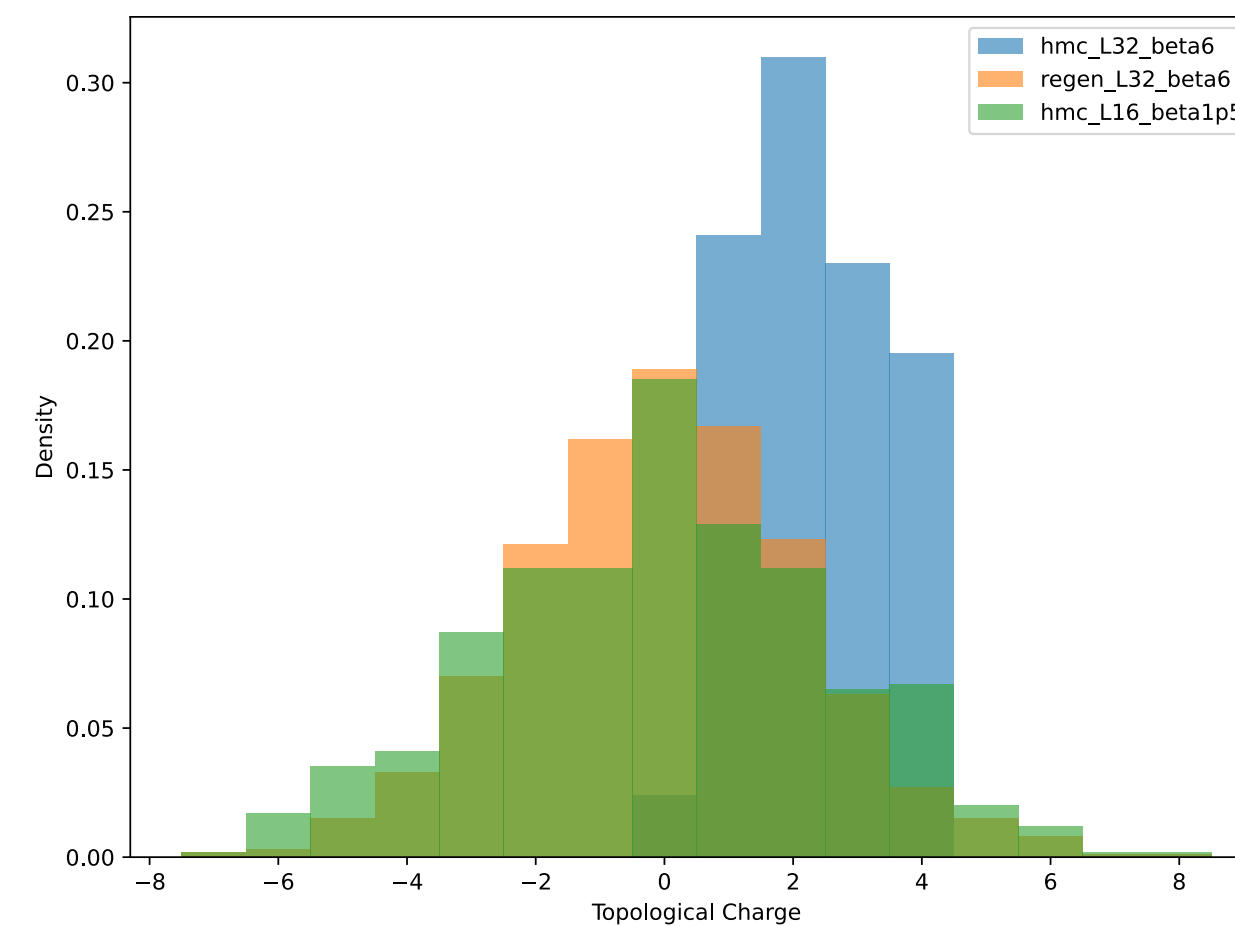
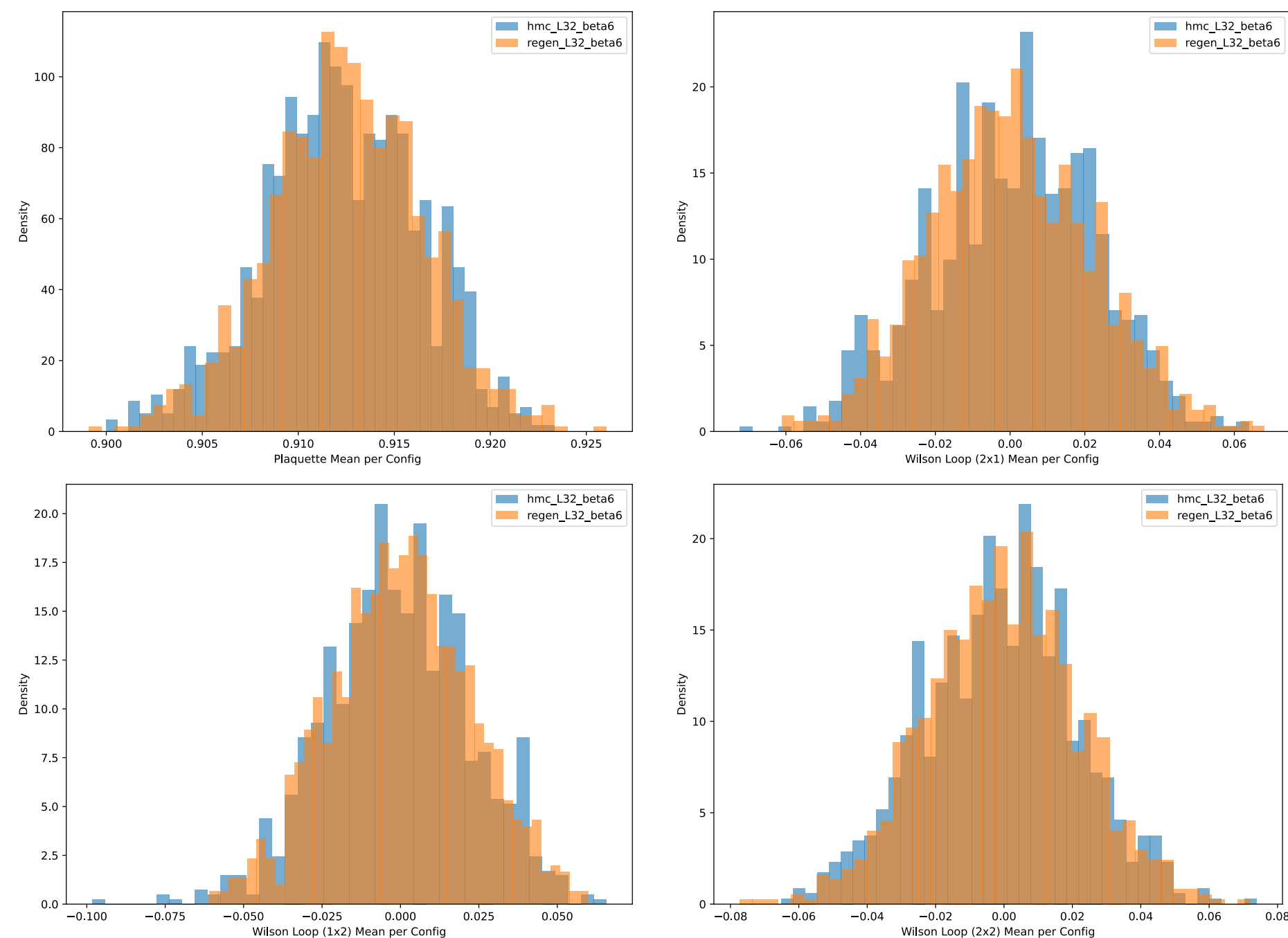
Three paths



Keywords: diffusion model, instanton model, PINN, ...

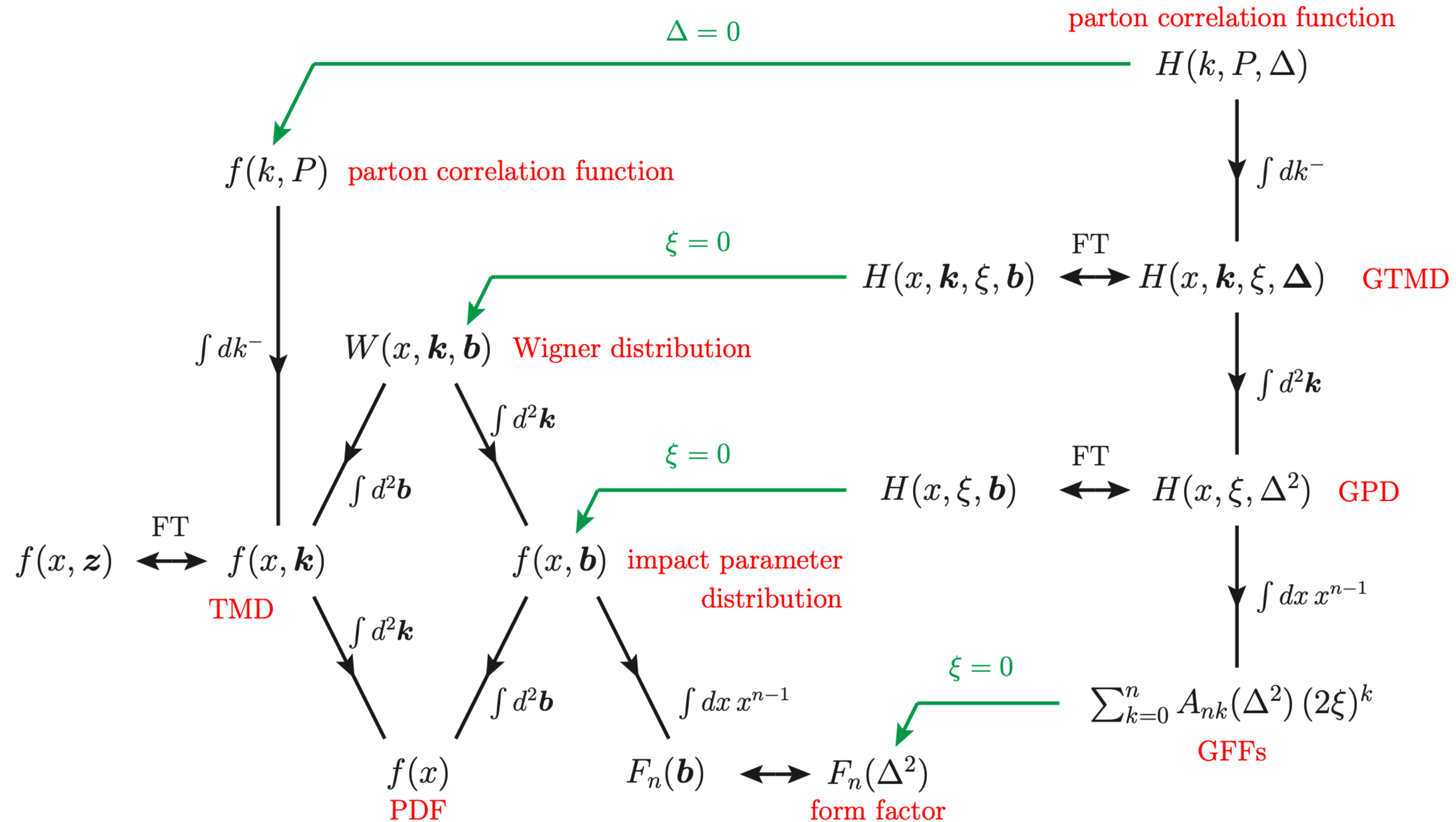
Inverse Renormalization Group Transformation

- Apply a naive IRG transformation on the coarse lattice ensemble to get a quasi fine ensemble;
- Use the quasi fine ensemble as the initial point for multi-chain HMC to get the regenerated fine ensemble;
- We test on L16-beta1.5 and L32-beta4 as below, 1000 configurations for an ensemble;
- The regenerated fine ensemble has **consistent distributions** on Wilson loops, with an **early** thermalization;



Backup

Partonic Structures



Soft Functions in TMD Factorization

$$\frac{d\sigma}{dQdYd^2\mathbf{q}_T} = \left(\frac{d\sigma^W}{dQdYd^2\mathbf{q}_T} + \frac{d\sigma^Y}{dQdYd^2\mathbf{q}_T} \right) \left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right) \right].$$

$$\frac{d\sigma^W}{dQdYd^2\mathbf{q}_T} = \sum_{\text{flavors } i} H_{i\bar{i}}(Q^2, \mu) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T \cdot \mathbf{q}_T} \tilde{f}_{i/p}(x_a, \mathbf{b}_T, \mu, \zeta_a) \tilde{f}_{\bar{i}/p}(x_b, \mathbf{b}_T, \mu, \zeta_b) \quad (2.29a)$$

$$= \sum_{\text{flavors } i} H_{i\bar{i}}(Q^2, \mu) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T \cdot \mathbf{q}_T} \tilde{B}_{i/p}(x_a, \mathbf{b}_T, \mu, \zeta_a/\nu^2) \tilde{B}_{\bar{i}/p}(x_b, \mathbf{b}_T, \mu, \zeta_b/\nu^2) \\ \times \tilde{S}_{n_a n_b}(b_T, \mu, \nu). \quad (2.29b)$$

$$\tilde{f}_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = \tilde{B}_{i/p}(x, \mathbf{b}_T, \mu, \zeta/\nu^2) \sqrt{\tilde{S}_{n_a n_b}(b_T, \mu, \nu)}, \quad (2.32)$$

$$\tilde{B}_{i/p}(x, \mathbf{b}_T, \mu, \zeta/\nu^2) = \lim_{\substack{\epsilon \rightarrow 0 \\ \tau \rightarrow 0}} \tilde{Z}_B^i(b_T, \mu, \nu, \epsilon, \tau, xP^+) \tilde{B}_{i/p}^0(x, \mathbf{b}_T, \epsilon, \tau, xP^+) \\ = \lim_{\substack{\epsilon \rightarrow 0 \\ \tau \rightarrow 0}} \tilde{Z}_B^i(b_T, \mu, \nu, \epsilon, \tau, xP^+) \frac{\tilde{f}_{i/p}^{0(u)}(x, \mathbf{b}_T, \epsilon, \tau, xP^+)}{\tilde{S}_{n_a n_b}^{0\text{subt}}(b_T, \epsilon, \tau)}, \quad (2.34)$$

$$\tilde{S}_{n_a n_b}(b_T, \mu, \nu) = \lim_{\substack{\epsilon \rightarrow 0 \\ \tau \rightarrow 0}} \tilde{Z}_S(b_T, \mu, \nu, \epsilon, \tau) \tilde{S}_{n_a n_b}^0(b_T, \epsilon, \tau). \quad (2.35)$$

$$\tilde{f}_\Gamma(x, b_\perp, P^z; \mu) = H_f(x, P^z; \mu) B(x, b_\perp, xP^+; \mu, \nu) \\ \times S_C^0(b_\perp; \mu, \nu), \quad (3)$$

$$\sqrt{S_I(b_\perp, y_n; \mu)} \cdot \tilde{f}_\Gamma(x, b_\perp, P^z; \mu) \\ = f(x, b_\perp; \mu, \zeta) H_f(x, P^z; \mu) + \text{p.c.}, \quad (5)$$

- Zero-bin subtracted beam function has no soft contribution;
- S_C^0 is the quasi-soft function with zero-bin contribution;
- The intrinsic soft function S_I is the combination of subtracted soft function and the quasi-soft function

Soft Function in TMD Factorization

$$\tilde{S}_{n_a n_b}^0(b_T, \epsilon, \tau) = \frac{1}{N_c} \langle 0 | \text{Tr} [W_{\gg}(b_T)]_{\tau} | 0 \rangle$$

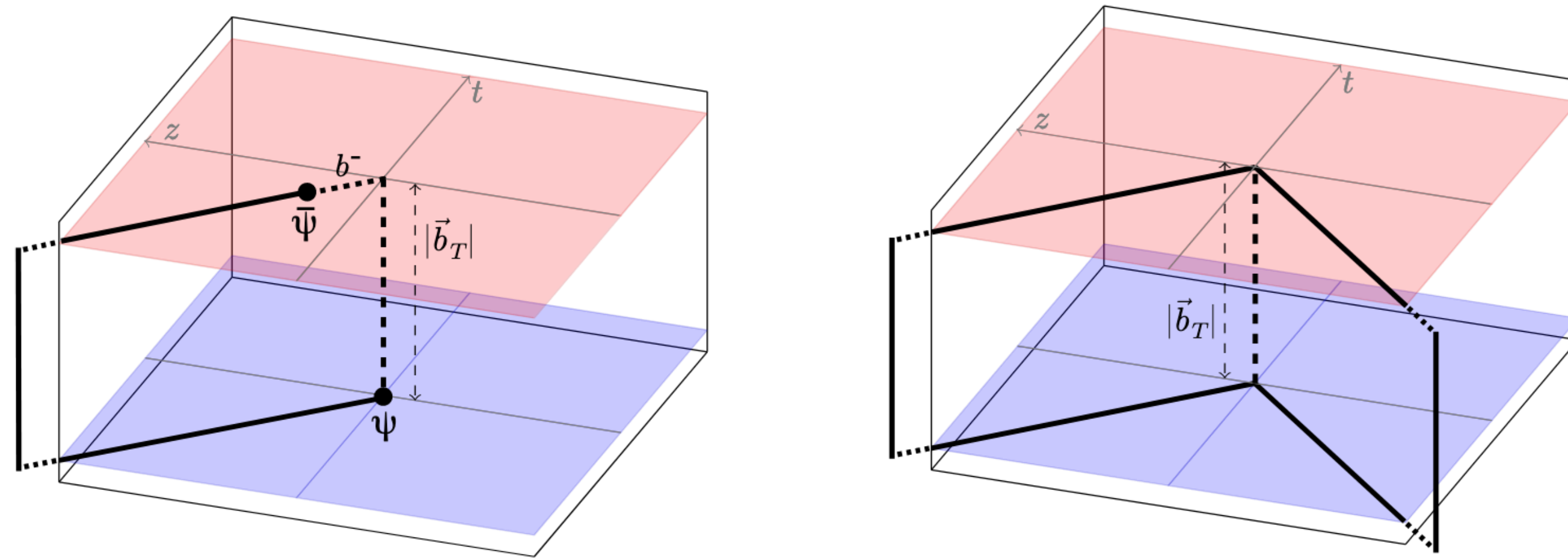


Figure 2.1: Graphs of the Wilson line structure $W_{\square}(b^{\mu}, 0)$ of the unsubtracted TMD PDF $f_{i/p}^{0(u)}$ (left) and of $W_{\gg}(b_T)$ for the soft function $S_{n_a n_b}^0$ (right), defined in Eqs. (2.37) and (2.38). The Wilson lines (solid) extend to infinity in the directions indicated. Adapted from [106].

R. Boussarie, et al., 2304.03302 (2023)

Phenomenological Extraction of TMDs

- Significant progress has been made in the phenomenological parameterizations of TMDs

Some of groups will incorporate neural network;

Some of groups will include lattice data;

- Collins-Soper kernel (CS kernel): rapidity evolution kernel of TMDs

A. Bacchetta, et al. (MAP) JHEP 08 (2024); V. Moos, et al., 2503.11201 ...

- Nucleon TMDs

- Unpolarized *M. Bury, et al., JHEP 10 (2022); A. Bacchetta, et al., JHEP 10 (2022); V. Moos, et al., JHEP 05 (2024) ...*

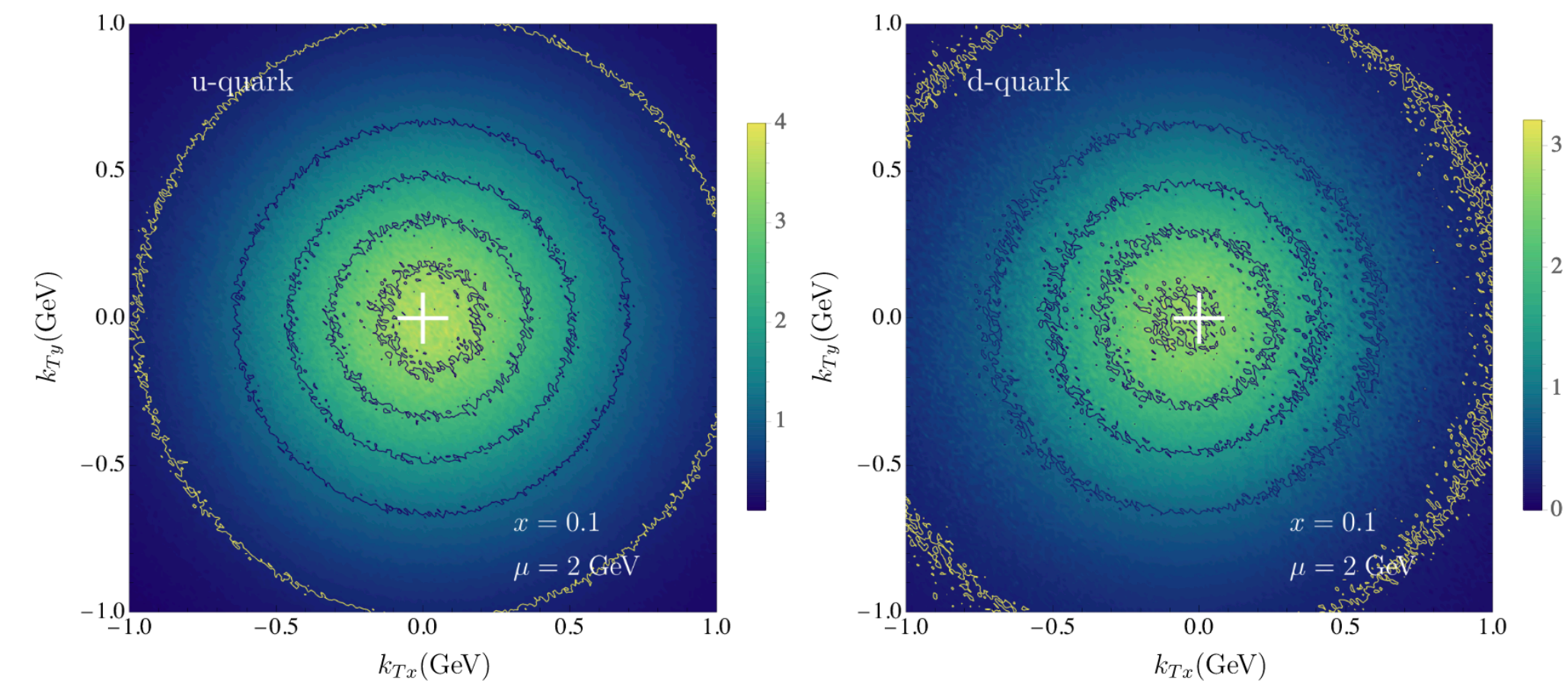
- Sivers *M. Bury, et al., PRL 126 (2021); I. P. Fernando, et al., Phys.Rev.D 108 (2023) ...*

- Boer-Mulders *Z. Lu, et al., Phys.Rev.D 81 (2010); X. Liu, et al., Eur.Phys.J.C 81 (2021) ...*

- **Pion TMDs: much less is known about the TMDs of the pion**

- Unpolarized *A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), Phys.Rev.D 107 (2023); P. C. Barry, et al. (JAM) Phys.Rev.D 108 (2023)*

Tomographic scan of the nucleon via quark density function



M. Bury, et al., JHEP 05 (2021)

As the lightest pseudo Nambu-Goldstone boson, the 3D structure of pion will help us understand the strong interaction, such as the origin of chiral-symmetry breaking.

Lattice QCD Calculation of TMDs

- As a first-principles non-perturbative method, Lattice QCD provides independent predictions of TMDs.

- Mellin Moments *B. Yoon, et al., 1601.05717; B. Yoon, et al., Phys. Rev. D 96 (2017)...*

- Large Momentum Effective Theory (LaMET)

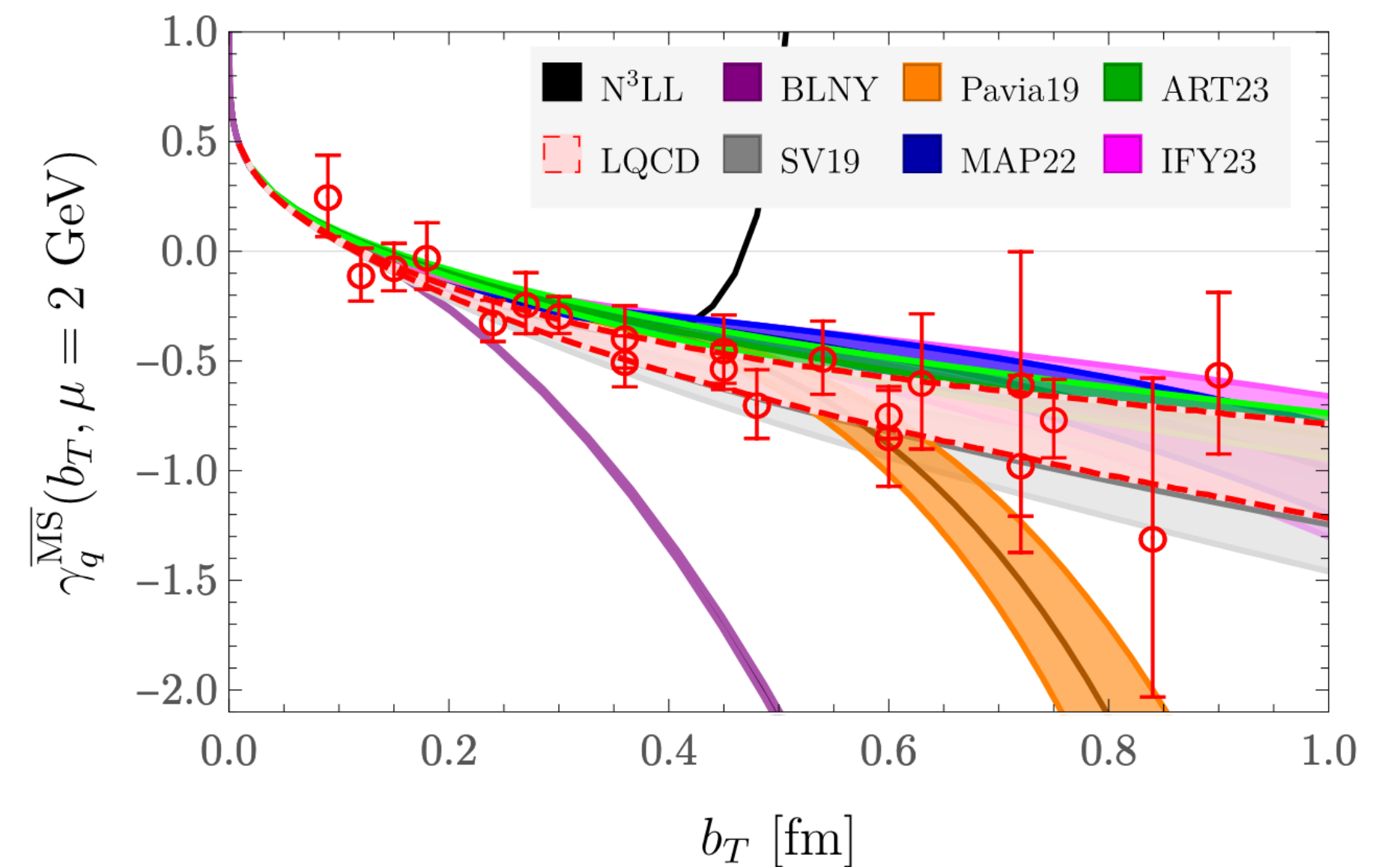
- CS kernel *M. H. Chu, et al. (LPC), JHEP 08 (2023); A. Avkhadiev et al., PRL 132 (2024); D. Bollweg, et al., Phys. Lett. B 852 (2024) ...*

- Intrinsic soft function *Q. A. Zhang, et al. (LPC), PRL 125 (2020); M. H. Chu, et al. (LPC), JHEP 08 (2023)*

- Unpolarized *JH, et al. (LPC), Phys.Rev.D 109 (2024)*

- Helicity *D. Bollweg, X. Gao, S. Mukherjee and Y. Zhao, 2505.18430*

- Boer-Mulders *L. Walter, et al. (LPC), 2412.19988; L. Ma, et al. (LPC), 2502.11807*



A. Avkhadiev et al., PRL 132 (2024)

Large-Momentum Effective Theory (LaMET)

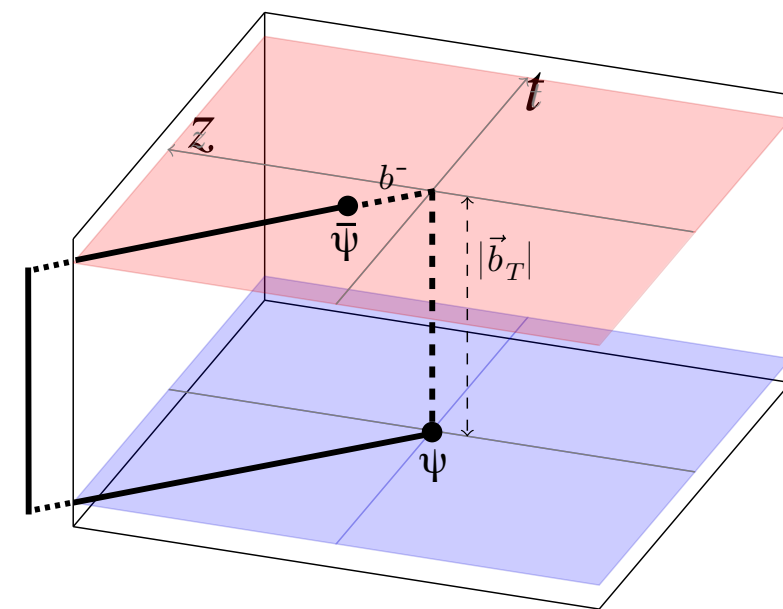
- TMDPDF is defined from a light-cone correlator in a hadron, which is Lorentz invariant.

$$f(x, b_{\perp}, \dots) = \int_{-\infty}^{\infty} \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \left\langle P \left| \bar{\psi}(b^{\mu}) W_{\square}(b^{\mu}, 0) \frac{\gamma^+}{2} \psi(0) \right| P \right\rangle \longleftrightarrow \left\langle \left| \vec{P} \right| = \infty \left| O(t=0) \right| \left| \vec{P} \right| = \infty \right\rangle$$

Parton model

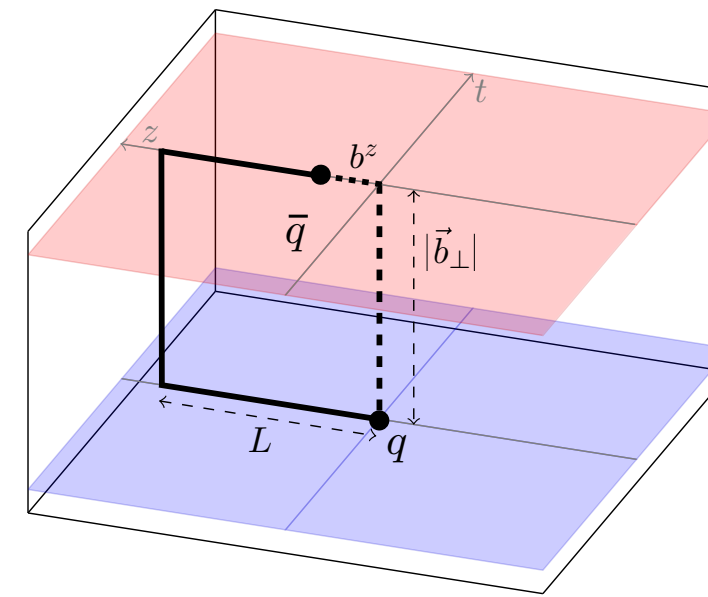
- Define a quasi distribution with large-momentum states and time-independent operators.

$$\tilde{f}_{\Gamma}^0(x, b_{\perp}, P^z, \mu) = P^z \int \frac{dz}{2\pi} e^{iz(xP^z)} \frac{1}{2P^t} \langle P | \bar{\psi}_0(z, b_{\perp}) W_{\square}(z, b_{\perp}; 0) \Gamma \psi_0(0) | P \rangle, \quad \Lambda_{\text{QCD}} \ll |\vec{P}| \ll \frac{\pi}{a}$$



X. Ji, *Phys.Rev.Lett.* 110 (2013)
X. Ji, et al., *Rev.Mod.Phys.* 93 (2021)
X. Ji, *Nucl. Phys. B* 1007 (2024)

← Lorentz boost & Matching



Different orders of limit, but pert.

Large-momentum expansion

Light-cone distribution:

Critical point with infinite correlation length
Cannot be directly calculated on the lattice

Quasi distribution:

Directly calculable on the lattice

$H_f(x, P^z; \mu) = |C_{\text{TMD}}(xP^z; \mu)|^2$ is the TMD hard kernel for matching.

- LaMET enables us to obtain the precision-controlled x-distribution of TMDs in $x \in [x_{\min}, x_{\max}]$.

$$\sqrt{S_I(b_{\perp}; \mu)} \cdot \tilde{f}_{\Gamma}(x, b_{\perp}, P^z; \mu) = f(x, b_{\perp}; \mu, \zeta) H_f(x, P^z; \mu) \exp \left[\frac{1}{2} \ln \frac{(2xP^z)^2}{\zeta} \gamma^{\overline{\text{MS}}}(b_{\perp}; \mu) \right] + \text{Power corrections}$$

Collins-Soper scale: $\zeta \sim 2(xP^+)^2$

Pion Form Factor

- The form factor is defined as

$$F(b_{\perp}, P^z, \Gamma) \equiv -4N_c \frac{\langle -P^z | \bar{q}(b_{\perp}) \Gamma q(b_{\perp}) \bar{q}(0) \Gamma q(0) | P^z \rangle}{f_{\pi}^2 ((P^t)^2 + (P^z)^2)}$$

We choose $\Gamma \in \{\gamma^{\perp}, \gamma^{\perp} \gamma^5\}$ to get leading-twist contribution, then take the Fierz rearrangement.

$$F(b_{\perp}, P^z) = \frac{1}{4} [F(b_{\perp}, P^z, \Gamma = \gamma^x \gamma^5) + F(b_{\perp}, P^z, \Gamma = \gamma^y \gamma^5) + F(b_{\perp}, P^z, \Gamma = \gamma^x) + F(b_{\perp}, P^z, \Gamma = \gamma^y)]$$

- It can be extracted from the ratio

$$R_F(b_{\perp}, P^z, \Gamma) \equiv -4N_c \frac{\langle -P^z | \bar{q}(b_{\perp}) \Gamma q(b_{\perp}) \bar{q}(0) \Gamma q(0) | P^z \rangle}{\langle 0 | \bar{q}(0) \gamma^{\mu} \gamma^5 q(0) | P^z \rangle \langle -P^z | \bar{q}(0) \gamma_{\mu} \gamma^5 q(0) | 0 \rangle}$$

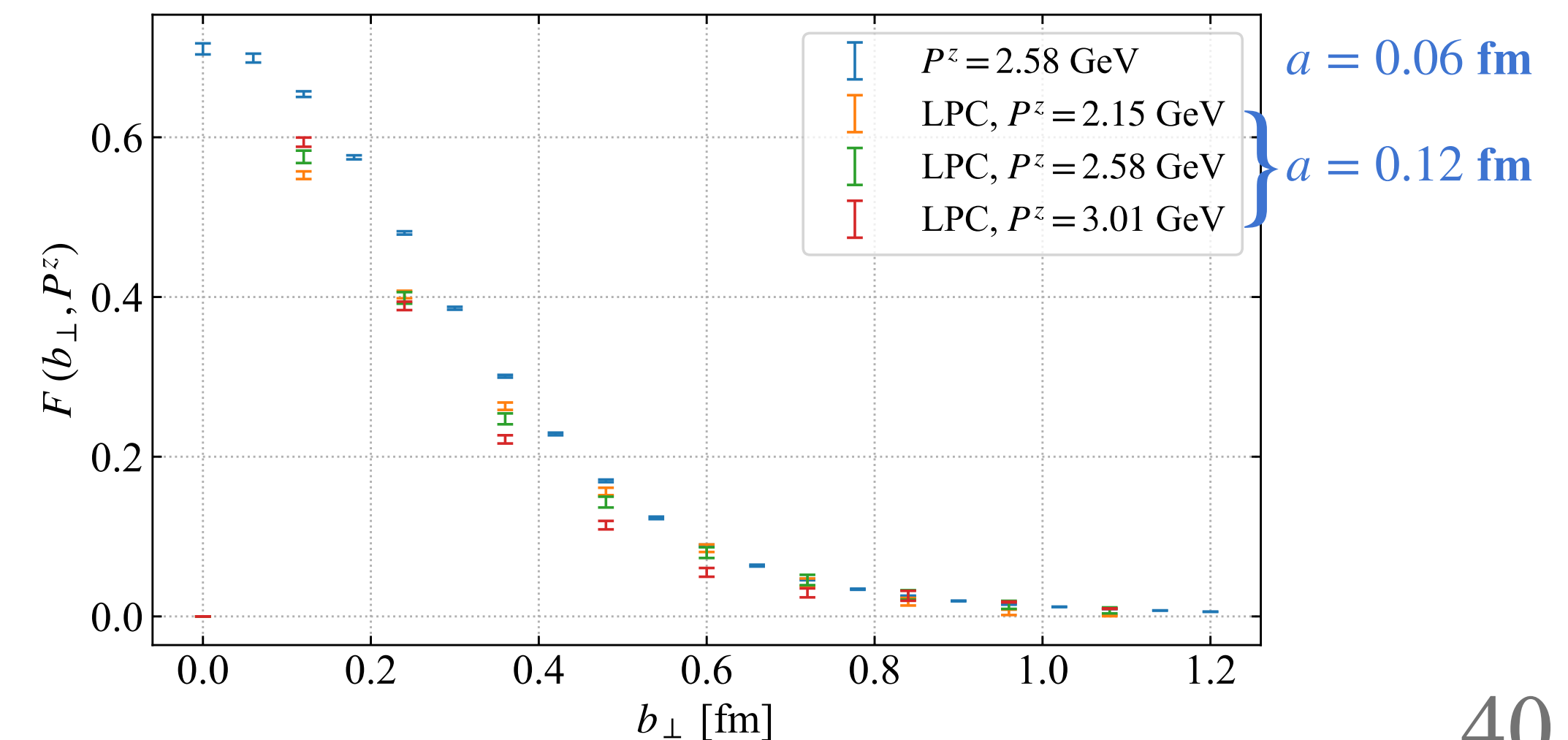
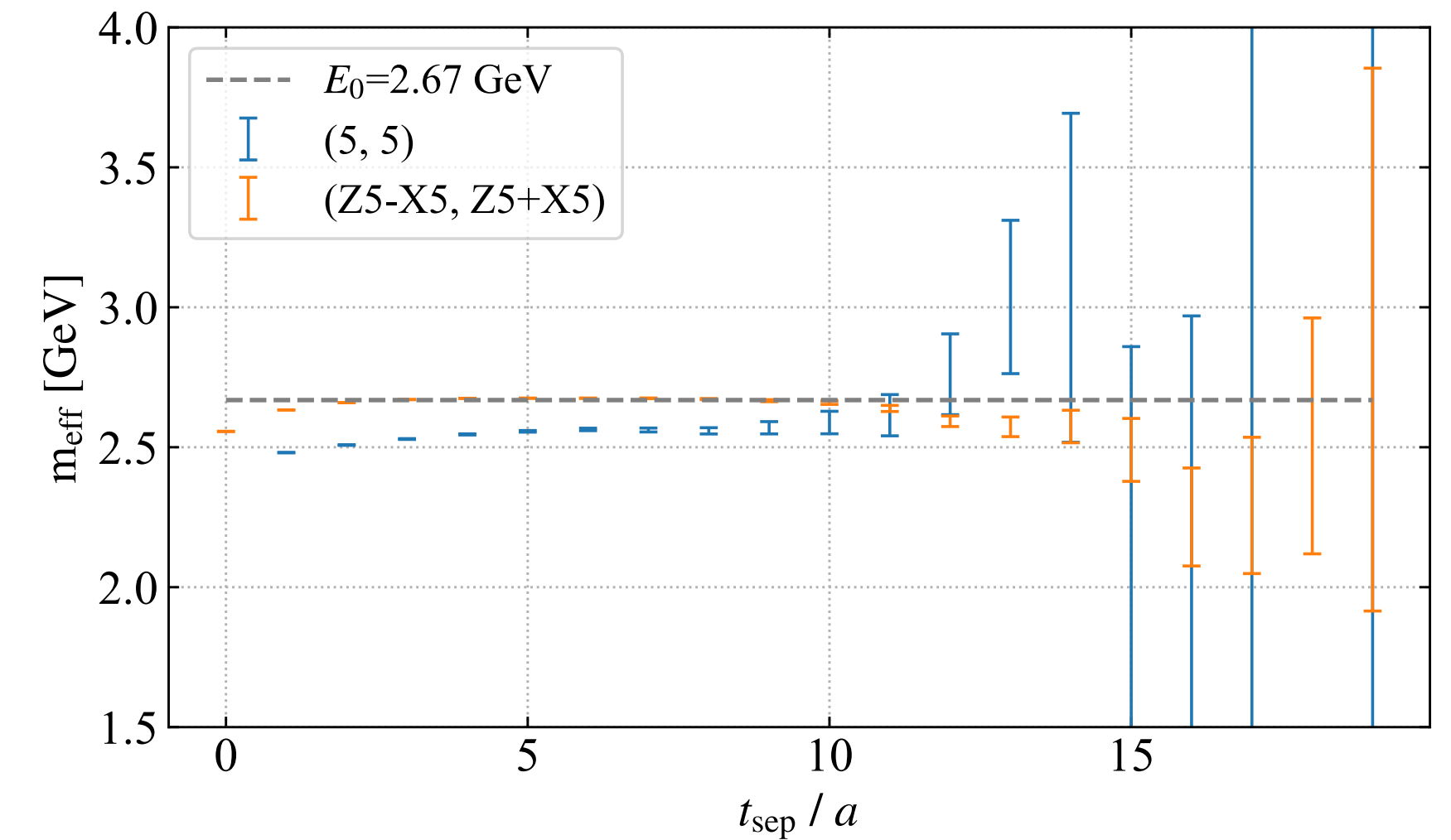
- The ratio in terms of correlators on lattice

$$R_F(t_{\text{sep}}, \tau) = \frac{-4N_c}{1 + (P^t/P^z)^2} \frac{C_F(t_{\text{sep}}, \tau)}{\left| C_{2\text{pt}}(t_{\text{sep}}/2) \right|^2}$$

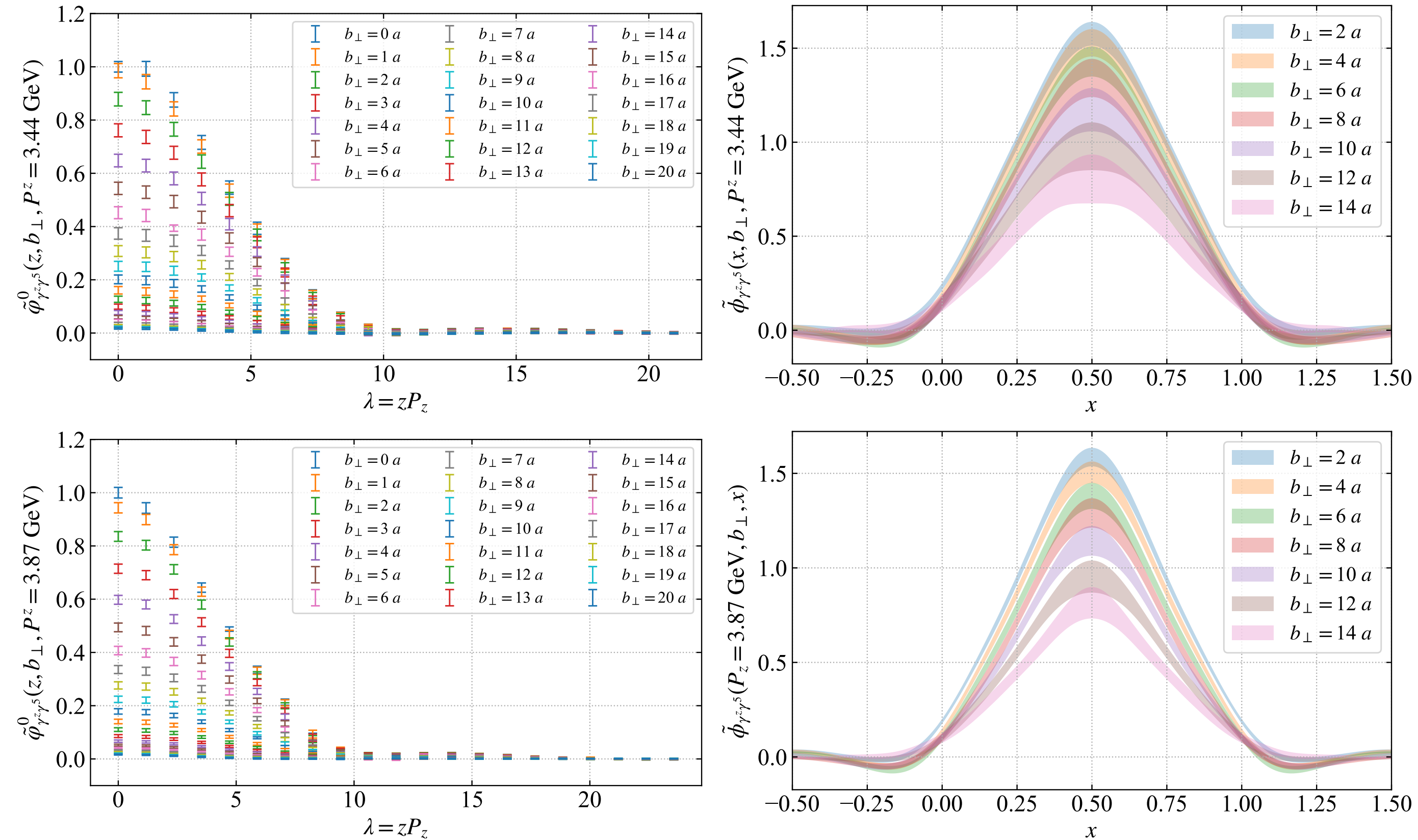
The 2pt is calculated using the new interpolating operator.

Kinematically-enhanced interpolating operator

R. Zhang, et al., Phys. Rev. D 112 (2025)

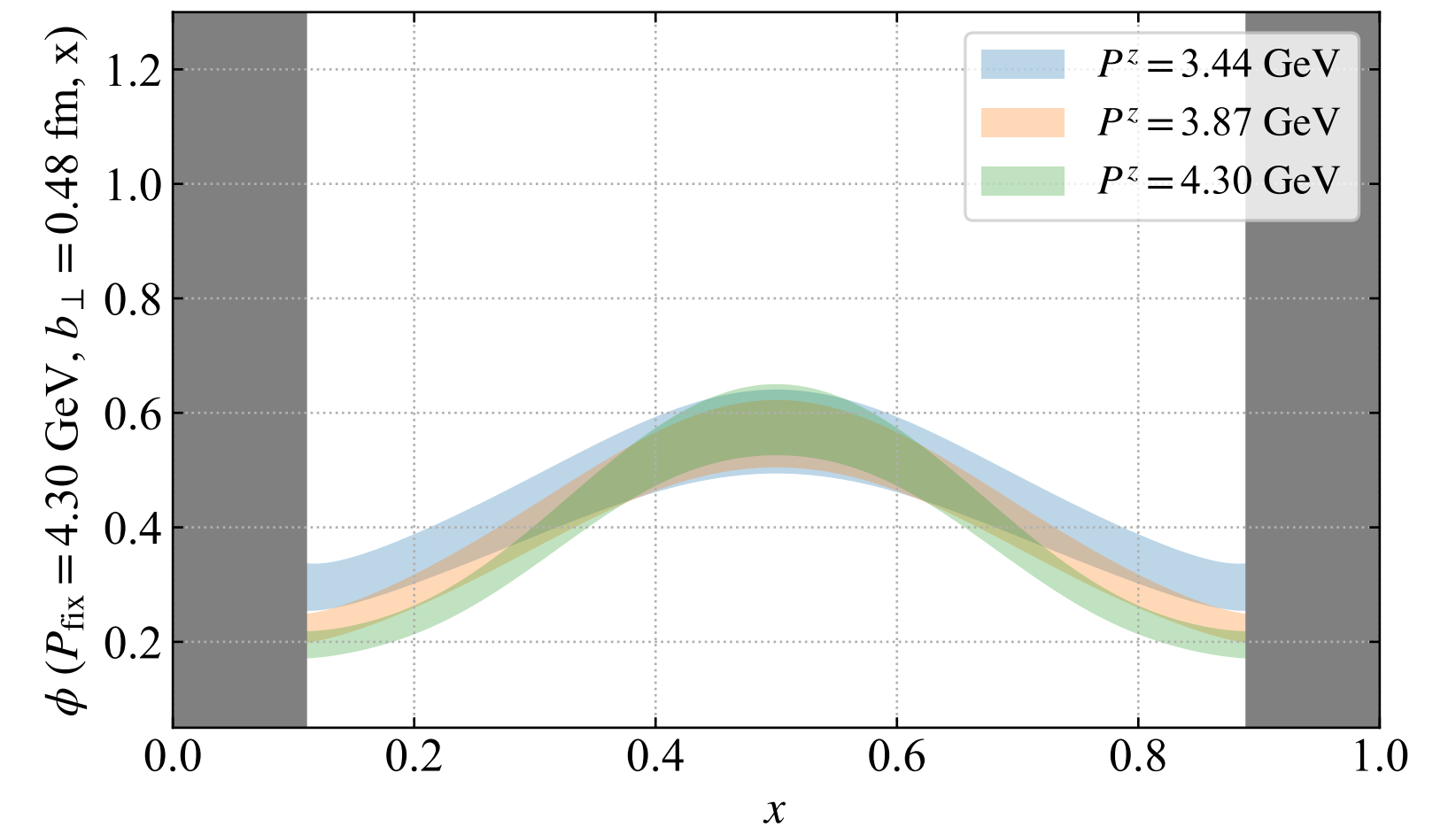
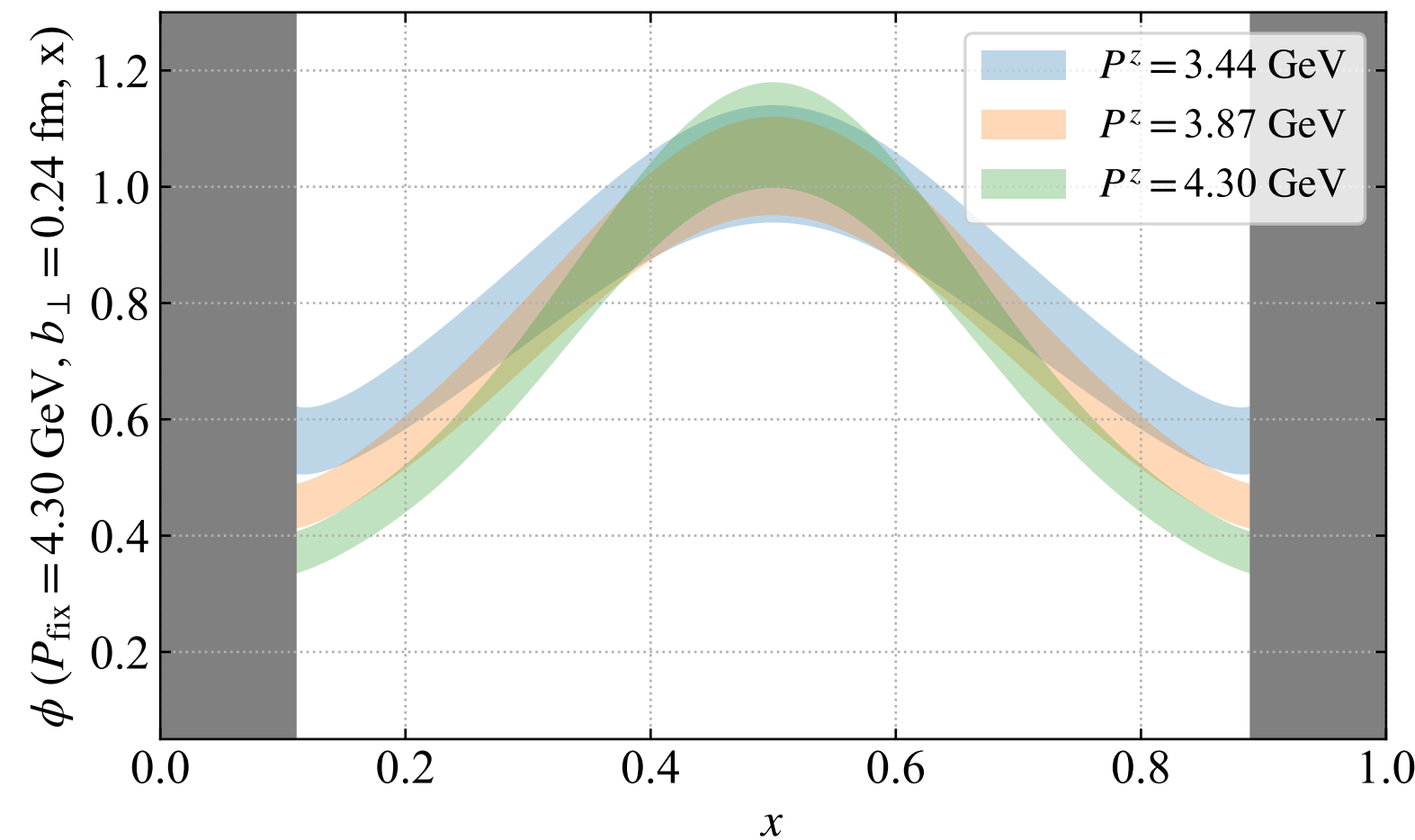
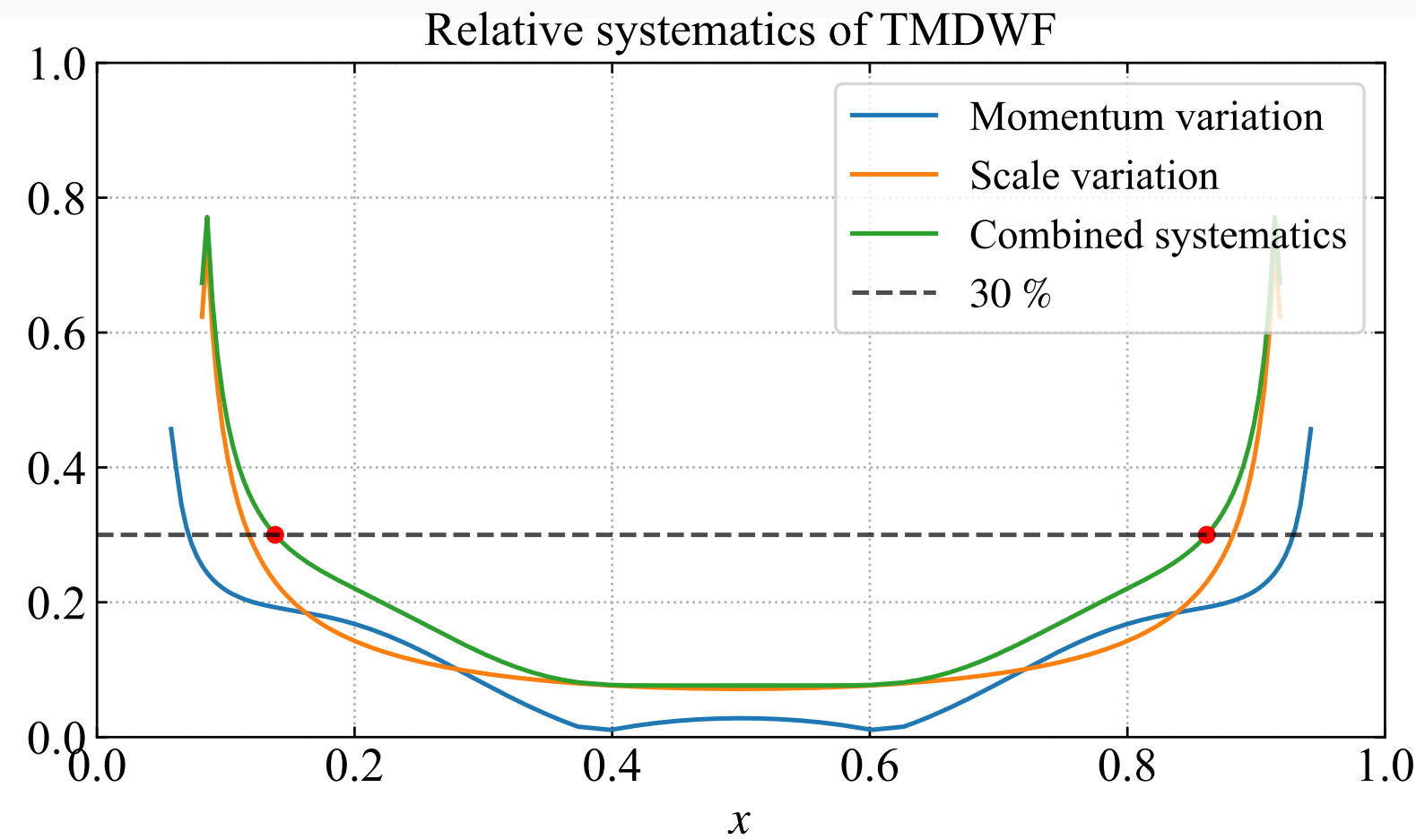


Pion quasi-TMD Wave Function



- We did a discretized Fourier transform because of the good convergence in the λ -space;
- The CG matrix elements decay to zero with the error bars remain almost constant, making the FT easy to be under control.

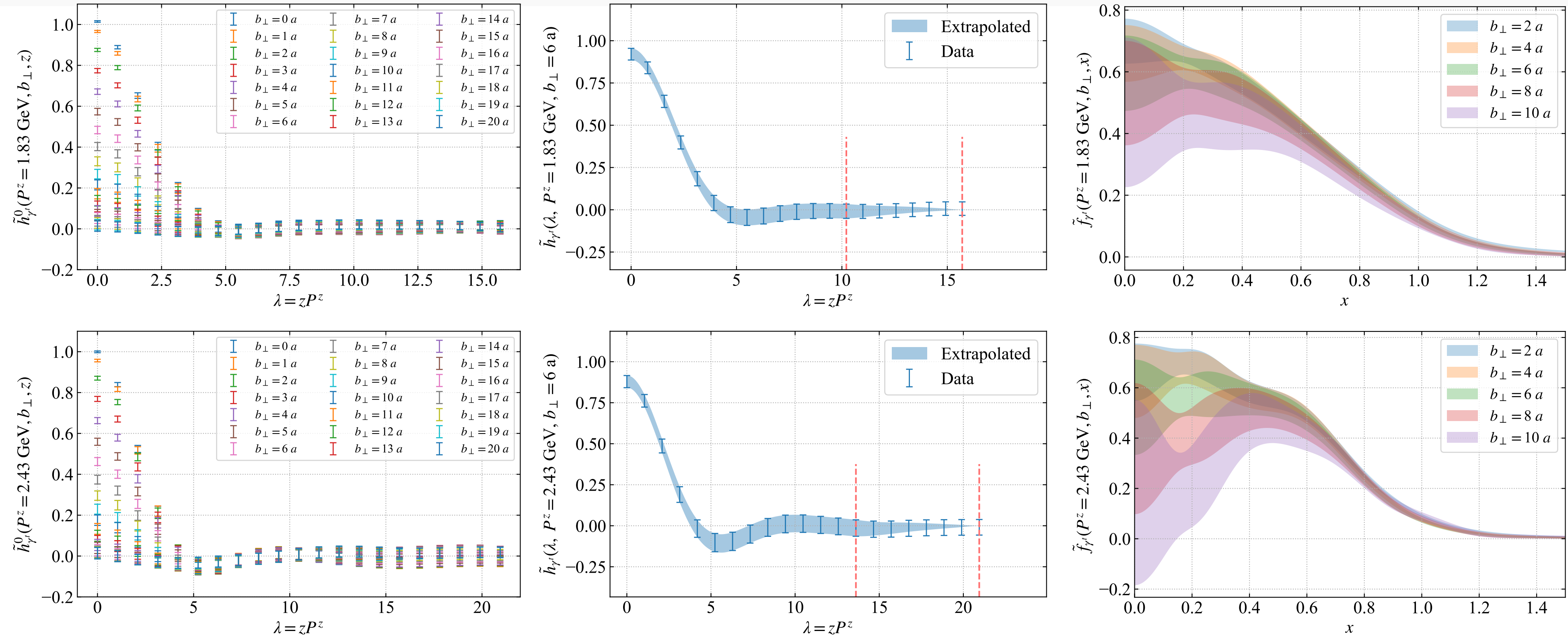
Pion TMD Wave Function



$$\sqrt{S_I(b_\perp; \mu)} \cdot \tilde{\phi}_\Gamma(x, b_\perp, P^z; \mu) = \phi(x, b_\perp; \mu, \zeta, \bar{\zeta}) H_\phi(x, \bar{x}, P^z; \mu) \exp \left[\frac{1}{2} \ln \frac{(2xP^z)^2}{\zeta} \gamma^{\overline{\text{MS}}}(b_\perp; \mu) \right] + \text{Power corrections}$$

- The variation between different momenta remains mild in the moderate x region, demonstrating the validity of power expansion in large P^z ;
- The combined systematics are estimated from two sources:
 - Momentum variation: spread of central values between three momenta / mean of central values of three momenta
 - Vary the initial scale in the RG resummation of matching kernel by a factor of $\sqrt{2}$;
- The 30 % combined systematics are used to quantify the moderate x region that LaMET can make reliable predictions;
- The convergence between three momenta near the endpoint regions can be improved with larger Lorentz boost factor.

Pion quasi-TMD Beam Function

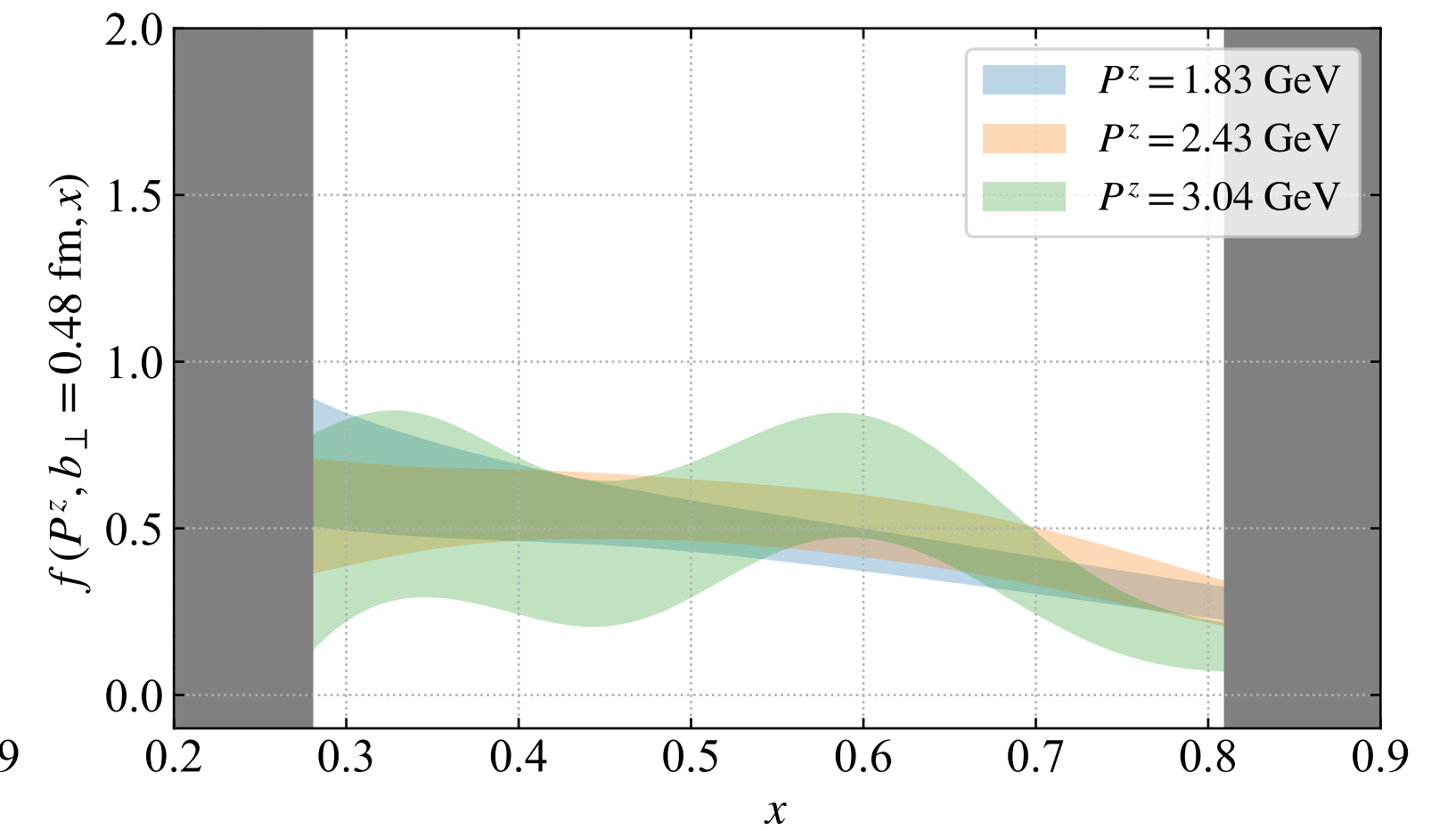
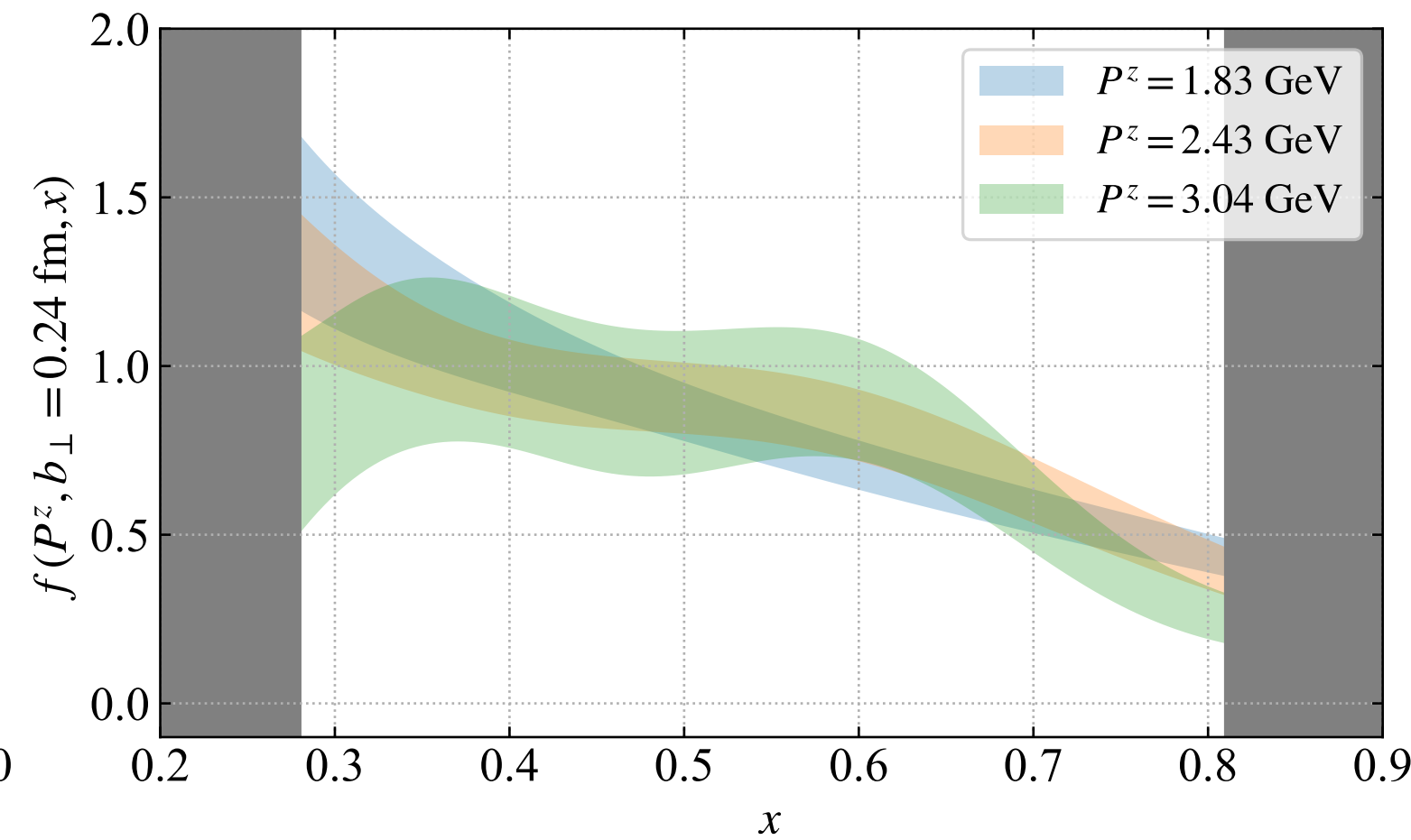
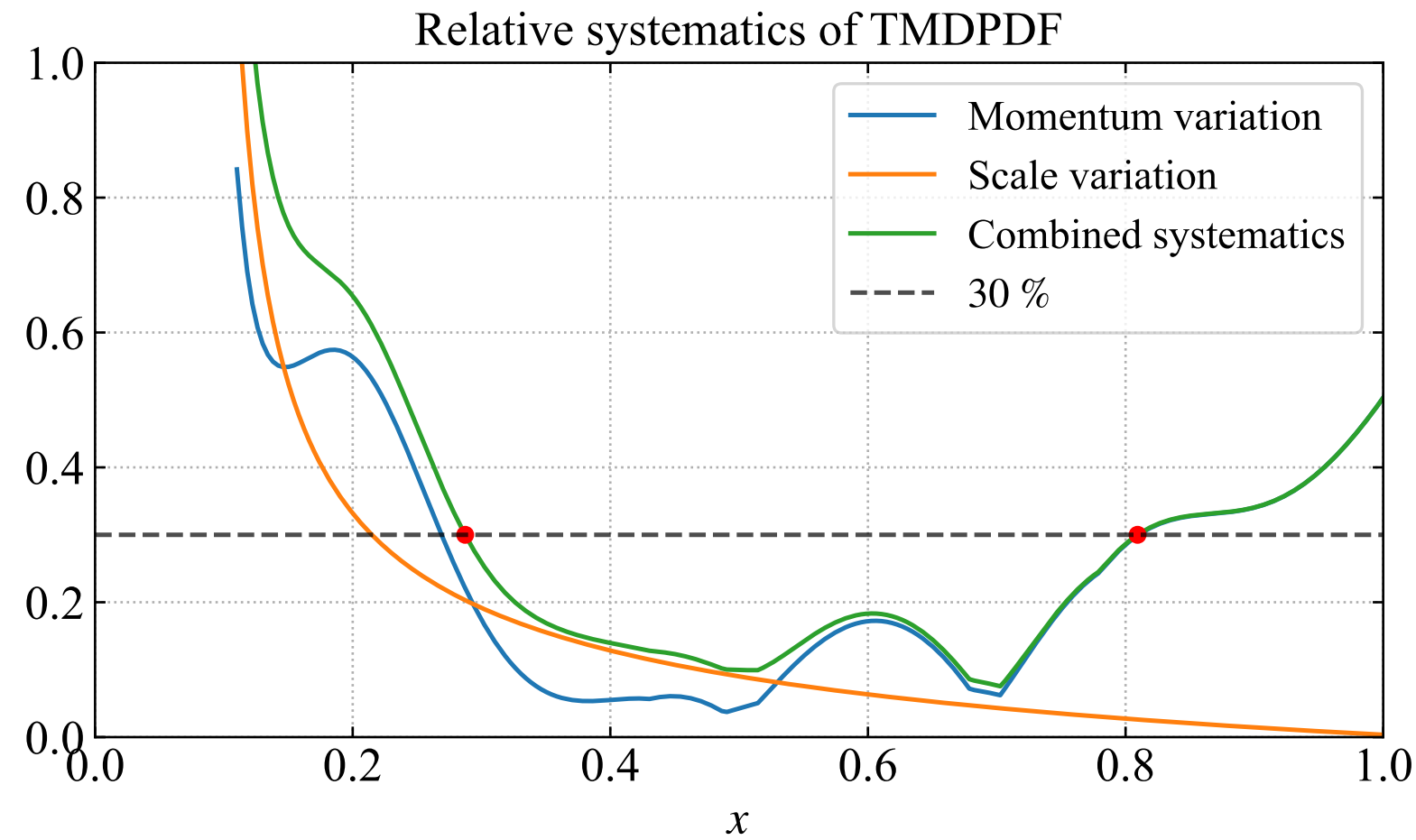


- The CG matrix elements decay to zero with the error bars remain almost constant, making the FT easy to be under control.
- To remove the non-physical oscillation, we apply the extrapolation to make error bars converge to zero smoothly.
- Since quasi-TMD (in moderate x) is insensitive to the extrapolation strategies, the non-fit extrapolation is adopted here:

A recent paper on asymptotic analysis in LaMET: *J. W. Chen, et al., 2505.14619*

$\tilde{h}^{\text{ext}} = w \cdot \tilde{h} + (1 - w) \cdot 0$, where the weight $w(z)$ linearly decays from 1 to 0 within two red dashed lines below.

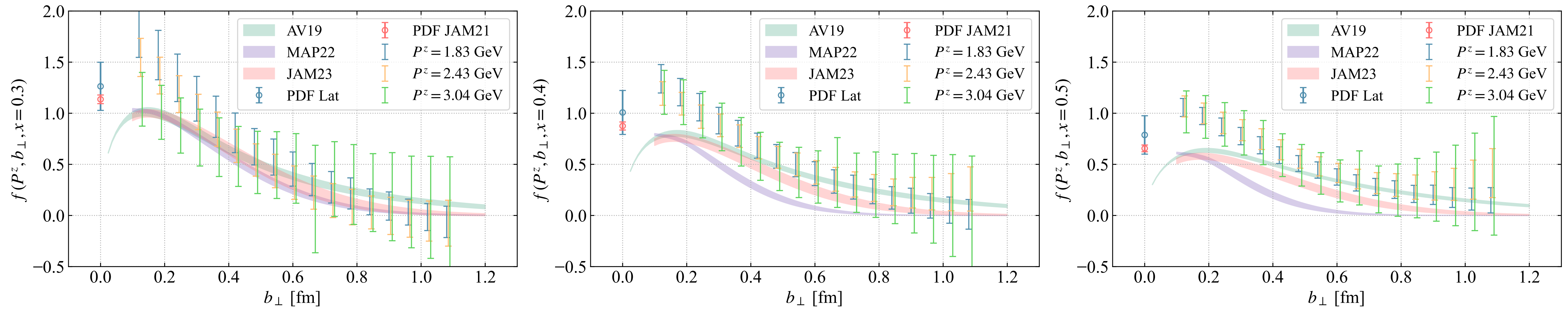
Pion TMDPDF in the x Space



$$\sqrt{S_I(b_{\perp}; \mu)} \cdot \tilde{f}_{\Gamma}(x, b_{\perp}, P^z; \mu) = f(x, b_{\perp}; \mu, \zeta) H_f(x, P^z; \mu) \exp \left[\frac{1}{2} \ln \frac{(2xP^z)^2}{\zeta} \gamma^{\overline{\text{MS}}}(b_{\perp}; \mu) \right] + \text{Power corrections}$$

- The variation between different momenta remains mild in the moderate x region, demonstrating the validity of power expansion in large P^z ;
- The combined systematics are estimated from two sources:
 - Momentum variation: spread of central values between three momenta / mean of central values of three momenta
 - Vary the initial scale in the RG resummation of matching kernel by a factor of $\sqrt{2}$;
- The 30 % combined systematics are used to quantify the moderate x region that LaMET can make reliable predictions;

Pion TMDPDF in the $b_\perp$ Space



A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), Phys.Rev.D 107 (2023); P. C. Barry, et al. (JAM) Phys.Rev.D 108 (2023)

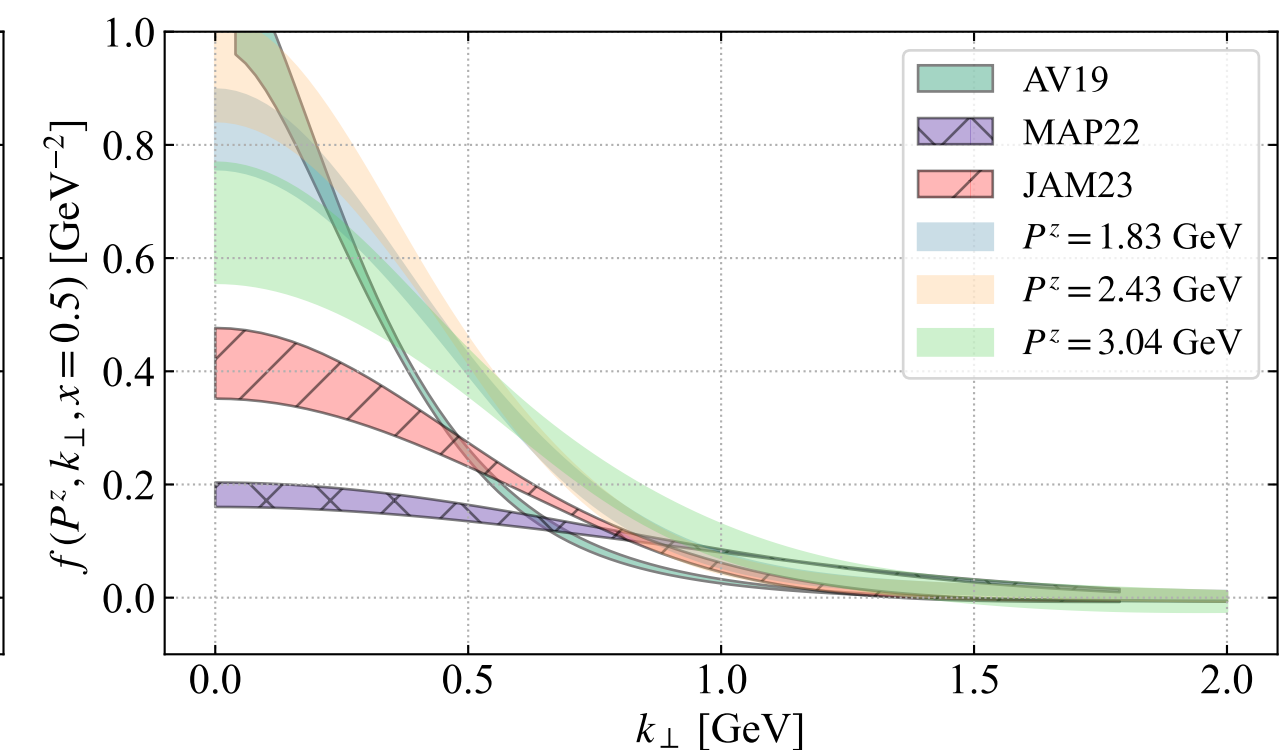
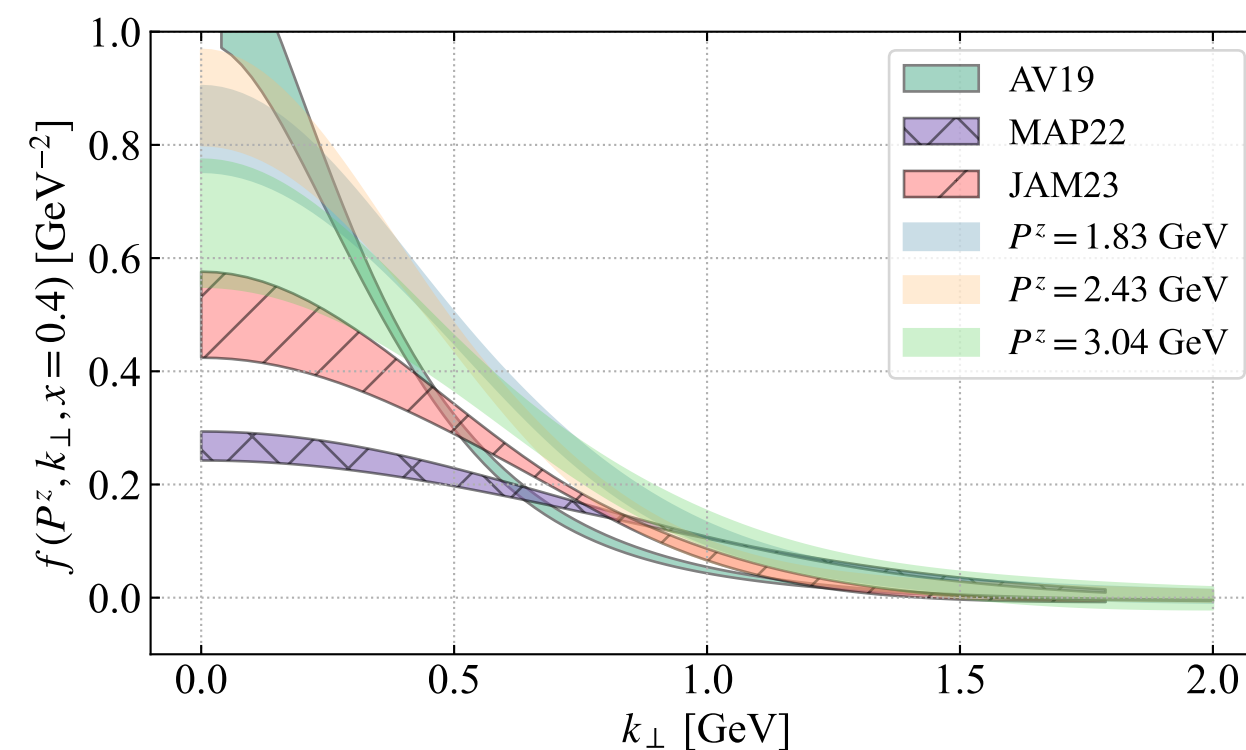
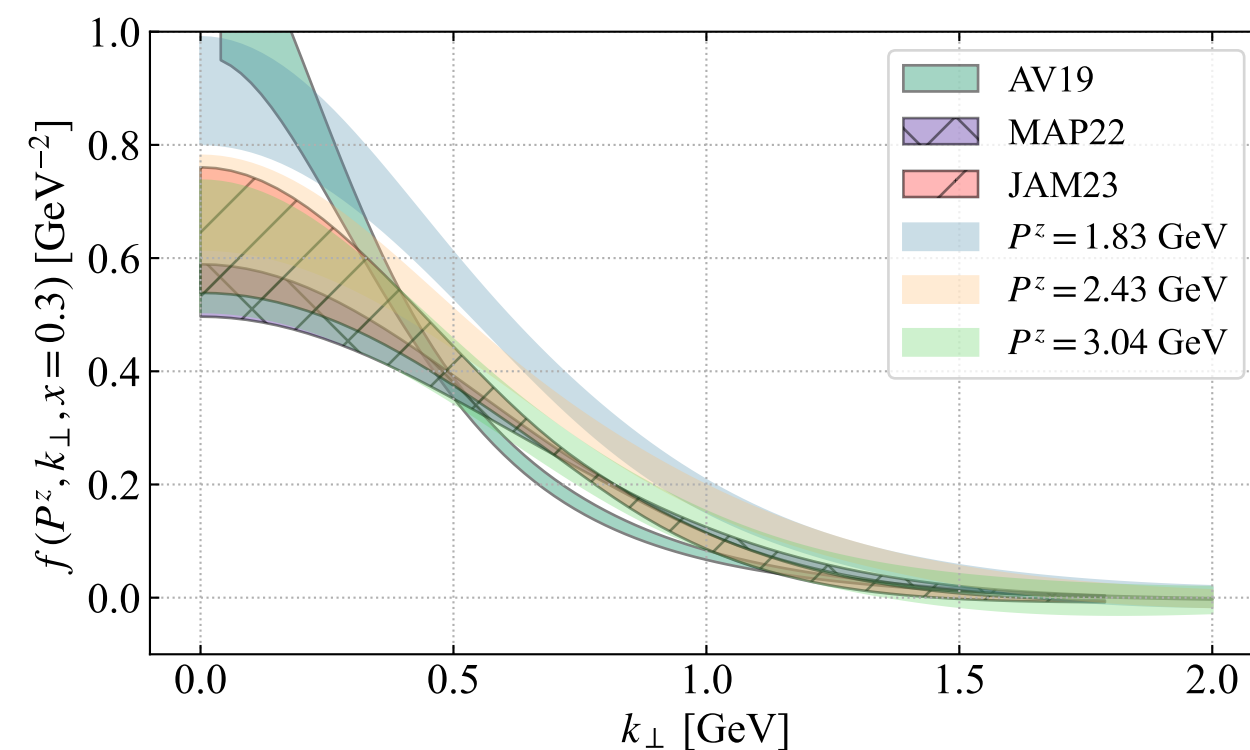
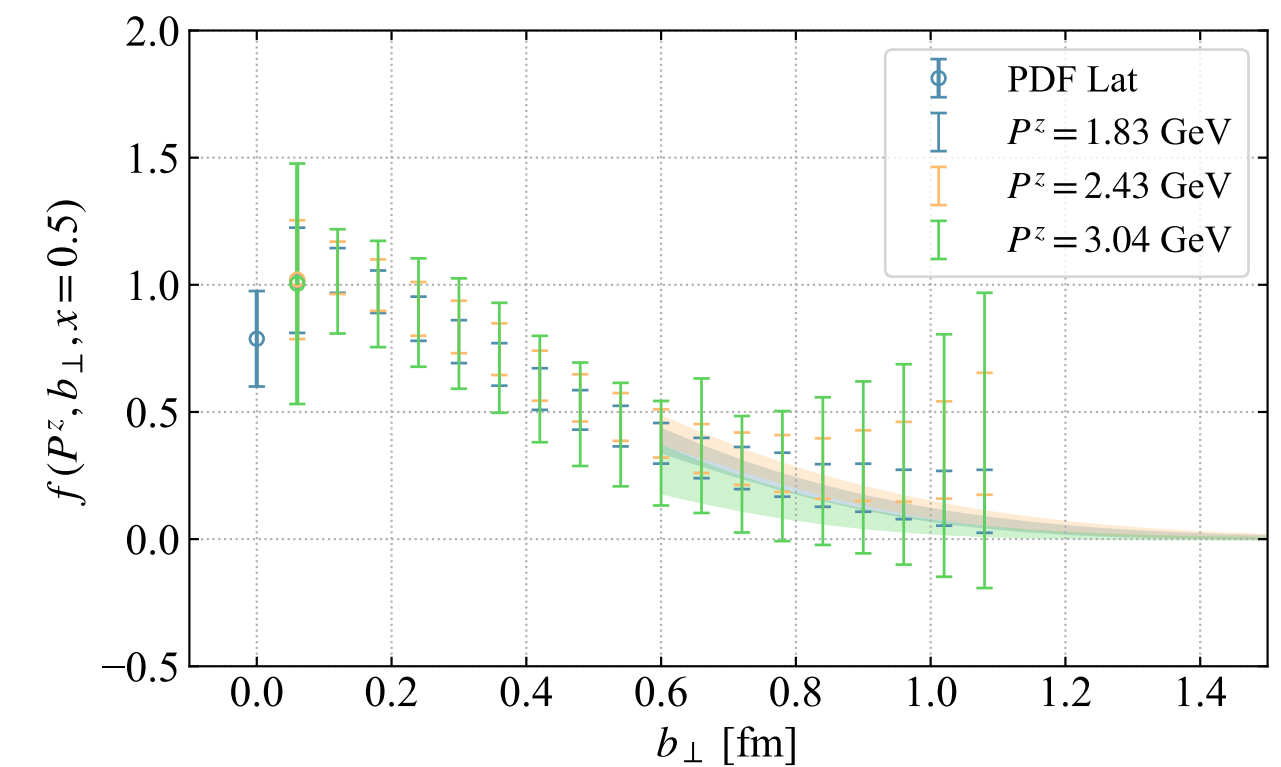
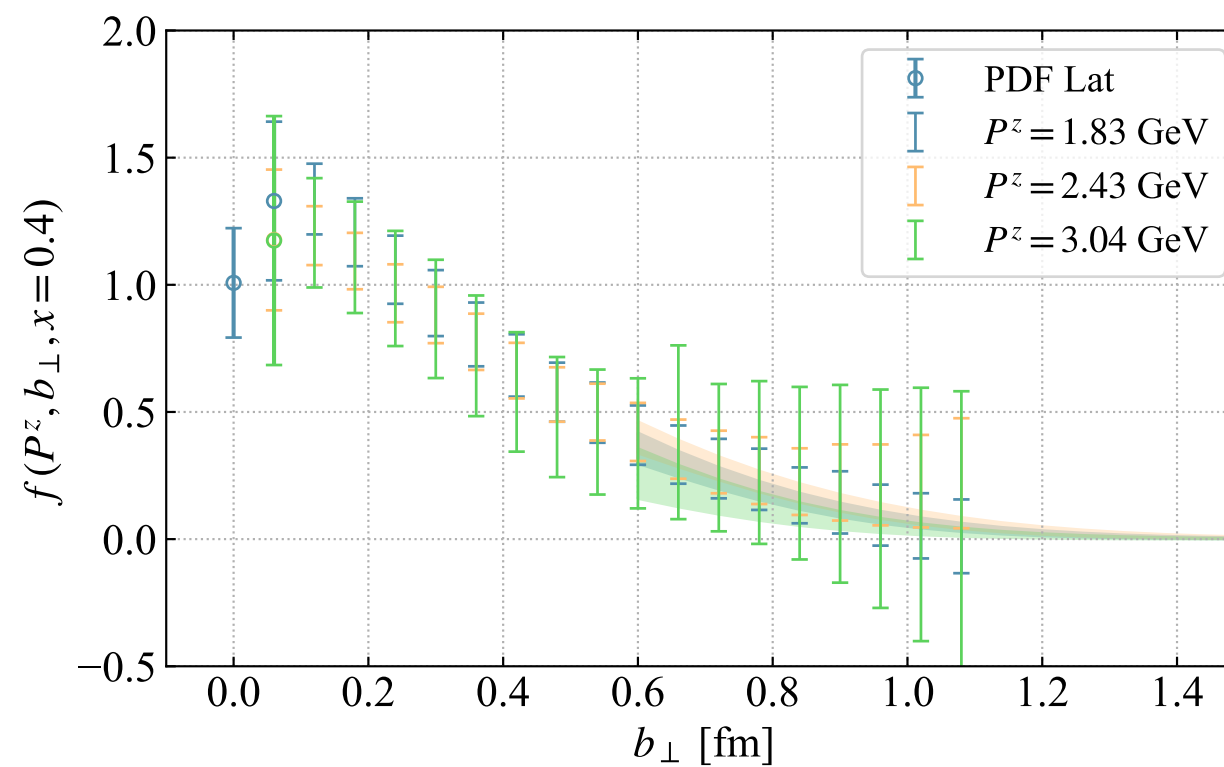
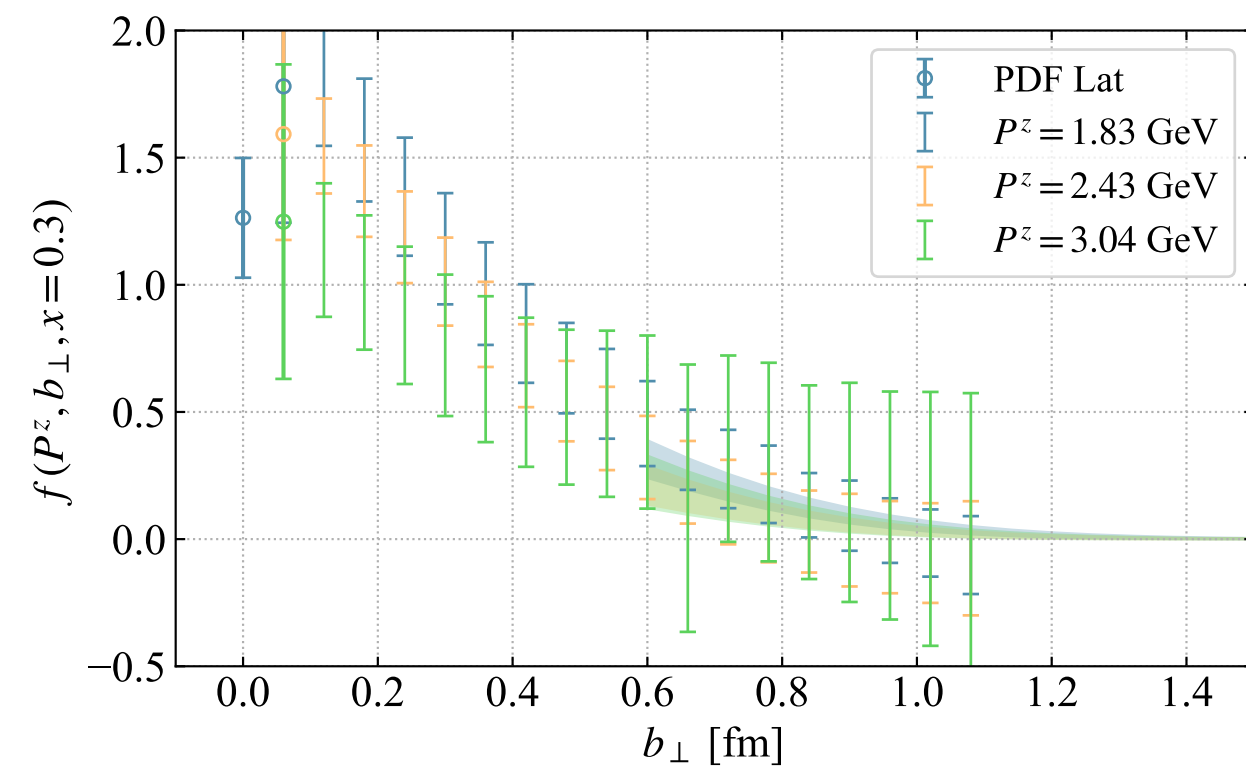
- Thanks to the absence of linear divergence, we can calculate pion TMDPDF up to $b_\perp > 1$ fm;
- When x gets larger, the amplitude of TMDPDF is decreasing, while the the transverse correlation length stays roughly the same;
- When x gets closer to $x = 0.5$, we can find that the variance across different momenta becomes smaller, indicating the suppression of power correction;
- Global analysis has a better control at relative small , where we saw consistency;
- Lattice provides predictions at relative large , where experimental data gives less constraints.

Pion TMDPDF in the $k_{\perp}$ Space

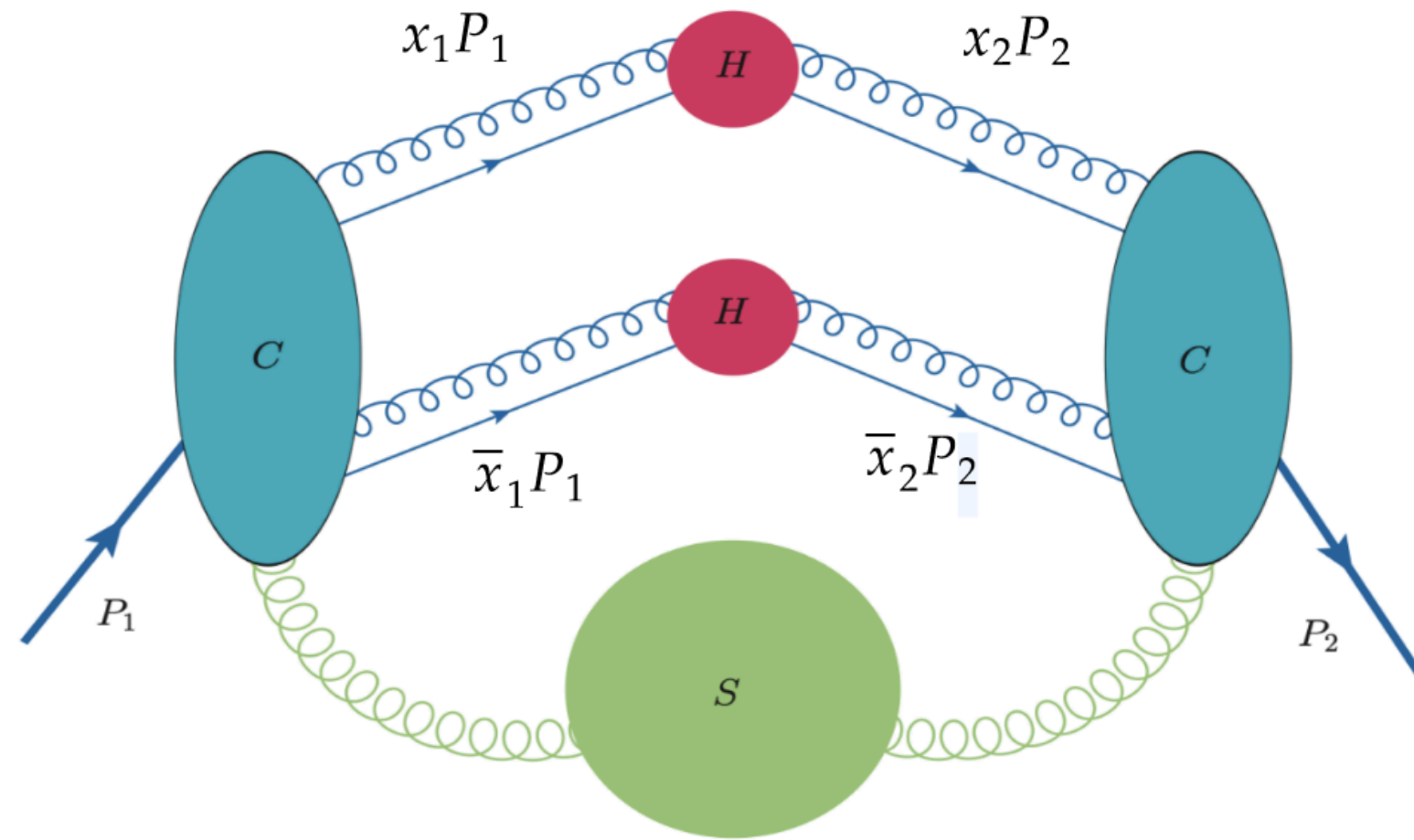
- We can give the $k_{\perp}$ -dependence thanks to the good SNR in CG;

- Extrapolate the large $b_{\perp}$ using a simple Gaussian form: $f(b_{\perp}) = Ae^{-mb_{\perp}^2}$;

- Fourier transform to the $k_{\perp}$ space: $\tilde{f}(k_{\perp}) = \int \frac{d^2\vec{b}_{\perp}}{(2\pi)^2} e^{i\vec{b}_{\perp} \cdot \vec{k}_{\perp}} f(b_{\perp}) = \int \frac{d|b_{\perp}|}{2\pi} |b_{\perp}| \cdot J_0(|b_{\perp}| \cdot |k_{\perp}|) \cdot f(b_{\perp})$.



Sudakov Kernel



Z. F. Deng, et al., JHEP 09 (2022)

$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 H_F(Q^2, \bar{Q}^2, \mu^2) \times \left[\frac{\psi_{\bar{q}q}^\pm(x_2, b_\perp, \mu, \delta'^+)}{\sqrt{S^\pm(b_\perp, \mu, \delta'^+, \delta^-)}} \right]^\dagger \left[\frac{\psi_{\bar{q}q}^\pm(x_1, b_\perp, \mu, \delta'^-)}{\sqrt{S^\pm(b_\perp, \mu, \delta'^+, \delta^-)}} \right] \times \frac{S^\pm(b_\perp, \mu, \delta'^+, \delta^-)}{\sqrt{S^\pm(b_\perp, \mu, \delta'^+, \delta^-) S^\pm(b_\perp, \mu, \delta'^+, \delta'^-)}} \quad (17)$$

Here $H_F(Q^2, \bar{Q}^2, \mu^2)$ is the hard kernel, $\psi_{\bar{q}q}^\pm$ is the TMDWF, S is the TMD soft function, $Q^2 = x_1 x_2 P_1 \cdot P_2$, $\bar{Q}^2 = \bar{x}_1 \bar{x}_2 P_1 \cdot P_2$. An integral over the momentum fractions x_1, x_2 is assumed.

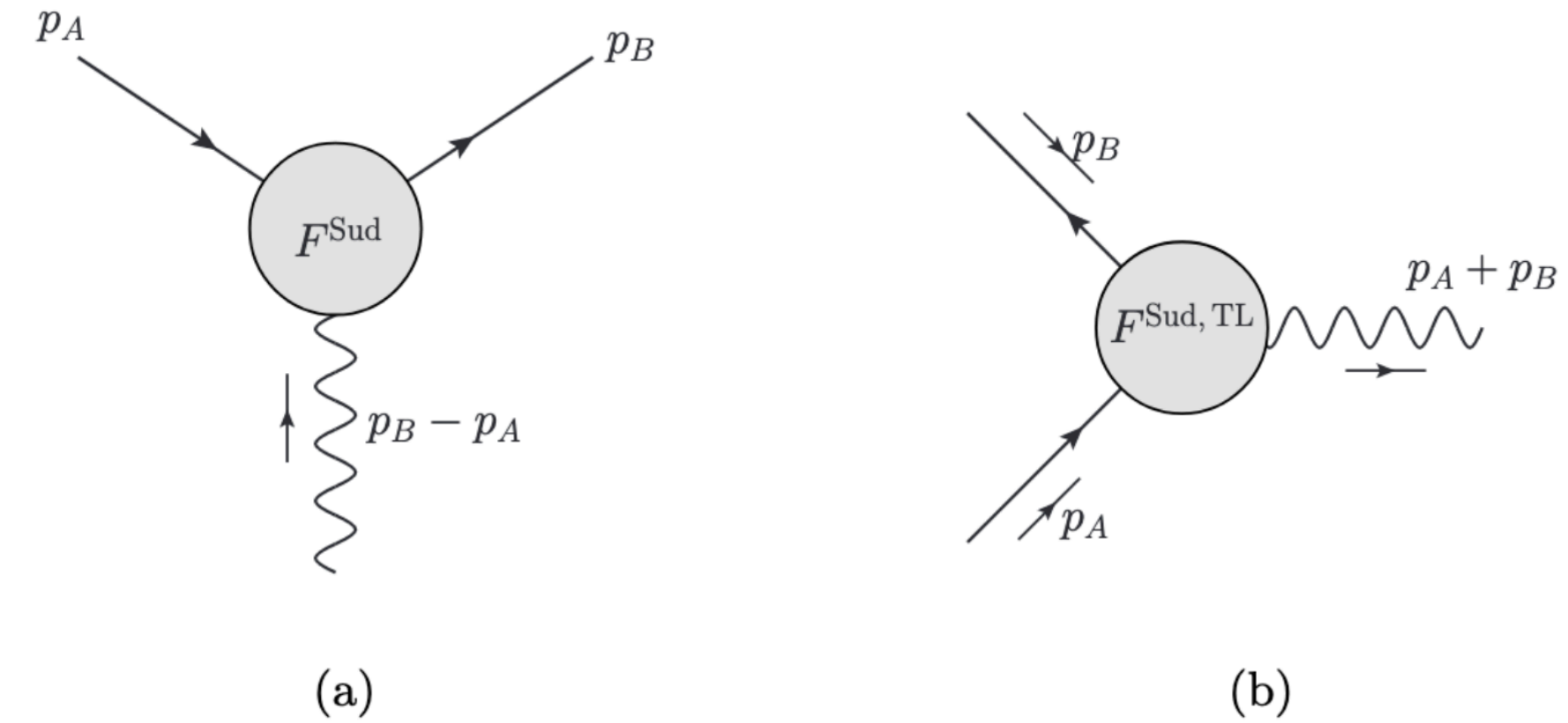


FIG. 1. Graphs for the space-like (a) and time-like (b) quark form factor.

J. Collins and T. C. Rogers, Phys. Rev. D 96 (2017)

$H_F(x_1, x_2, P^z; \mu) = C_{\text{Sud}}(x_1, x_2, P^z; \mu) \cdot C_{\text{Sud}}(\bar{x}_1, \bar{x}_2, P^z; \mu)$, where C_{Sud} is the Sudakov kernel.

$$p_A = (x_1 P_1, 0, 0, x_1 P_1), \quad p_B = (x_2 P_2, 0, 0, -x_2 P_2)$$

$$Q^2 = -(p_B - p_A)^2 = 4x_1 x_2 P_1 P_2$$

$$\bar{Q}^2 = 4\bar{x}_1 \bar{x}_2 P_1 P_2$$

Faddeev-Popov operator

- The existence of Gribov copies is related to the zero mode of the Faddeev-Popov operator

Gauge condition: $\mathcal{F} = \vec{\nabla} \cdot \vec{A} = 0$

Infinitesimal transformation: $\delta \vec{A} = -\vec{D}\omega$

Variation: $\delta \mathcal{F} = \vec{\nabla} \cdot \delta \vec{A} = -\vec{\nabla} \cdot (\vec{D}\omega)$

Faddeev-Popov: $\mathcal{M} \equiv \frac{\delta \mathcal{F}}{\delta \omega} = -\vec{\nabla} \cdot \vec{D}$ For QED, there is no Gribov copy in Landau gauge.

if $\mathcal{M}\theta = 0$, then $\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot (\vec{A} - \vec{D}\theta) \equiv \vec{\nabla} \cdot \vec{A}' = 0$

$$* D_{\mu ab} \omega_b = \partial_\mu \omega_a - g f^{cab} A_\mu^c \omega_b$$

Criteria of Gauge Fixing

- **Variation of the functional**

$$\delta F/F < 10^{-8}$$

- **Residual gradient of the functional**

$$\theta^G \equiv \frac{1}{V} \sum_x \theta^G(x) \equiv \frac{1}{V} \sum_x \text{Tr} \left[\Delta^G(x) (\Delta^G)^\dagger(x) \right], \Delta^G(x) \equiv \sum_\mu \left(A_\mu^G(x) - A_\mu^G(x - \hat{\mu}) \right)$$

