3D Imaging of the Pion on a Fine Lattice

Based on arXiv:2504.04625

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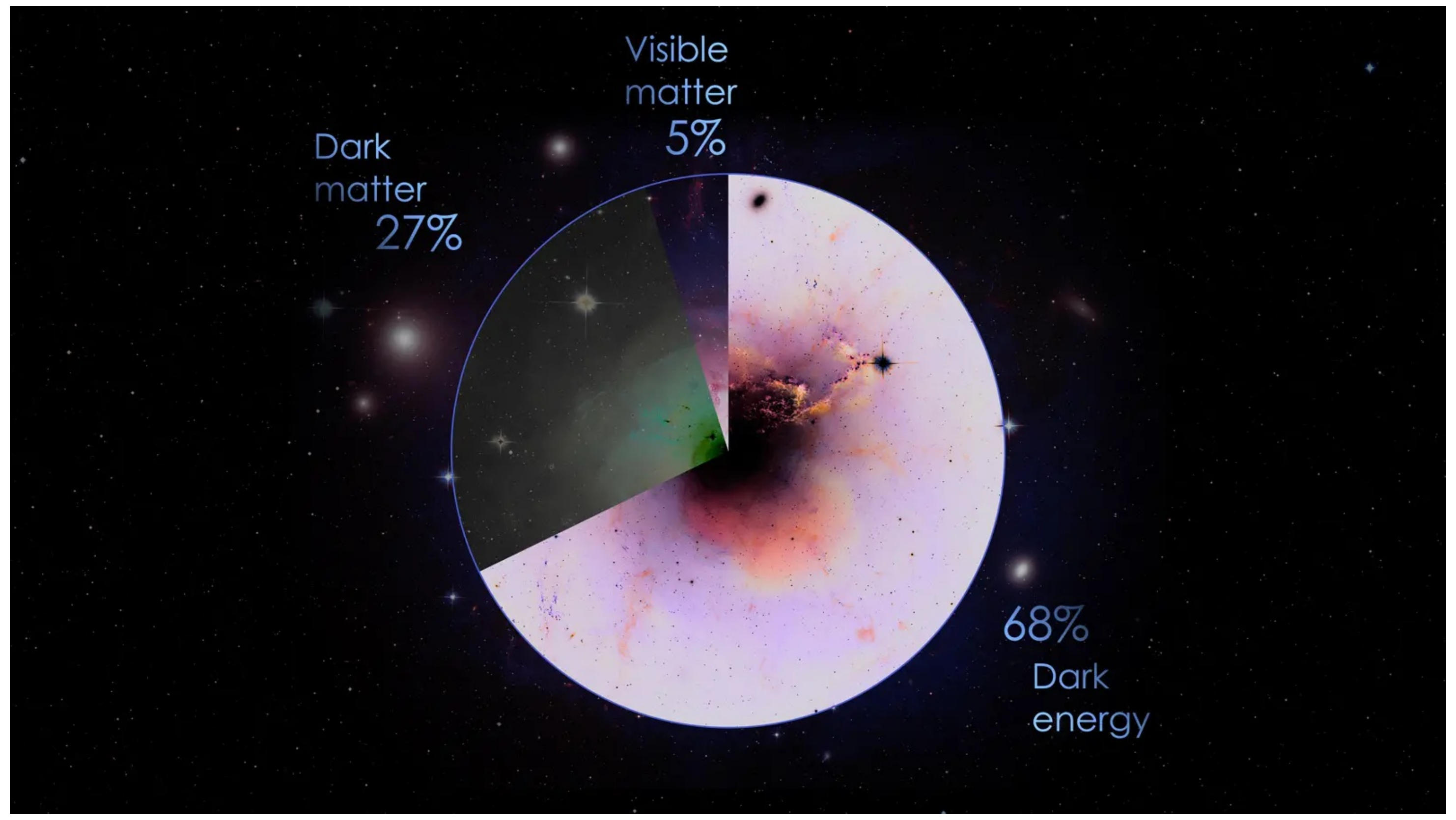
Contents

Introduction

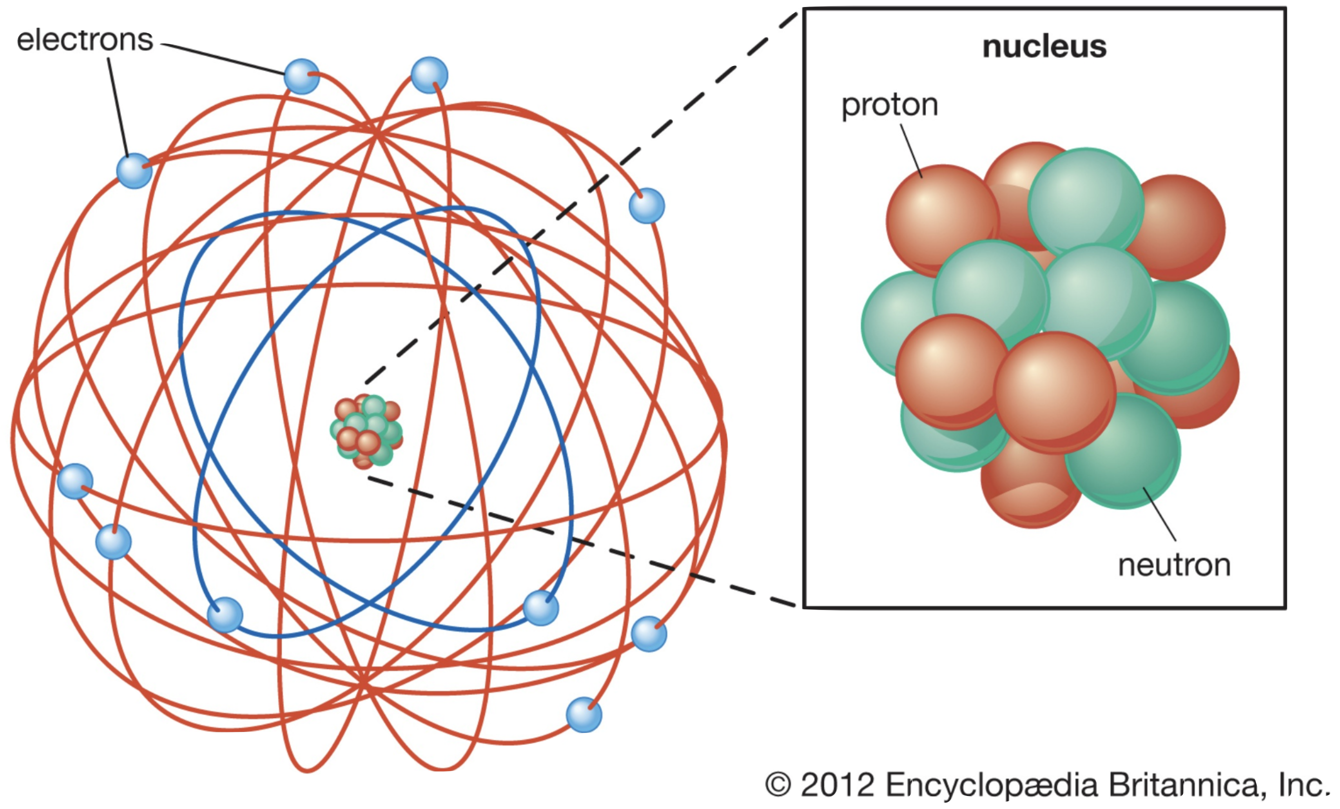
Visible Universe

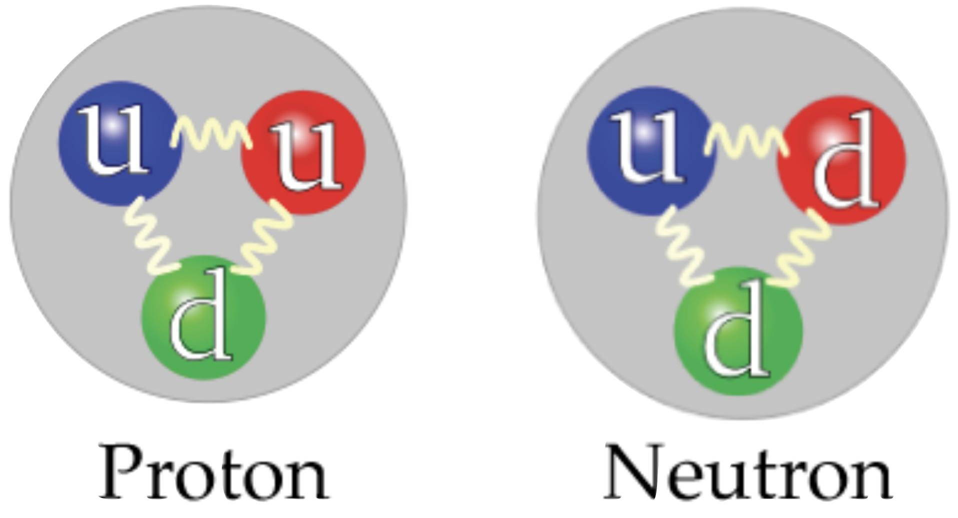
- Only 5% of the universe is visible. *Spergel, David N. "The dark side of cosmology: Dark matter and dark energy." Science 347.6226 (2015): 1100-1102.*

- The visible universe is made up of protons and neutrons, the inner structure of hadrons are sophisticated if we step closer.



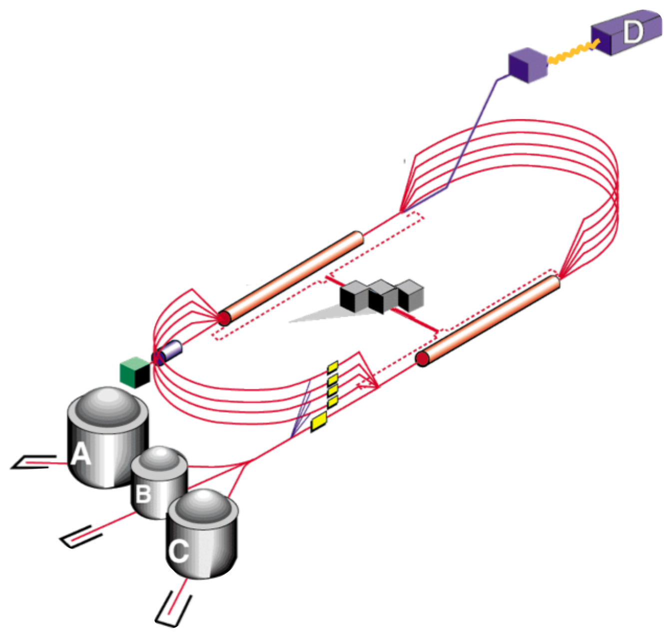
Cr. NASA's Goddard Space Flight Center





- Many experiments have been designed to probe the internal structure of hadrons. *P.S. The list of experiments here is not complete.*

CEBAF(JLab)



Cr. Dave Gaskell

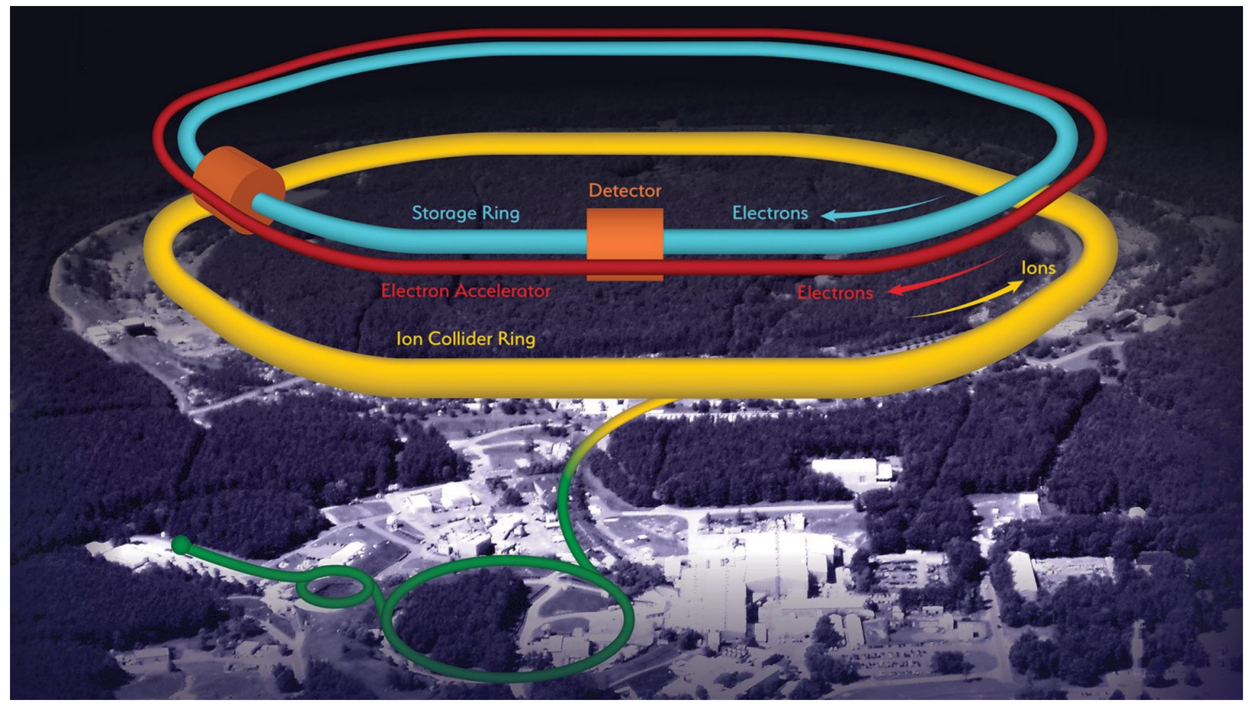
HERA

Cr. DESY

LHC

Cr. CERN

EIC



Cr. BNL

3D Imaging of Hadron

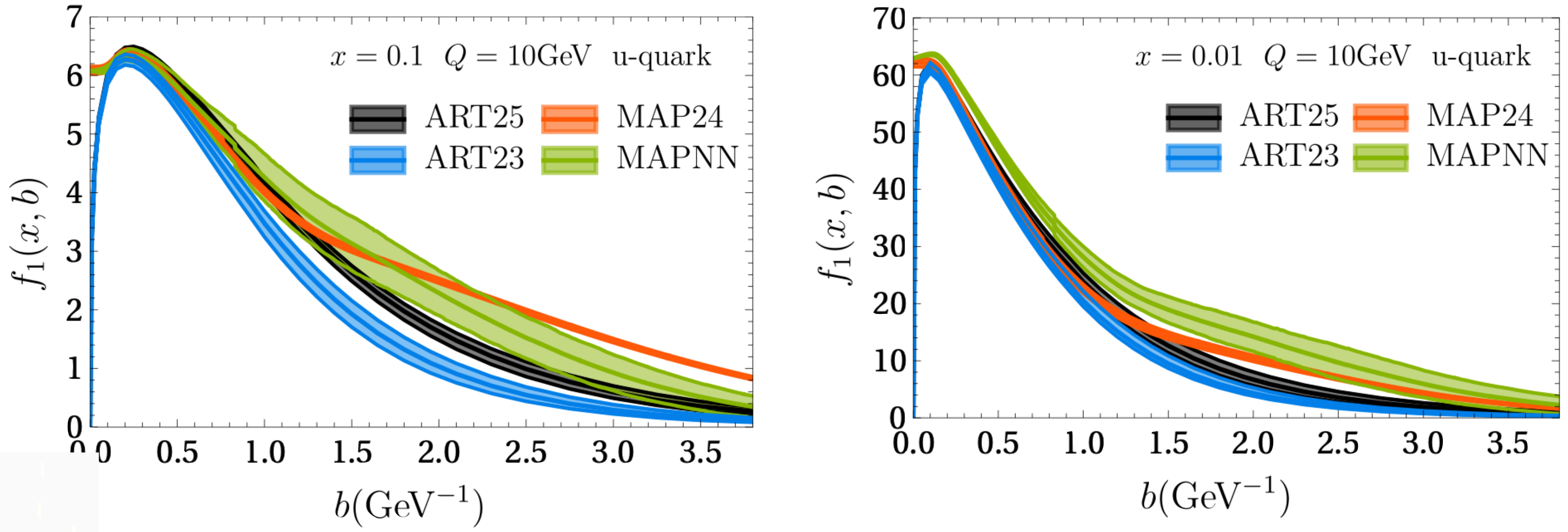
x is the momentum fraction in the longitudinal (hadron momentum) direction.

Wigner Distribution / GTMD

$W(x, \vec{r}_\perp, \vec{k}_\perp)$

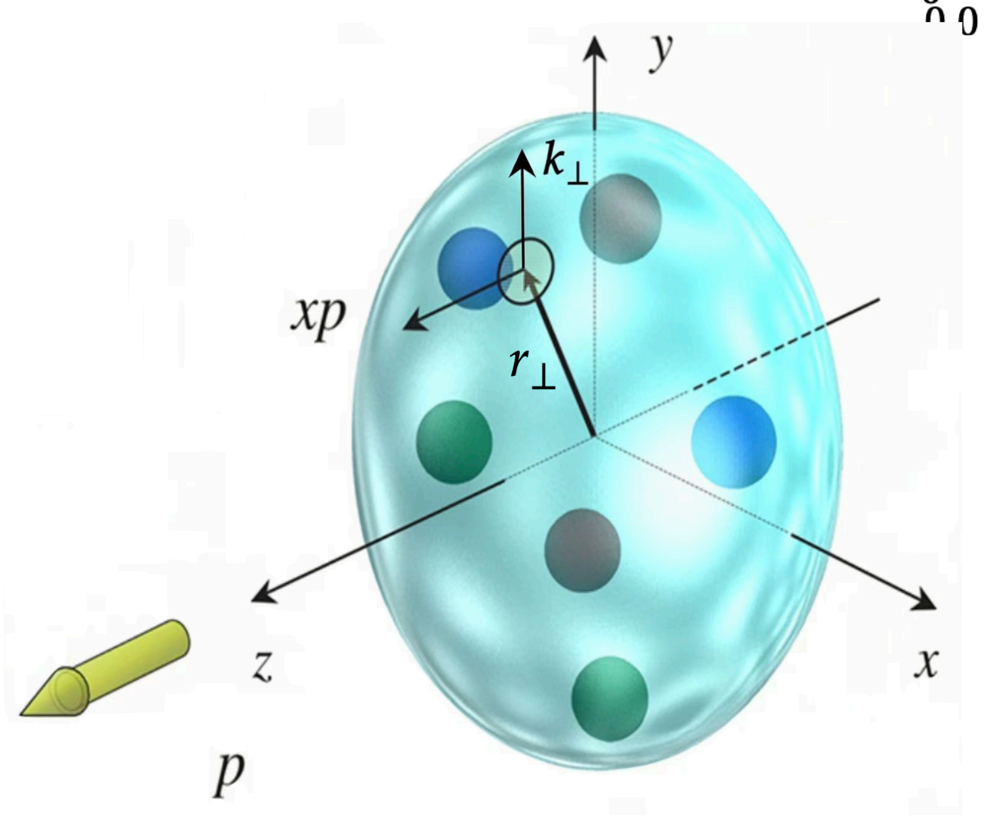
$\int d^2\vec{r}_\perp$

Transverse-Momentum-Dependent distributions (TMDs)



V. Moos, et al., 2503.11201 (2025)

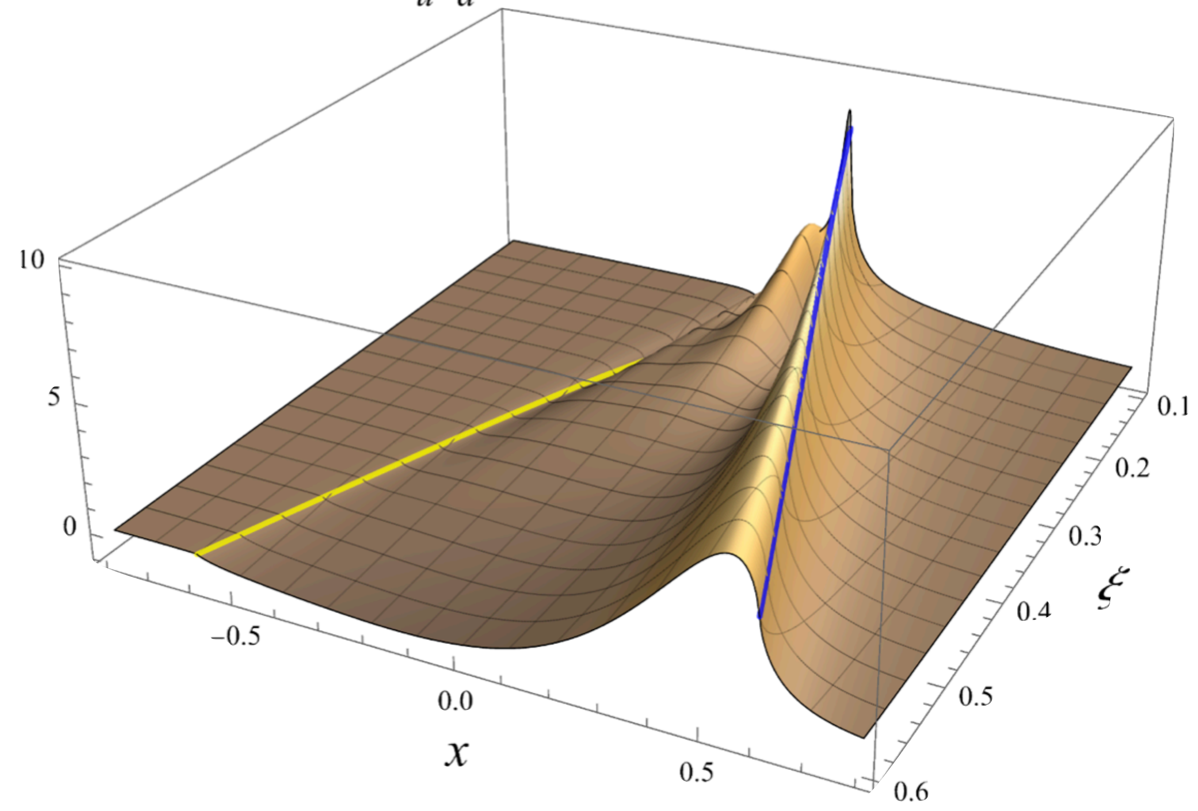
$\int d^2\vec{k}_\perp$



$\int d^2\vec{k}_\perp$

Generalized Parton Distributions (GPDs)

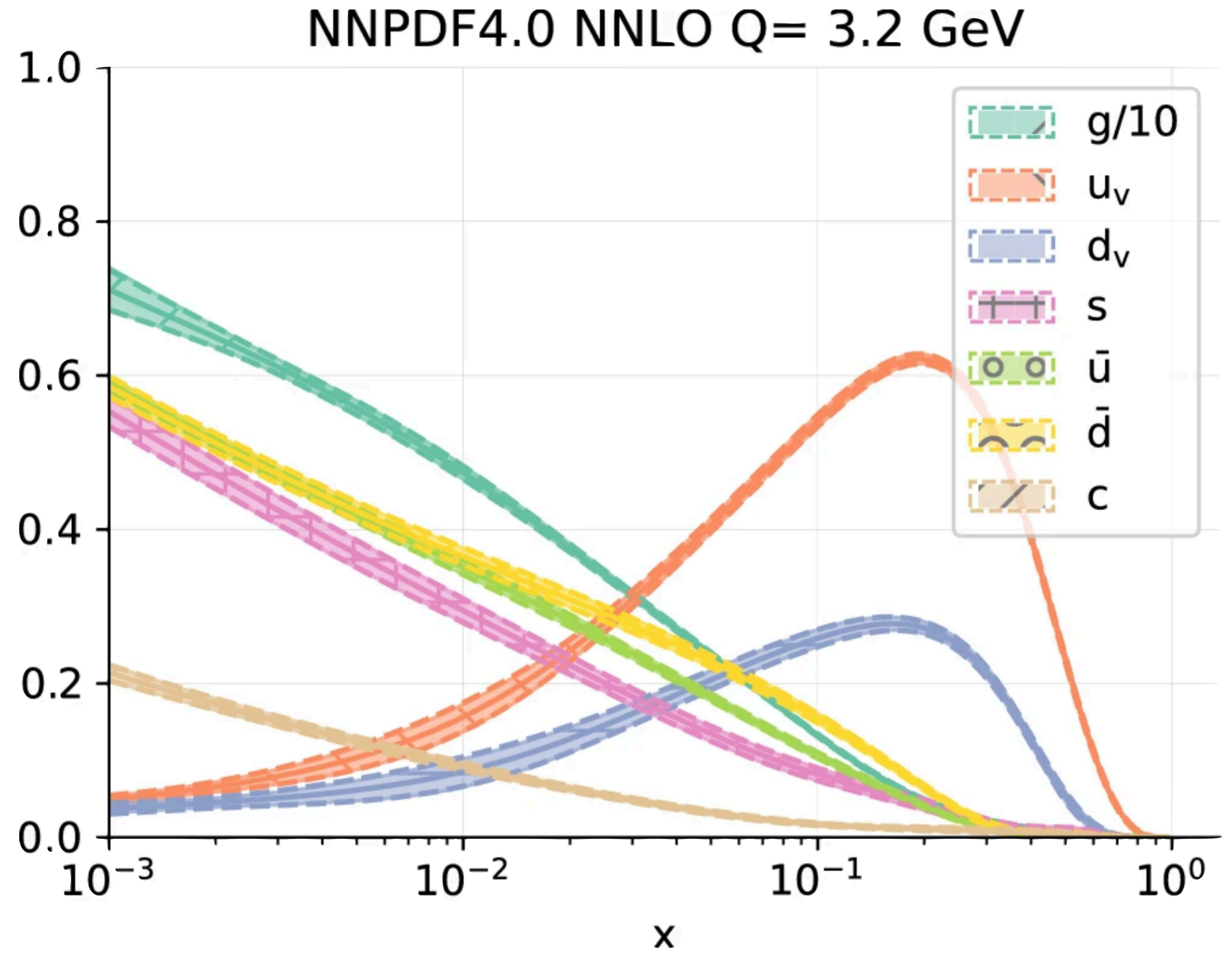
The isovector GPD H_{u-d} at $-t = 0.69$ GeV^2 tuned with DA terms



Y. Guo, et al., JHEP 09 (2022)

$\int d^2\vec{r}_\perp$

Parton Distribution Functions (PDFs)

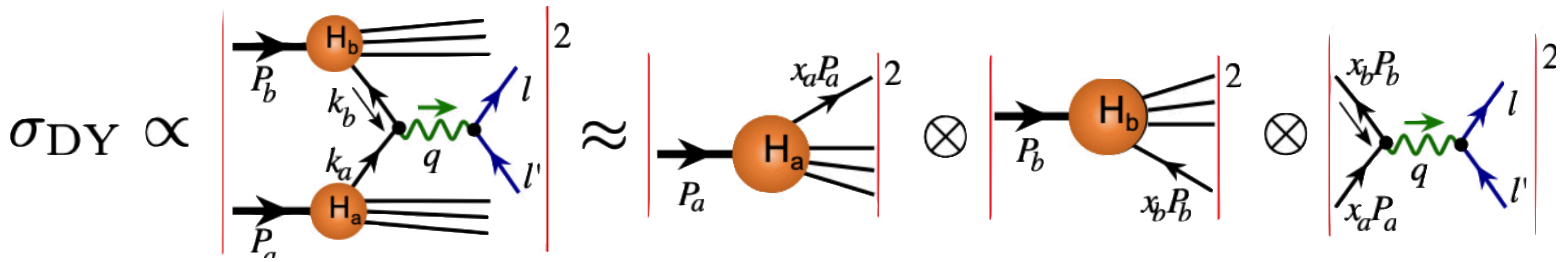


R. D. Ball, et al. [NNPDF], Eur. Phys. J. C 82 (2022)

TMDs in Experiments

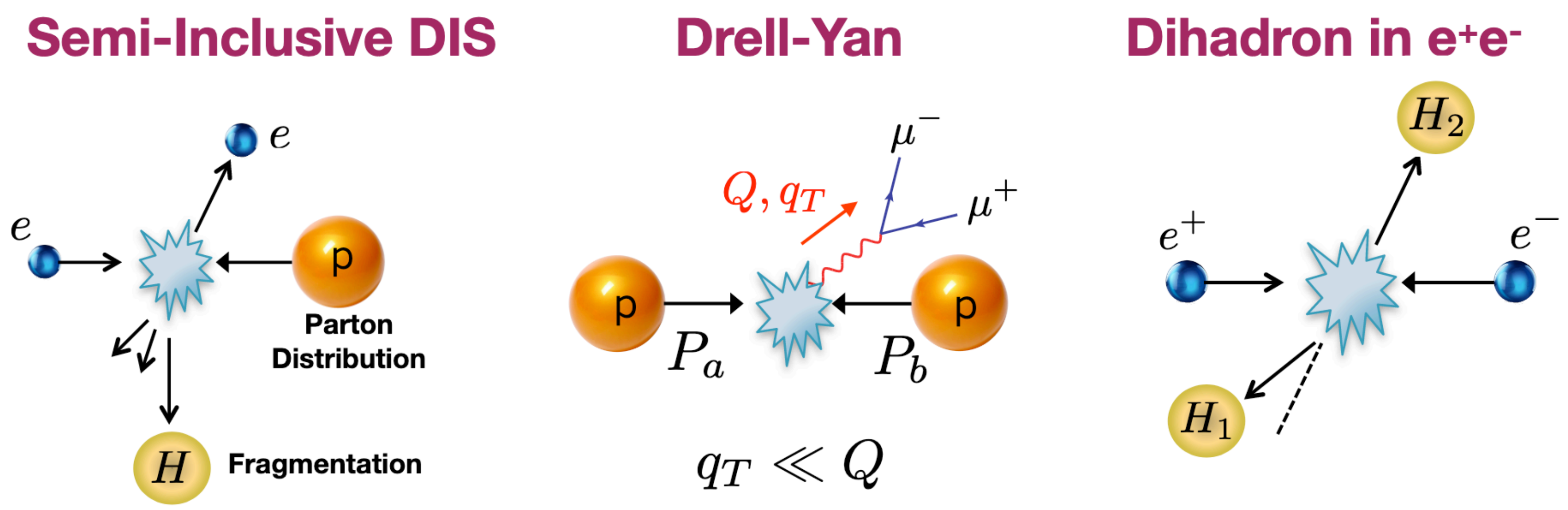
- **TMDPDFs: the distribution densities of finding a parton carrying a longitudinal momentum fraction x and transverse momentum $k_\perp$ in a hadron;**

- **TMD processes are important processes in high energy collisions, like Drell-Yan process on LHC and Semi-Inclusive DIS on EIC;**



$$\frac{d\sigma_{H_a+H_b\to l\bar{l}+X}}{dQ^2\, dY\, d^2\vec{q}_T} = \frac{4\pi\alpha^2}{3N_c Q^2 S}\sum_i e_i^2 \int d^2\vec{k}_{a\perp}\, d^2\vec{k}_{b\perp}\delta^{(2)}\left(\vec{q}_T - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right)$$

$$\times f_{1(i/H_a)}\left(x_a, \vec{k}_{a\perp}\right) \times f_{1(\bar{i}/H_b)}\left(x_b, \vec{k}_{b\perp}\right)$$



Leading Quark TMDPDFs — Nucleon Spin — Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

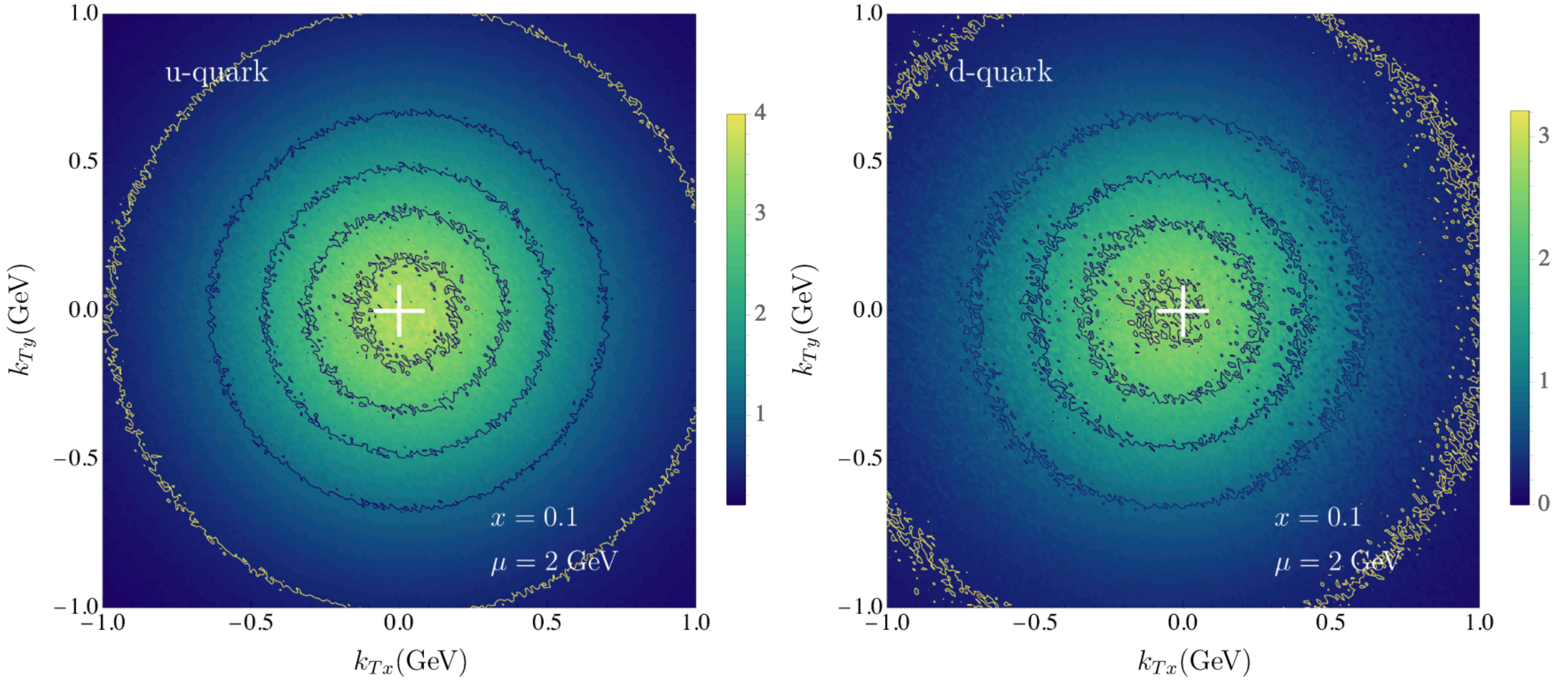
R. Boussarie, et al., 2304.03302 (2023)

Phenomenological Extraction of TMDs

- **Significant progress has been made in the phenomenological parameterizations of TMDs**
  - **Collins-Soper kernel (CS kernel): rapidity evolution kernel of TMDs**

    A. Bacchetta, et al. (MAP) JHEP 08 (2024); V. Moos, et al., 2503.11201 …
  - **Nucleon TMDs**
    - **Unpolarized** *M. Bury, et al., JHEP 10 (2022); A. Bacchetta, et al., JHEP 10 (2022); V. Moos, et al., JHEP 05 (2024) …*
    - **Sivers** *M. Bury, et al., PRL 126 (2021); I. P. Fernando, et al., Phys.Rev.D 108 (2023) …*
    - **Boer-Mulders** *Z. Lu, et al., Phys.Rev.D 81 (2010); X. Liu, et al., Eur.Phys.J.C 81 (202*
  - **Pion TMDs: much less is known about the TMDs of the pion**
    - **Unpolarized** *A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), Phys.Rev.D 107 (2023); P. C. Barry, et al. (JAM) Phys.Rev.D 108 (2023)*

Tomographic scan of the nucleon



M. Bury, et al., JHEP 05 (2021)

As the lightest pseudo Nambu-Goldstone boson, the 3D structure of pion will help us understand the strong interaction, such as the origin of chiral-symmetry breaking.

Lattice QCD

- **Path integral formalism**

$$Z = \int \mathscr{D}A\ \mathscr{D}\psi\ \mathscr{D}\bar{\psi}\ e^{iS_{\mathrm{QCD}}[A,\psi,\bar{\psi}]} \xrightarrow[t \to -it_{\mathrm{E}}]{U_\mu(x) = e^{igaA^\mu(x)}} Z_{\mathrm{E}} = \int \mathscr{D}U e^{-S_{\mathrm{E}}^g[U]} \int \mathscr{D}\psi \mathscr{D}\bar{\psi} e^{-\bar{\psi}M[U]\psi} = \int \mathscr{D}U \boxed{e^{-S_{\mathrm{E}}^g[U]} \det M[U]}$$

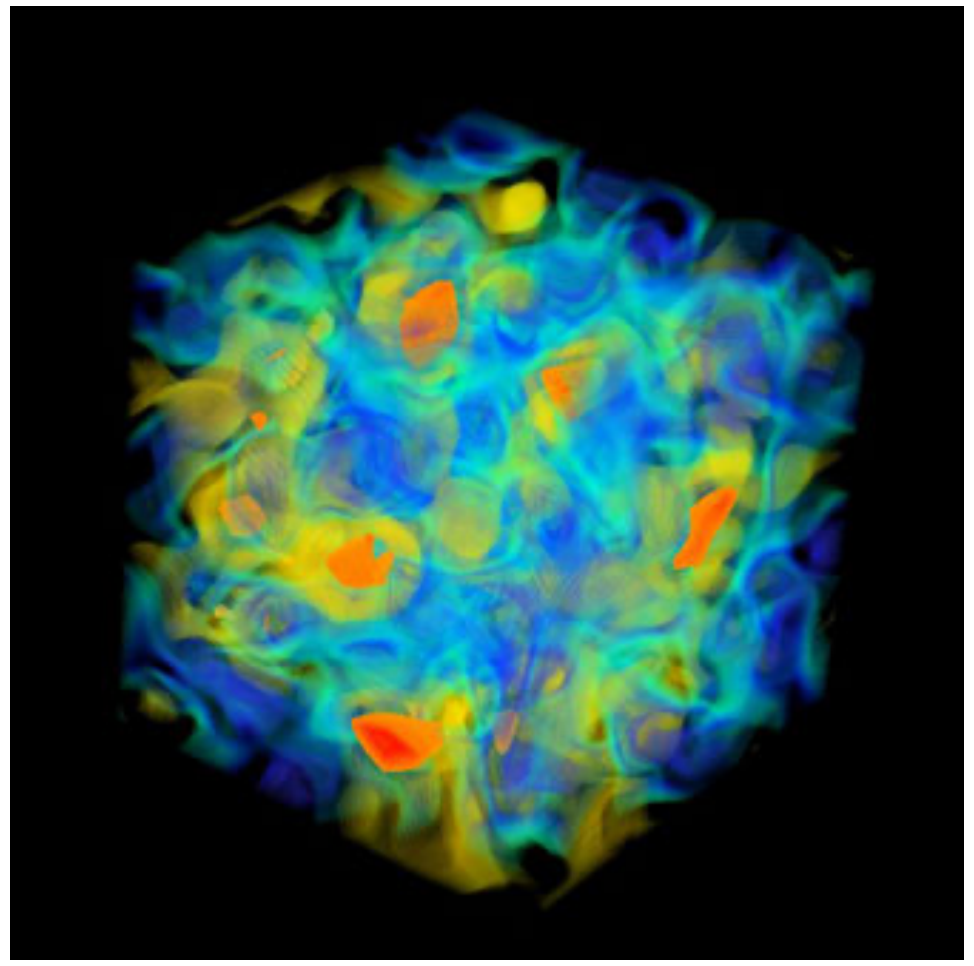
Wilson link

Wick rotation

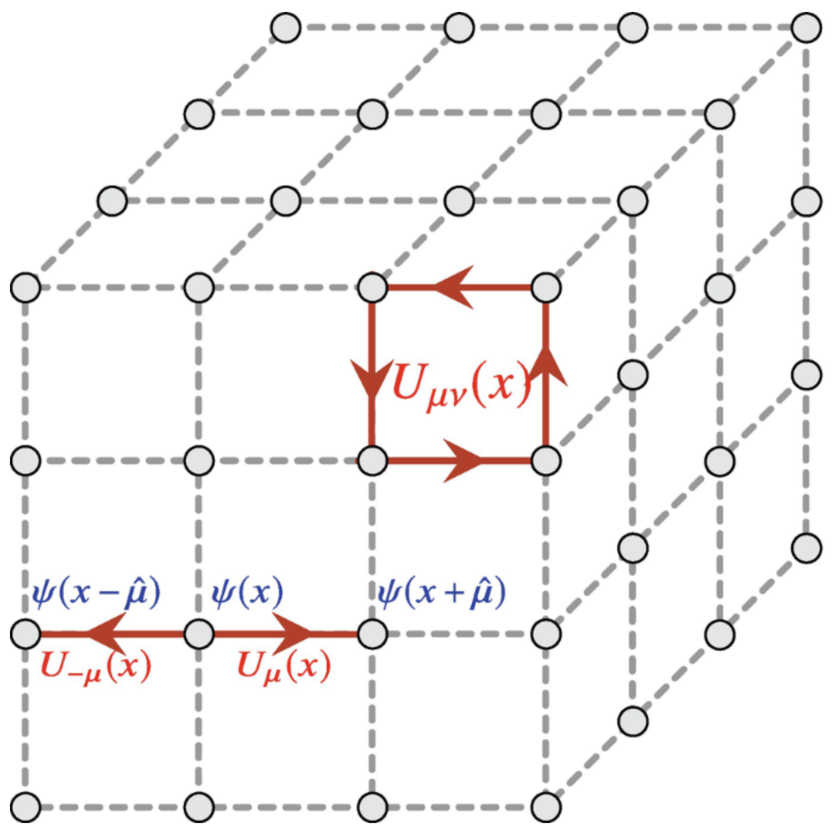
Sampling probability for configuration U

- **Monte Carlo sampling**

$$\langle \hat{O} \rangle = \frac{1}{Z_{\mathrm{E}}} \int \mathscr{D}U\ \mathscr{D}\psi\ \mathscr{D}\bar{\psi}\ \hat{O}\ e^{-S_{\mathrm{E}}^{\mathrm{QCD}}[A,\psi,\bar{\psi}]} = \frac{1}{N} \sum_{i=1}^{N} O[U^{(i)}]$$



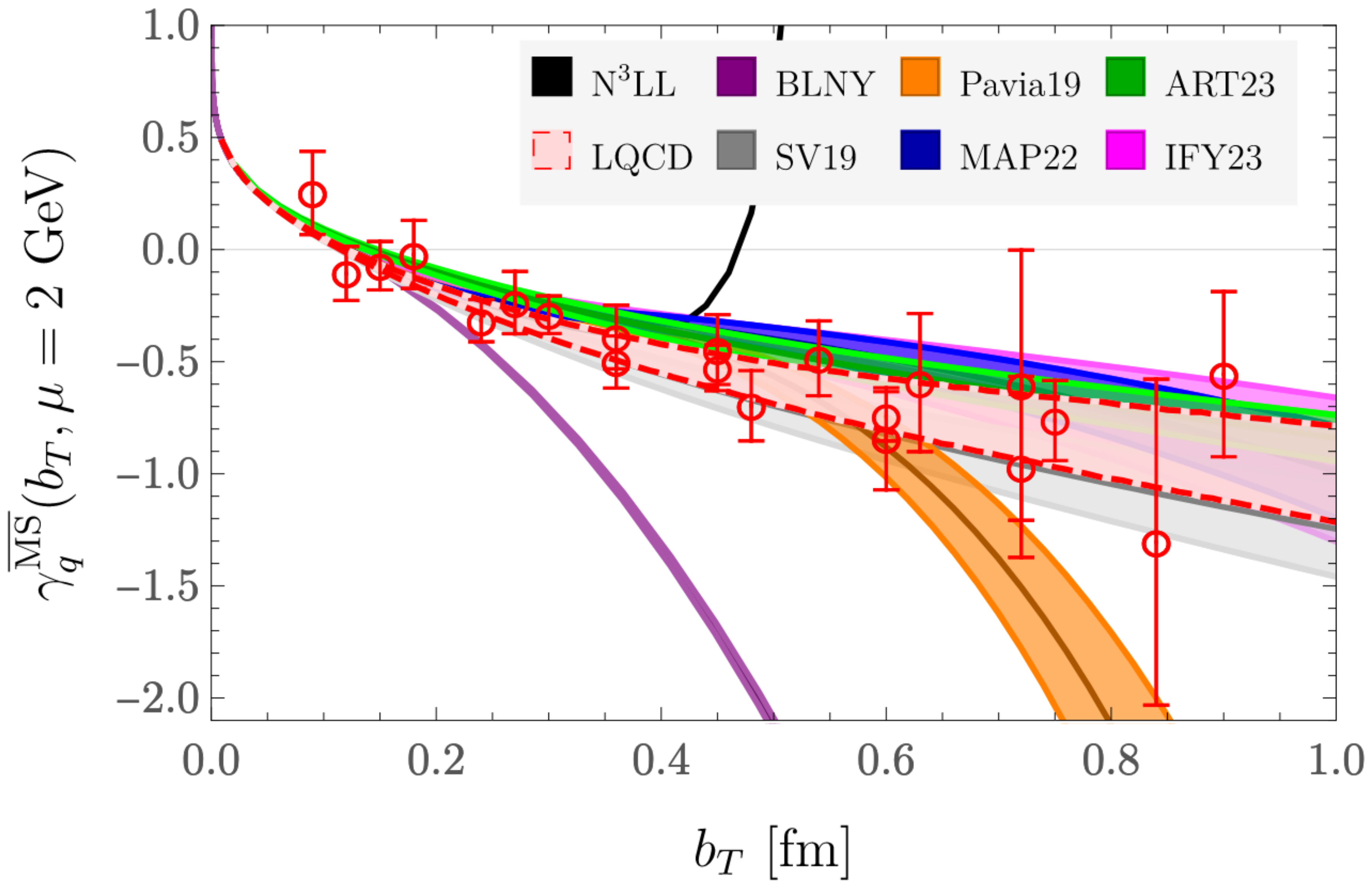
Cr. BNL



Cr. Olaf Kaczmarek

Lattice QCD Calculation of TMDs

- As a first-principle non-perturbative method, Lattice QCD provides independent predictions of TMDs.
  - Mellin Moments — *B. Yoon, et al., 1601.05717; B. Yoon, et al., Phys. Rev. D 96 (2017)…*
  - Large Momentum Effective Theory (LaMET)
    - CS kernel — *M. H. Chu, et al. (LPC), JHEP 08 (2023); A. Avkhadiev et al., PRL 132 (2024); D. Bollweg, et al., Phys. Lett. B 852 (2024) …*
    - Intrinsic soft function — *Q. A. Zhang, et al. (LPC), PRL 125 (2020); M. H. Chu, et al. (LPC), JHEP 08 (2023)*
    - Unpolarized — *JH, et al. (LPC), Phys.Rev.D 109 (2024)*
    - Boer-Mulders — *L. Walter, et al. (LPC), 2412.19988; L. Ma, et al. (LPC), 2502.11807*



A. Avkhadiev et al., PRL 132 (2024)

Methodology

Large-Momentum Effective Theory (LaMET)

- TMDPDF is defined from a light-cone correlator in a hadron, which is Lorentz invariant.

Parton model

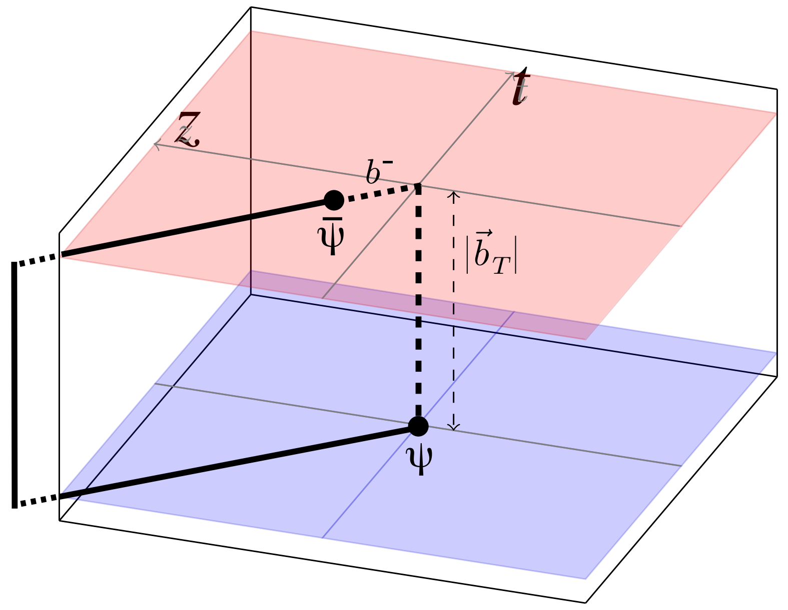
$$f(x,b_\perp,\ldots)=\int_{-\infty}^{\infty}\frac{db^-}{2\pi}e^{-ib^-(xP^+)}\left\langle P\left|\bar{\psi}\left(b^\mu\right)W_\sqsubset\left(b^\mu,0\right)\frac{\gamma^+}{2}\psi\left(0\right)\right|P\right\rangle \longleftrightarrow \left\langle |\vec{P}|=\infty\left|O(t=0)\right||\vec{P}|=\infty\right\rangle$$

Different orders of limit, but pert.

- Define a quasi distribution with large-momentum states and time-independent operators.

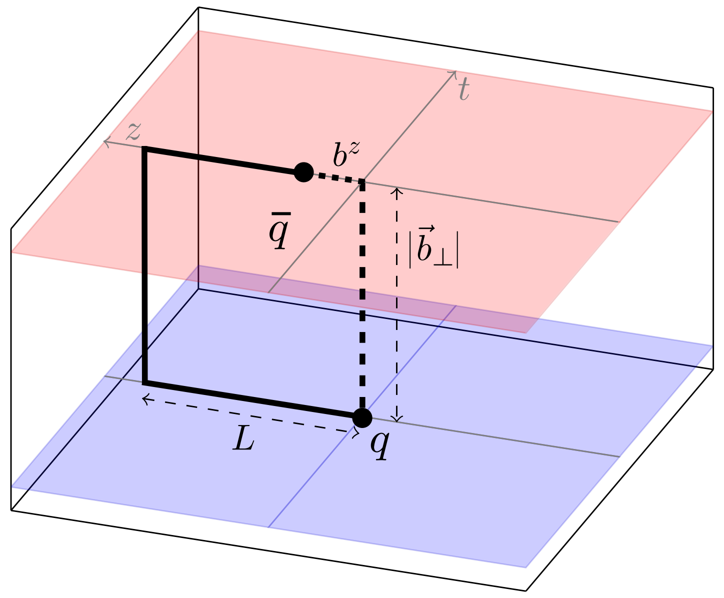
$$\tilde{f}^0_\Gamma(x,b_\perp,P^z,\mu)=P^z\int\frac{dz}{2\pi}e^{iz(xP^z)}\frac{1}{2P^t}\langle P|\bar{\psi}_0\left(z,b_\perp\right)W_\sqsubset\left(z,b_\perp;0\right)\Gamma\psi_0\left(0\right)|P\rangle,\quad \Lambda_{\mathrm{QCD}}\ll|\vec{P}|\ll\frac{\pi}{a}$$

Large-momentum expansion



X. Ji, Phys.Rev.Lett. 110 (2013)
X. Ji, et al., Rev.Mod.Phys. 93 (2021)
X. Ji, Nucl. Phys. B 1007 (2024)

Lorentz boost & Matching



Light-cone distribution:

Cannot be directly calculated on the lattice

Quasi distribution:

Directly calculable on the lattice

$H_f\left(x,P^z;\mu\right)=|C_{\mathrm{TMD}}(xP^z;\mu)|^2$ **is the TMD hard kernel for matching.**

- LaMET enables us to obtain the precision-controlled x-distribution of TMDs in $x\in[x_{\min},\,x_{\max}]$.

Collins-Soper scale: $\zeta\sim 2(xP^+)^2$

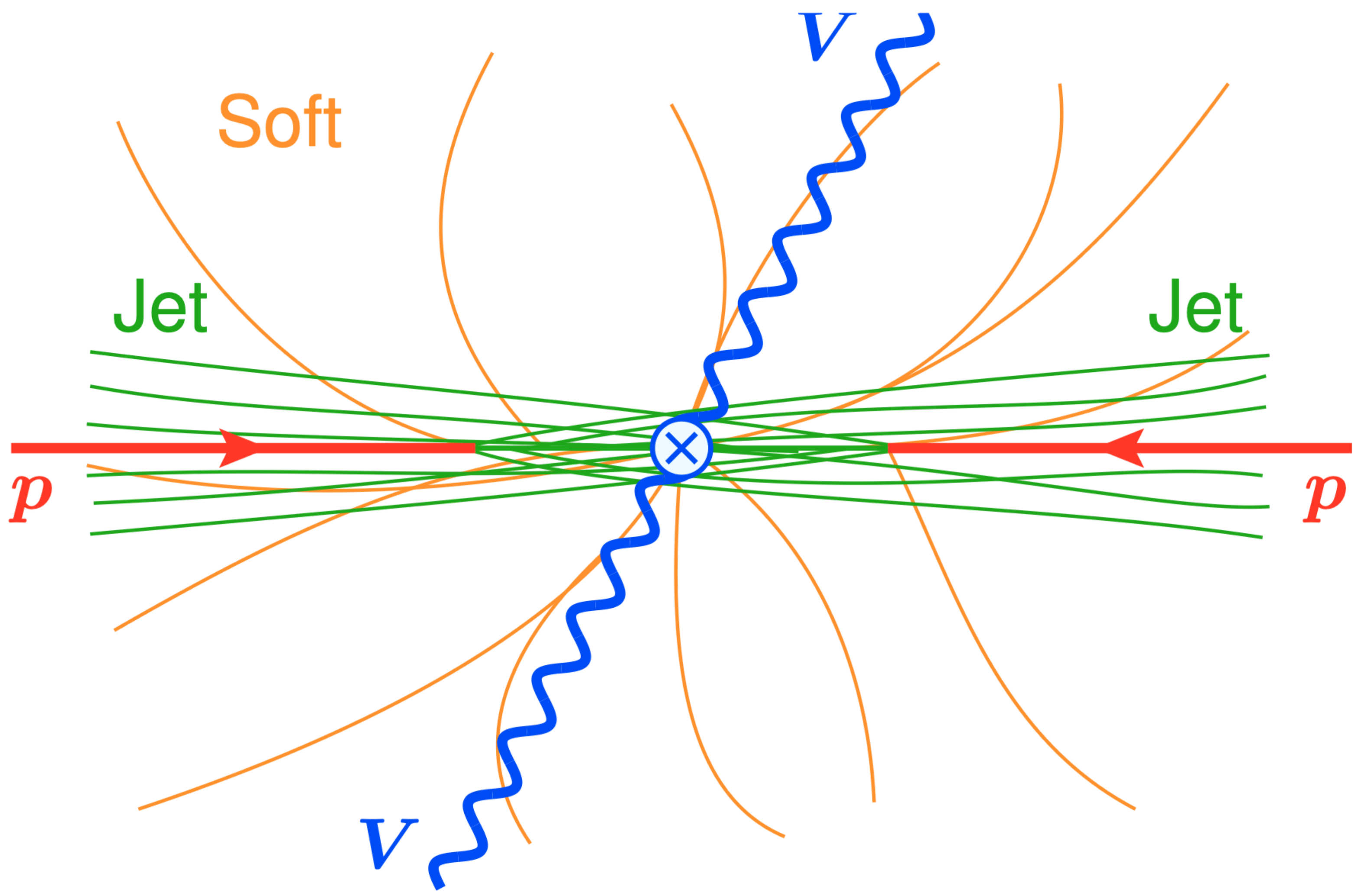
$$\sqrt{S_I\left(b_\perp;\mu\right)}\cdot\tilde{f}_\Gamma\left(x,b_\perp,P^z;\mu\right)=f\left(x,b_\perp;\mu,\zeta\right)H_f\left(x,P^z;\mu\right)\exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}\left(b_\perp;\mu\right)\right]+\text{Power corrections}$$

Soft Function

$$\sqrt{S_I\left(b_\perp;\mu\right)}\cdot\tilde{f}_\Gamma\left(x,b_\perp,P^z;\mu\right)=f\left(x,b_\perp;\mu,\zeta\right)H_f\left(x,P^z;\mu\right)\exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}\left(b_\perp;\mu\right)\right]+\text{Power corrections}$$

After regularization, the rapidity evolution is controlled by Collins-Soper scale: $\zeta\sim 2(xP^+)^2$

- **The soft gluon radiation will lead to the existence of soft functions;**
- **Due to the gluon radiation in the collinear mode, the soft function contains the well-known rapidity divergence;** $\underbrace{\int_{q_T}^{Q}\frac{\mathrm{d}k}{k}}_{\text{full}}=\lim_{\tau\to 0}[\underbrace{\int_0^Q\frac{\mathrm{d}k}{k}R_c(k,\tau)}_{\text{collinear}}+\underbrace{\int_{q_T}^{\infty}\frac{\mathrm{d}k}{k}R_s(k,\tau)}_{\text{soft}}]=\ln\frac{Q}{q_T}$
- **The soft function can be separated into two parts:**
  - **Rapidity evolution kernel: CS kernel** $\gamma^{\overline{\mathrm{MS}}}\left(b_\perp;\mu\right)$
  - **Rapidity independent part: intrinsic soft function** $S_I(b_\perp;\mu)$
- **CS kernel can be extracted from the rapidity evolution of TMDs**



M. Ebert, PhD Thesis (2017)

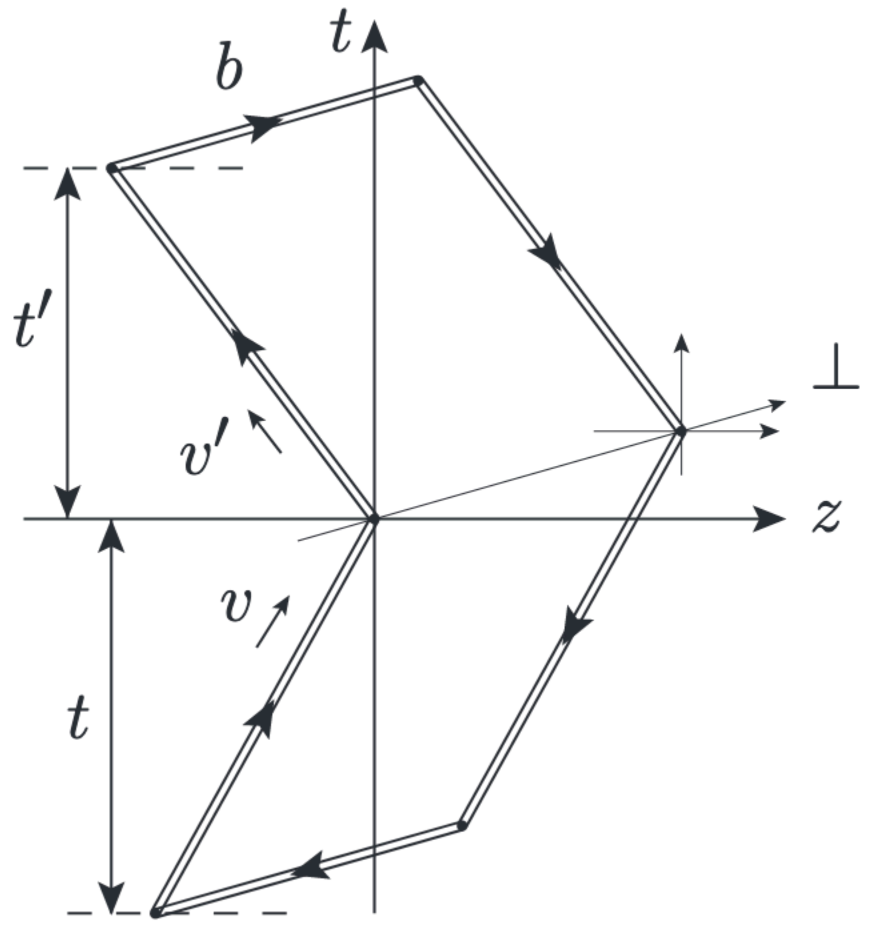
$$\gamma^{\overline{\mathrm{MS}}}(b_\perp,P_1,P_2;\mu)=\frac{1}{\ln\left(P_2/P_1\right)}\ln\frac{H_f\left(x,\bar{x},P_1;\mu\right)\tilde{f}_{\gamma^t}\left(x,b_\perp,P_2;\mu\right)}{H_f\left(x,\bar{x},P_2;\mu\right)\tilde{f}_{\gamma^t}\left(x,b_\perp,P_1;\mu\right)}$$

$H_f\left(x,P^z;\mu\right)=|C_{\mathrm{TMD}}(xP^z;\mu)|^2$ **is the TMD hard kernel for matching.**

Soft Function

- The intrinsic soft function cannot be directly calculated on lattice because of two light-like Wilson lines in different directions
  - Fortunately, it can be extracted from the meson form factor *X. Ji, et al., Nucl.Phys.B* 955 (2020)

$$F\left(b_\perp, P_1, P_2, \Gamma, \Gamma'\right) \equiv -4N_c \frac{\left\langle P_2 \middle| \bar{q}\left(b_\perp\right)\Gamma q\left(b_\perp\right)\bar{q}(0)\Gamma' q(0) \middle| P_1 \right\rangle}{f_\pi^2 (P_1 \cdot P_2)}$$



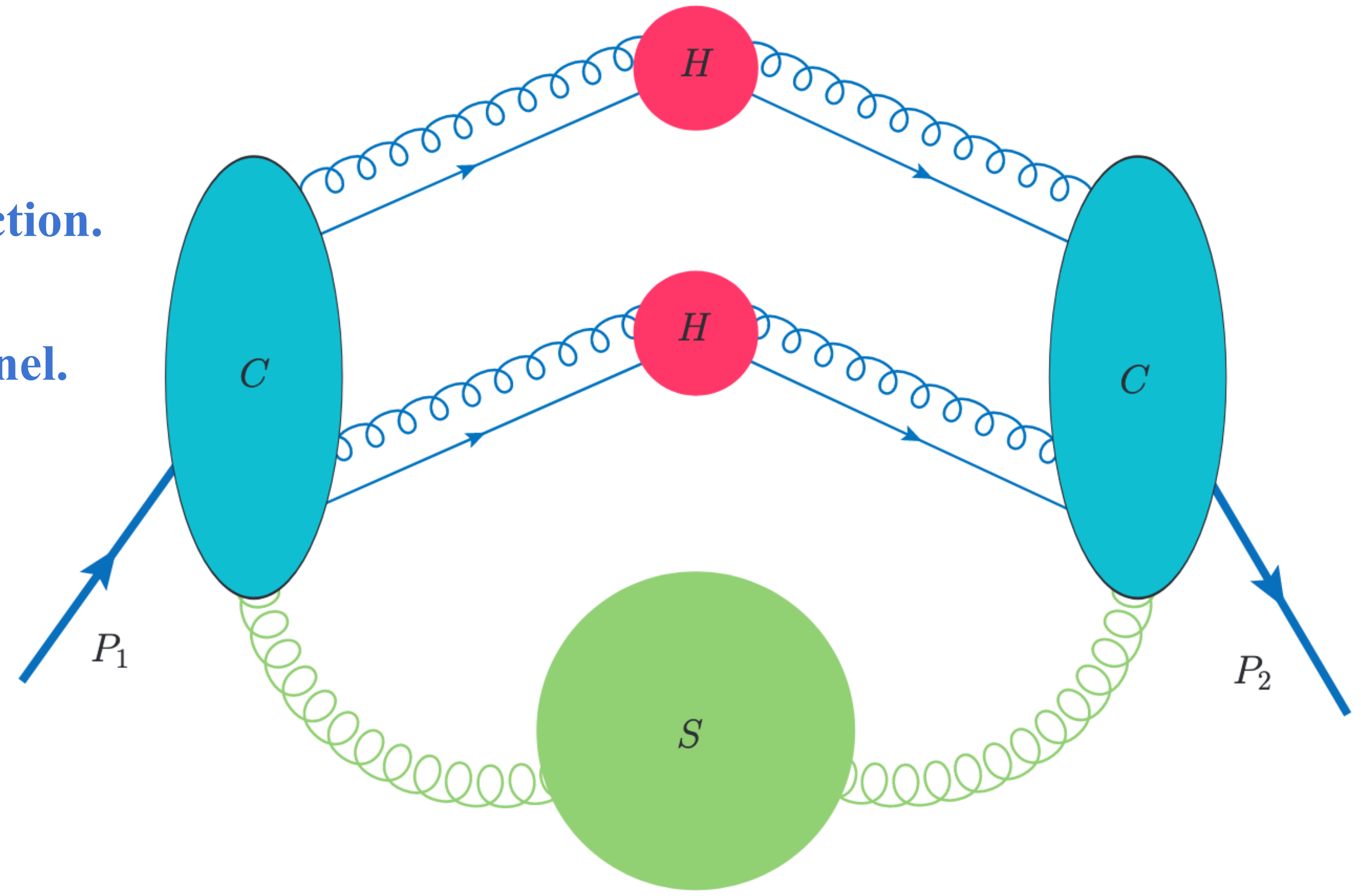
X. Ji, et al., Nucl.Phys.B 955 (2020)

  - The form factor satisfies the factorization formula

$$F(b_\perp, P^z) = \int dx_1 dx_2\; H_F(x_1, x_2, P^z; \mu)\; \phi^\dagger(x_1, b_\perp, y_n; \mu, \zeta_1, \bar{\zeta}_1)\; \phi(x_2, b_\perp, -y_n; \mu, \zeta_2, \bar{\zeta}_2)$$

$\phi(x, b_\perp, \ldots) = \int_{-\infty}^{\infty} \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \left\langle 0 \middle| \bar{\psi}\left(b^\mu\right) W_\sqsubset\left(b^\mu, 0\right) \gamma^+\gamma^5 \psi(0) \middle| P \right\rangle$ is TMD wave function.

$H_F(x_1, x_2, P^z; \mu) = C_{\rm Sud}(x_1, x_2, P^z; \mu) \cdot C_{\rm Sud}(\bar{x}_1, \bar{x}_2, P^z; \mu)$, where $C_{\rm Sud}$ is the Sudakov kernel.



  - Therefore, the intrinsic soft function can be extracted via

$$S_I(b_\perp; \mu) = \frac{F(b_\perp, P^z)}{\int dx_1 dx_2 H_F(x_1, x_2, P^z; \mu) \tilde{\Phi}^\dagger(x_1) \tilde{\Phi}(x_2)} \text{ with } \tilde{\Phi}(x) \equiv \frac{\tilde{\phi}_\Gamma\left(x, b_\perp, P^z; \mu\right)}{H_\phi\left(x, \bar{x}, P^z; \mu\right)}$$

$\tilde{\phi}_\Gamma\left(x, b_\perp, P^z; \mu\right)$ is quasi-TMD wave function.

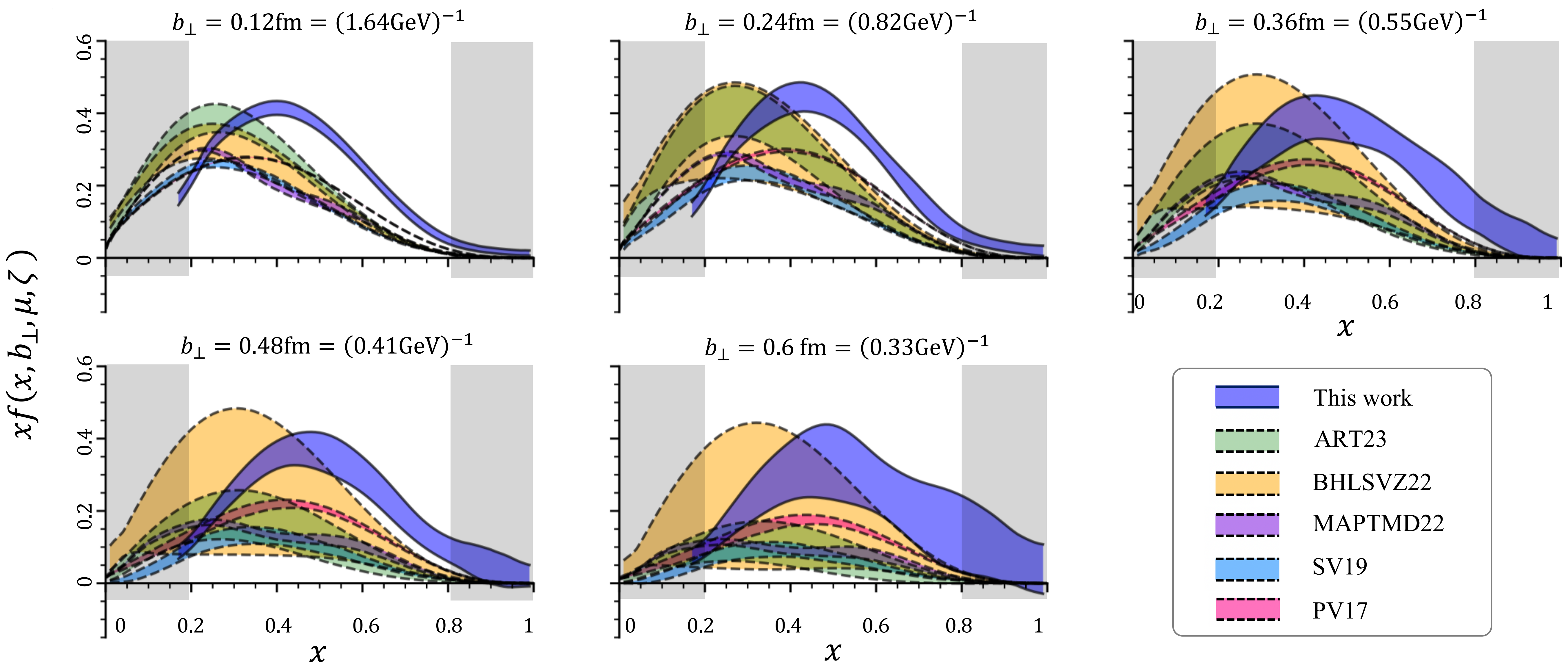
Z. F. Deng, et al., JHEP 09 (2022)

Unpolarized TMD via LaMET

- In recent years, a lot of improvements of renormalization and matching has been developed in LaMET;

Y. Su, et al., Nucl. Phys. B 991 (2023);
R. Zhang, et al., Phys. Lett. B 844 (2023);
X. Ji, et al., 2410.12910 [hep-ph]

Unpolarized nucleon TMDPDF



JH, et al. (LPC), Phys.Rev.D 109 (2024)

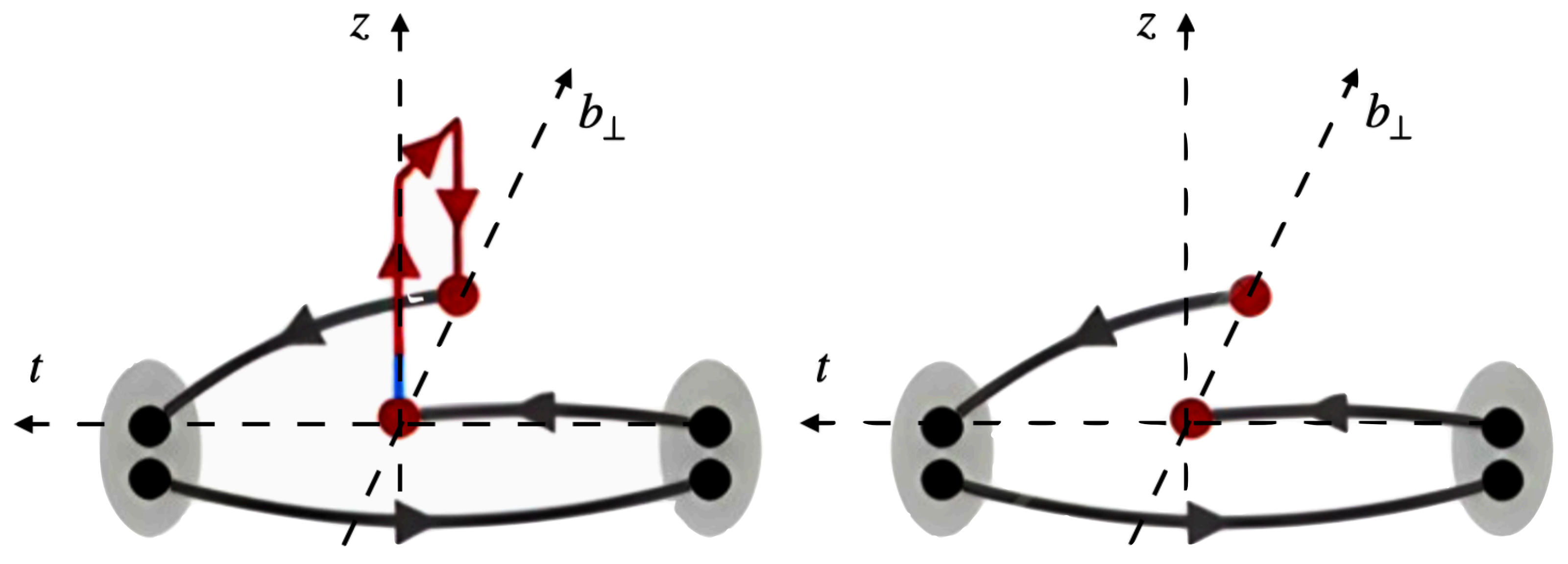
- ✦ $a = 0.12$ fm
- ✦ $P^z_{\text{max}} = 2.58$ GeV
- ✦ **Physical limit of m_π^{val}**
- ✦ **N3LL matching**
- ✦ **NLO soft function**

- **Existing lattice calculations of the nucleon TMD still suffer from some systematics:**
  - **Discretization effects;**
  - **Excited-state contamination;**
  - **Hadron momentum is not large enough …**
- **Due to the bad signal-to-noise ratio (SNR), it is hard to probe the large $b_\perp$ region.**

Coulomb Gauge Method

- **Define a quasi distribution in CG without Wilson line:** *X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)* *Y. Zhao, PRL 133 (2024)*

$$\tilde{f}^0_{\rm CG}(x, b_\perp, P^z, \mu) = P^z \int \frac{dz}{2\pi} e^{iz(xP^z)} \frac{1}{2P^t} \langle P | \bar{\psi}_0(z, b_\perp)\, \Gamma \psi_0(0) \Big|_{\vec{\nabla}\cdot\vec{A}=0} | P \rangle$$



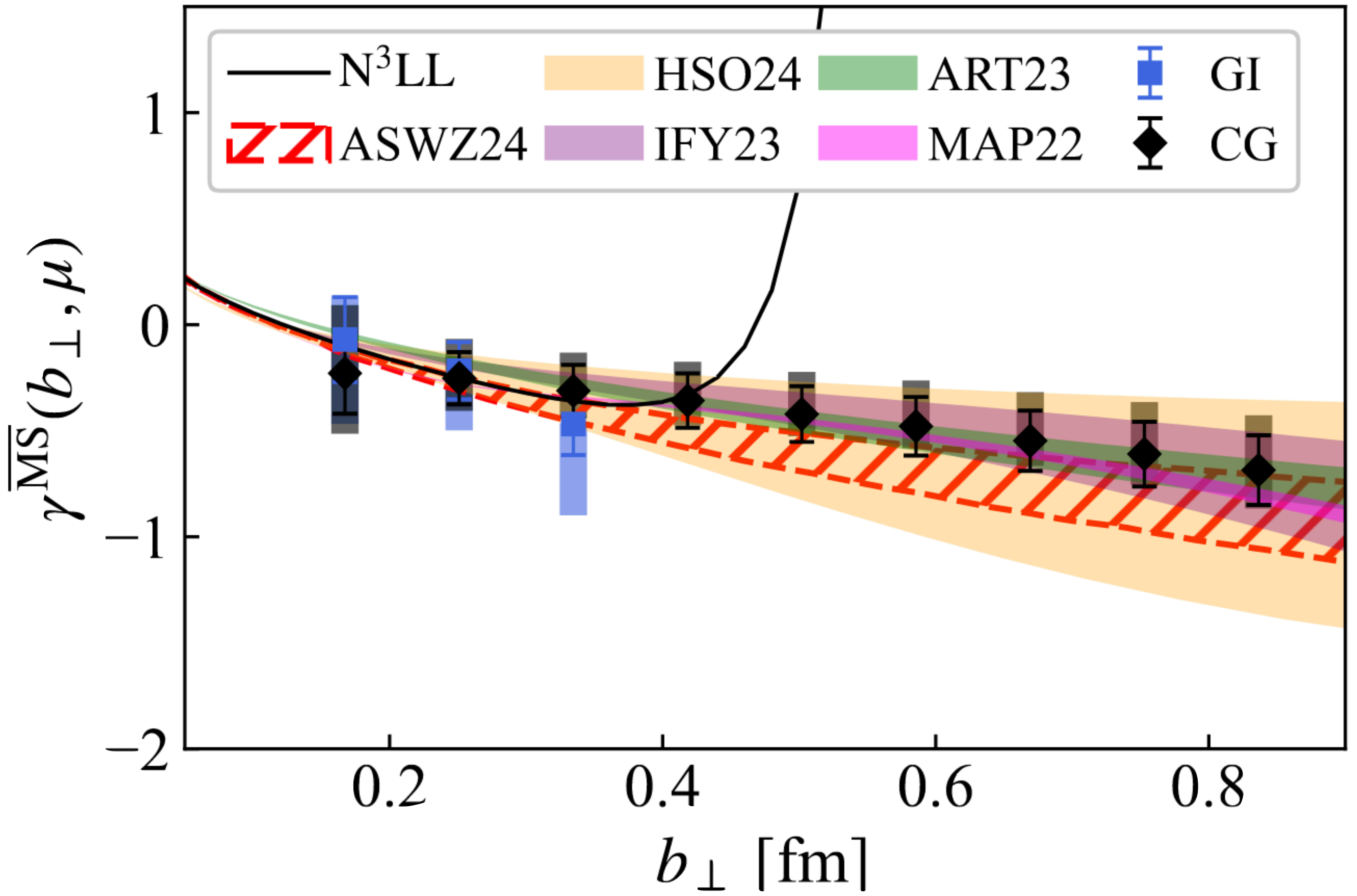
Gauge Invariant (GI) **Coulomb Gauge (CG)**

- **Why choose CG?**

X. Ji, Y. S. Liu, Y. Liu, J. H. Zhang and Y. Zhao, RMP 93 (2021)

  - $\vec{\nabla}\cdot\vec{A} = 0$ **becomes** $A^+ = 0$ **in the infinite boost, so the quasi distribution in CG belongs to the universality class in LaMET;**
  - **No linear divergence from the Wilson link, improving the SNR significantly;**
  - **Simplified renormalization** $\bar{\psi}_0(z, b_\perp)\Gamma\psi_0(0) = Z_\psi(a)\left[\bar{\psi}(z, b_\perp)\Gamma\psi(0)\right]$**;**
  - **Larger off-axis momenta (3D rotational symmetry).**

Collins-Soper kernel in CG v.s. GI



D. Bollweg, et al., Phys.Lett.B 852 (2024)

The results in CG and GI are consistent with the same lattice setup;

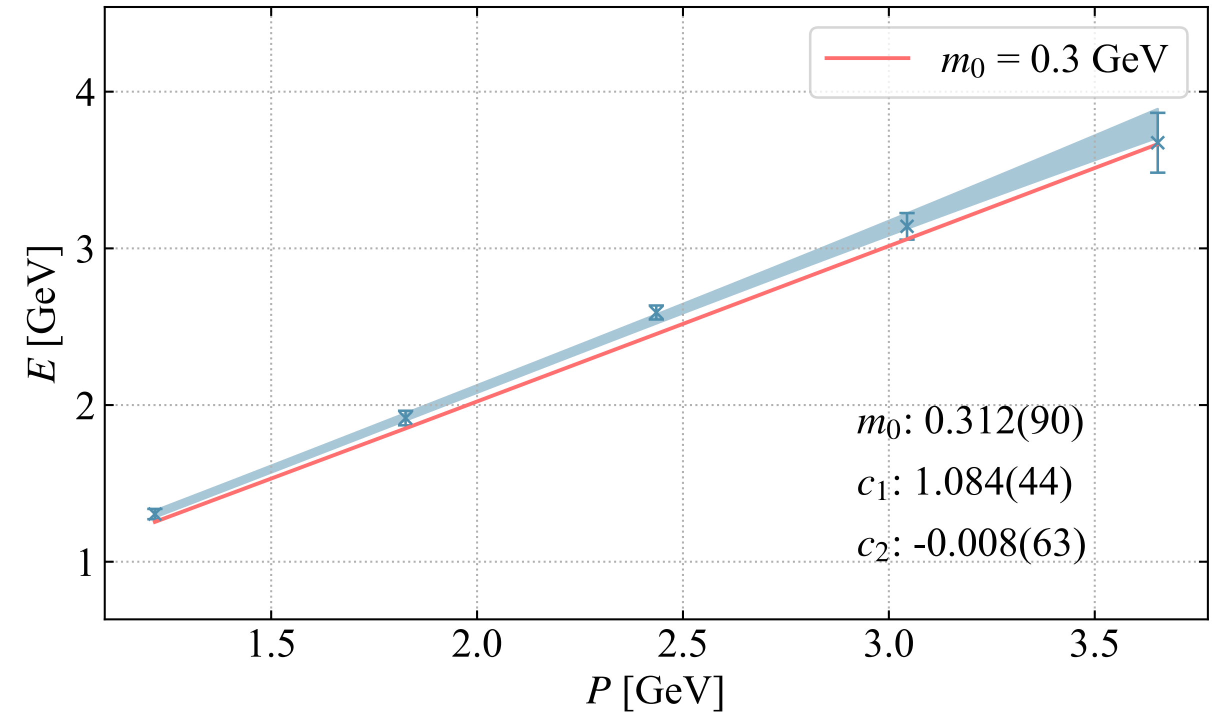
Compared with the GI method, CG method has much better SNR.

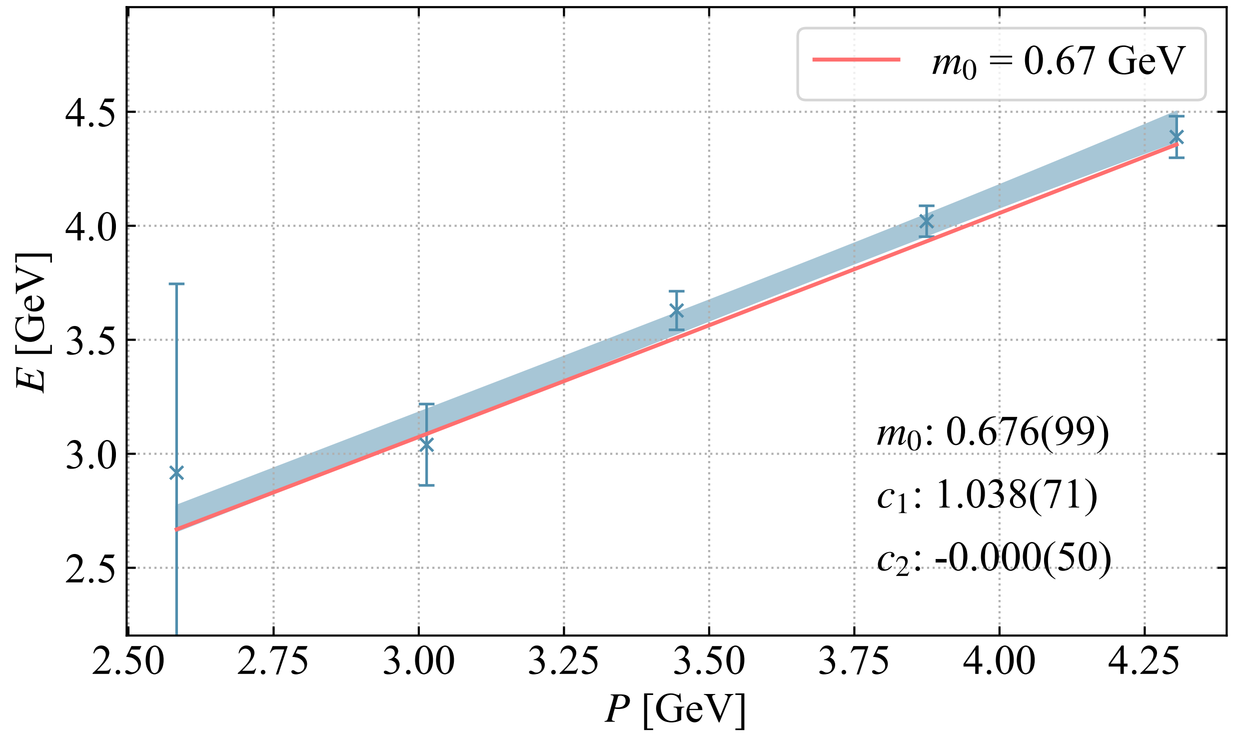
Numerical Results

Lattice Setup

- 2+1 flavor HISQ ensemble by HotQCD with volume $L_s \times L_t = 48^3 \times 64$;
- Lattice spacing is $a = 0.06$ fm;
- Pion mass of sea quark: $m_\pi^{\text{sea}} = 160$ MeV;
- Pion mass of valence quark for quasi-TMD: $m_\pi^{\text{val}} = 300$ MeV;
- Off-axis ($\vec{n} = (1,1,0)$) hadron momenta for quasi-TMD: 1.83 GeV, 2.43 GeV and 3.04 GeV;
- Pion mass of valence quark for qTMDWF and meson form factor: $m_\pi^{\text{val}} = 670$ MeV;
- On-axis hadron momenta for qTMDWF: 3.44 GeV, 3.87 GeV and 4.30 GeV;
- On-axis hadron momenta for meson form factor: 2.58 GeV.

Dispersion relation: $E^2 = m_0^2 + c_1 P^2 + c_2 a^2 P^4$





Quasi-TMD

TMDPDF

Hard kernel

$$\boxed{\sqrt{S_I\left(b_\perp;\mu\right)}} \cdot \boxed{\tilde{f}_\Gamma\left(x, b_\perp, P^z;\mu\right)} = \boxed{f\left(x, b_\perp;\mu,\zeta\right)}\boxed{H_f\left(x, P^z;\mu\right)} \exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\boxed{\gamma^{\overline{\text{MS}}}\left(b_\perp;\mu\right)}\right] + \text{Power corrections}$$

Intrinsic soft function

Quasi-TMDWF

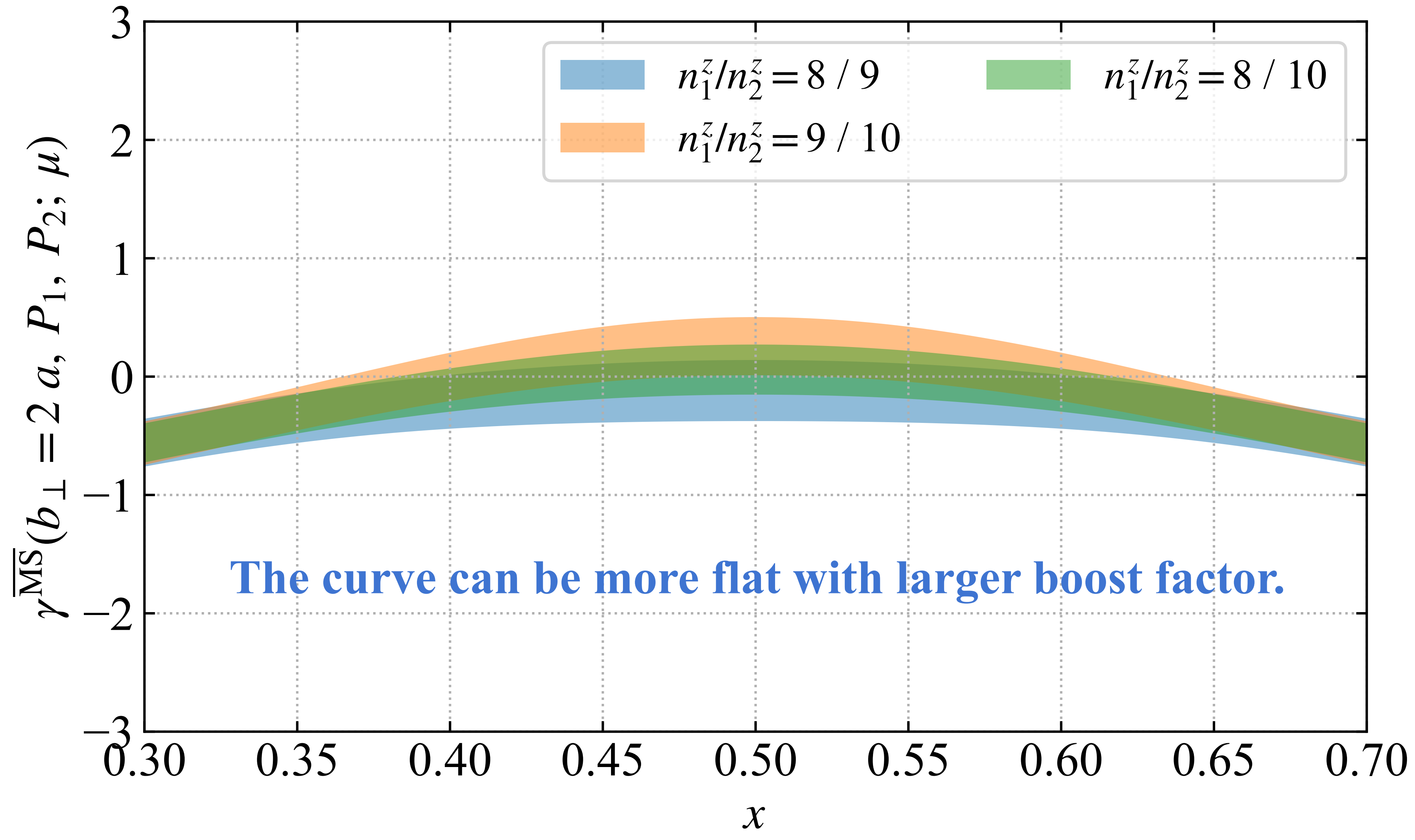
Meson Form Factor

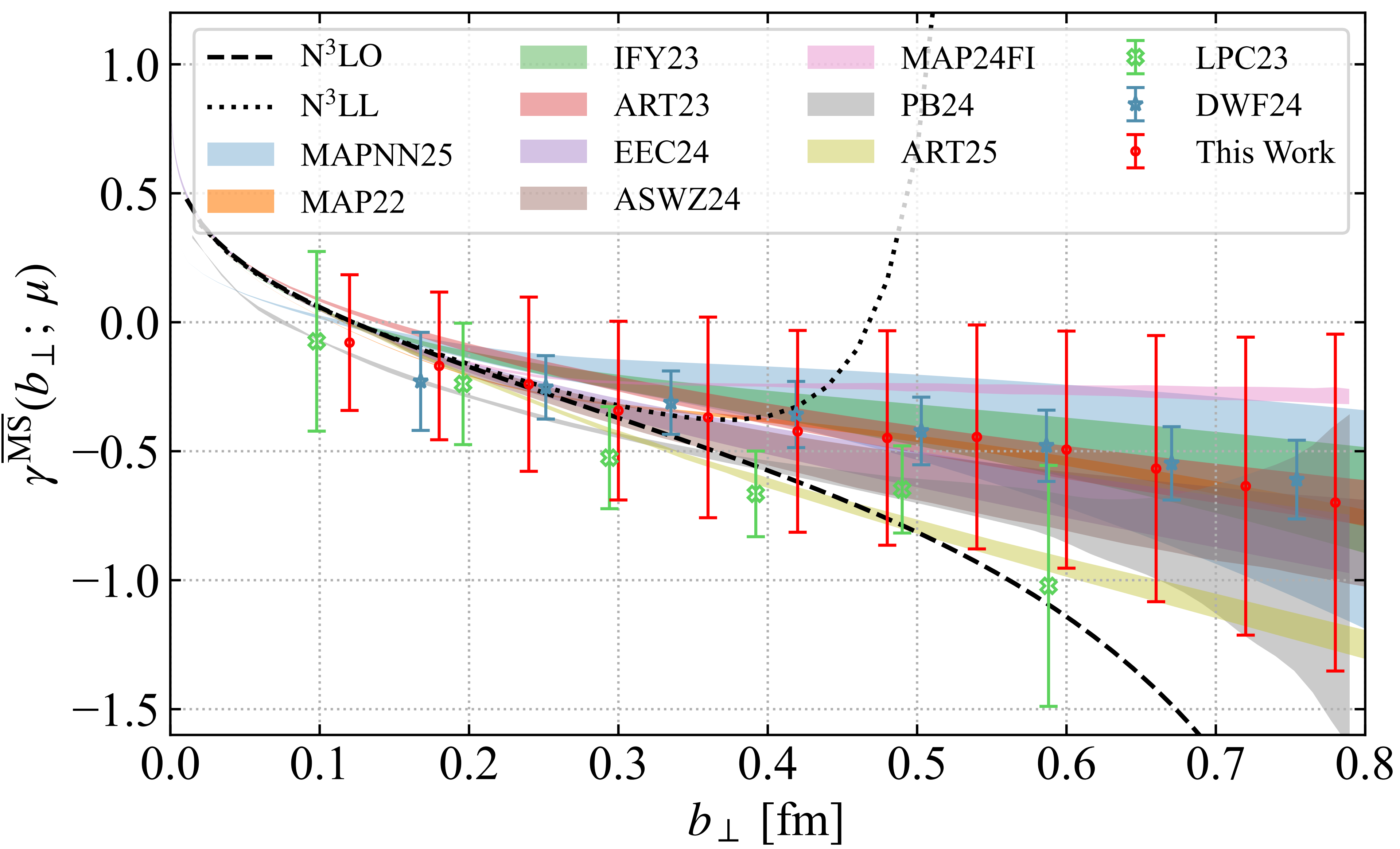
CS kernel

Collins-Soper Kernel

$$\gamma^{\overline{\mathrm{MS}}}(b_\perp, P_1, P_2; \mu) = \frac{1}{\ln\left(P_2/P_1\right)} \ln \frac{H_\phi\left(x, \bar{x}, P_1; \mu\right) \tilde{\phi}_{\gamma^z\gamma_5}\left(x, b_\perp, P_2; \mu\right)}{H_\phi\left(x, \bar{x}, P_2; \mu\right) \tilde{\phi}_{\gamma^z\gamma_5}\left(x, b_\perp, P_1; \mu\right)}$$

$H_\phi(x, \bar{x}, P^z; \mu) = C_{\mathrm{TMD}}(xP^z; \mu) \cdot C_{\mathrm{TMD}}(\bar{x}P^z; \mu)$ **is the TMD hard kernel for matching.**





- **DWF24 is another lattice calculation using the CG method on chirally symmetric domain-wall fermion configurations;**
- **There is a notable tension among recent results in phenomenology (MAP24FI & ART25);**
- **Both remain consistent with this work due to the large uncertainty;**
- **The large uncertainty is mainly caused by the small Lorentz boost factor at such a heavy pion mass ($m_\pi^{\mathrm{val}} = 670$ MeV);**

Pion Form Factor

- **The form factor is defined as**

$$F\left(b_\perp, P^z, \Gamma\right) \equiv -4N_c \frac{\langle -P^z \,|\, \bar{q}\left(b_\perp\right)\Gamma q\left(b_\perp\right)\bar{q}(0)\Gamma q(0) \,|\, P^z\rangle}{f_\pi^2\left((P^t)^2+(P^z)^2\right)}$$

We choose $\Gamma \in \{\gamma^\perp, \gamma^\perp\gamma^5\}$ to get leading-twist contribution, then take the Fierz rearrangement.

- **It can be extracted from the ratio**

$$R_F\left(b_\perp, P^z, \Gamma\right) \equiv -4N_c \frac{\langle -P^z \,|\, \bar{q}\left(b_\perp\right)\Gamma q\left(b_\perp\right)\bar{q}(0)\Gamma q(0) \,|\, P^z\rangle}{\langle 0 \,|\, \bar{q}(0)\gamma^\mu\gamma^5 q(0) \,|\, P^z\rangle \langle -P^z \,|\, \bar{q}(0)\gamma_\mu\gamma^5 q(0) \,|\, 0\rangle}$$

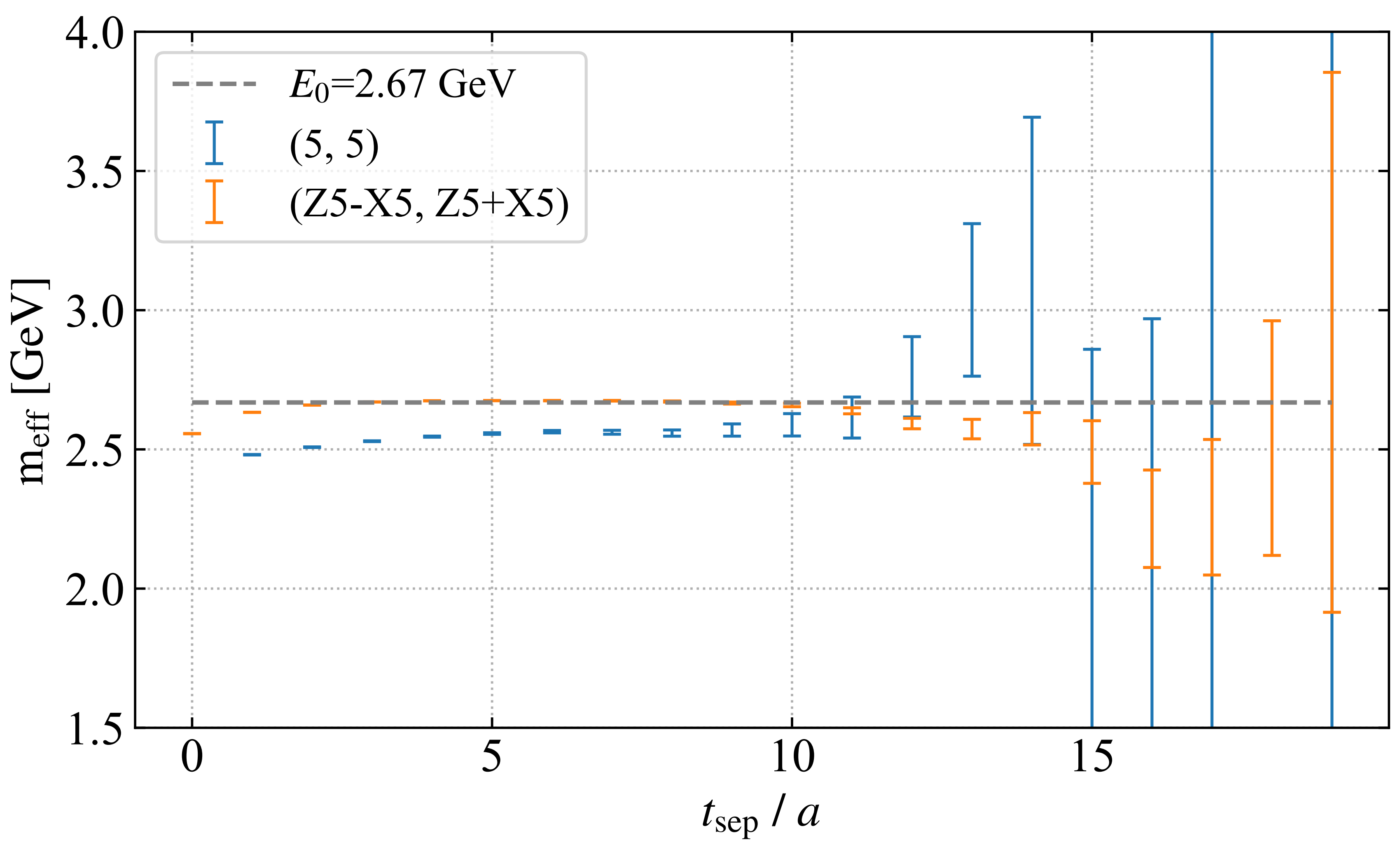
- **The ratio in terms of correlators on lattice**

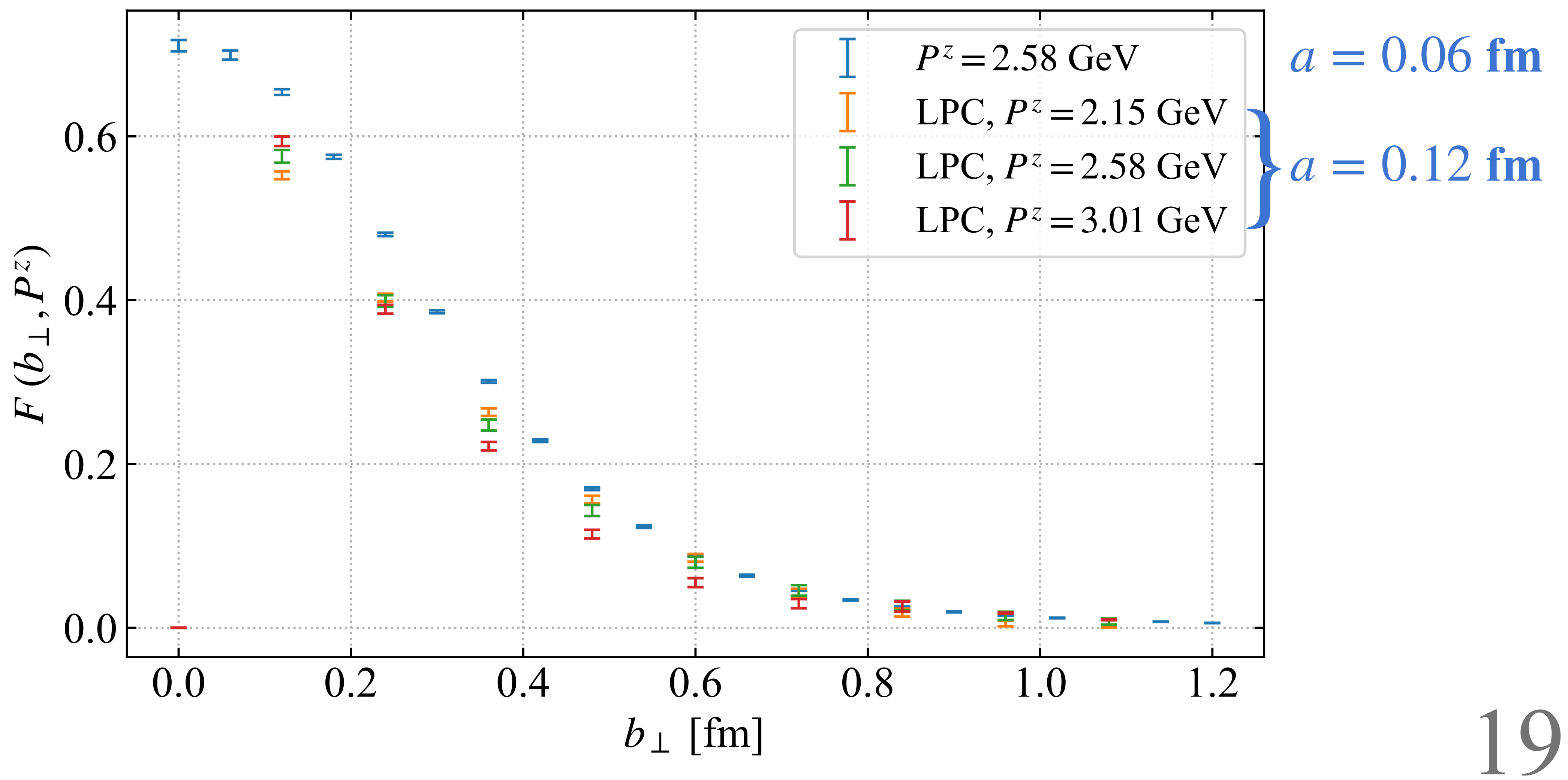
$$R_F\left(t_{\rm sep}, \tau\right) = \frac{-4N_c}{1+(P^t/P^z)^2} \frac{C_{\rm F}\left(t_{\rm sep}, \tau\right)}{\left|C_{\rm 2pt}\left(t_{\rm sep}/2\right)\right|^2}$$

The 2pt is calculated using the new interpolating operator.

Kinematically-enhanced interpolating operator

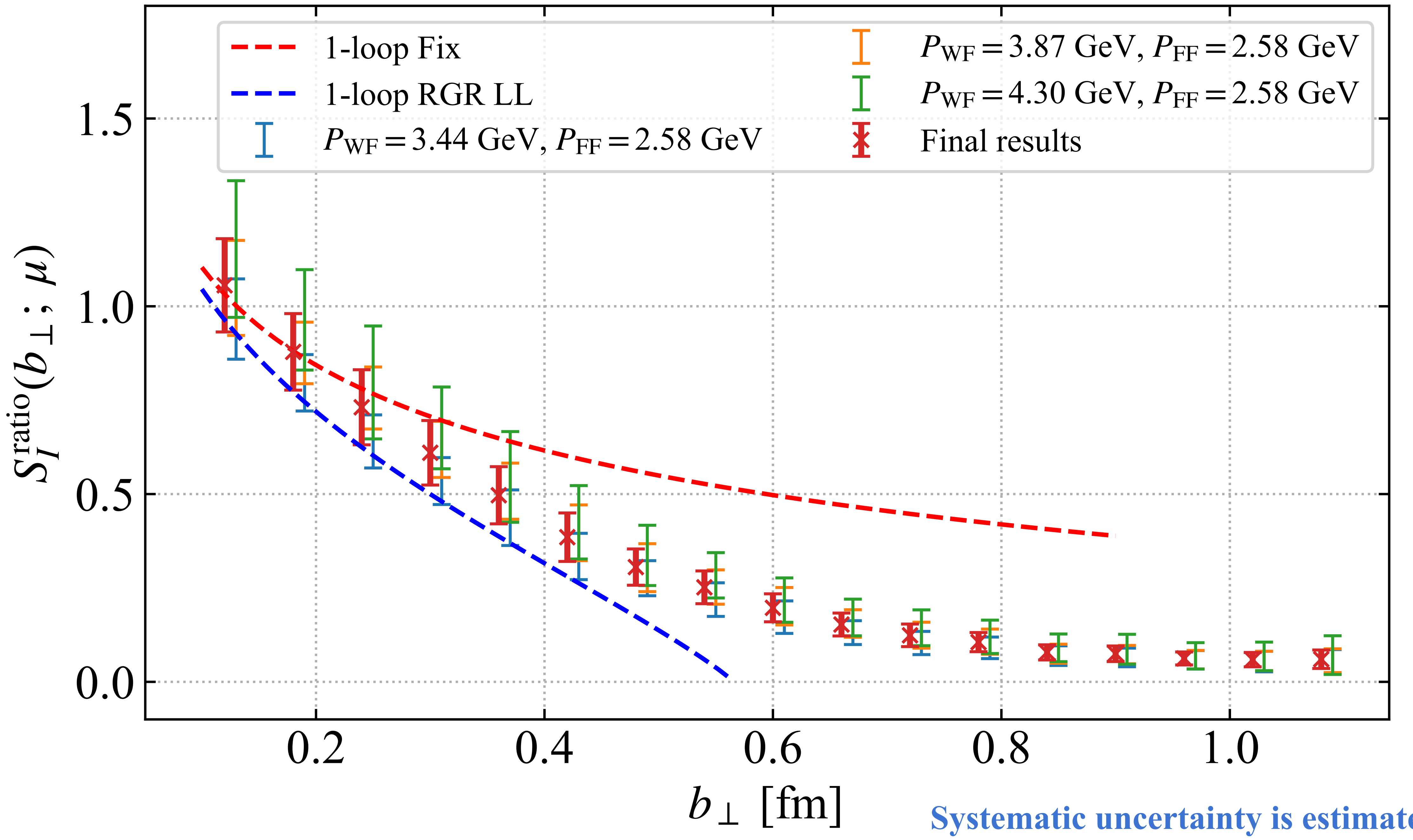
R. Zhang, et al., 2501.00729





Intrinsic Soft Function

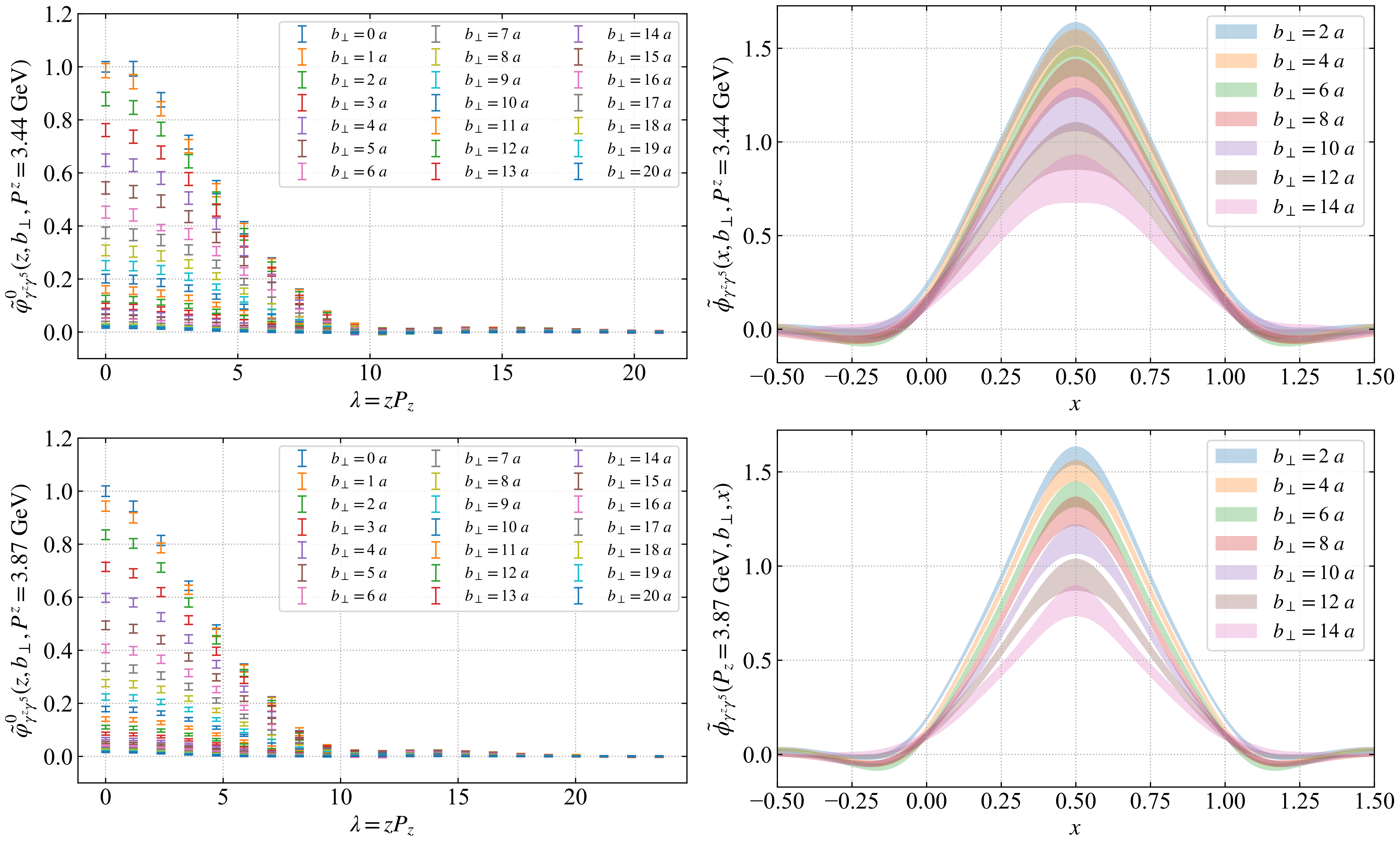
$$S_I(b_\perp;\mu) = \frac{F(b_\perp, P^z)}{\int dx_1 dx_2 H_F(x_1, x_2, P^z;\mu)\tilde{\Phi}^\dagger(x_1)\tilde{\Phi}(x_2)} \text{ with } \tilde{\Phi}(x) \equiv \frac{\tilde{\phi}_\Gamma\left(x, b_\perp, P^z;\mu\right)}{H_\phi\left(x, \bar{x}, P^z;\mu\right)}$$



Systematic uncertainty is estimated by the spread of the mean values between these three momentum pairs.

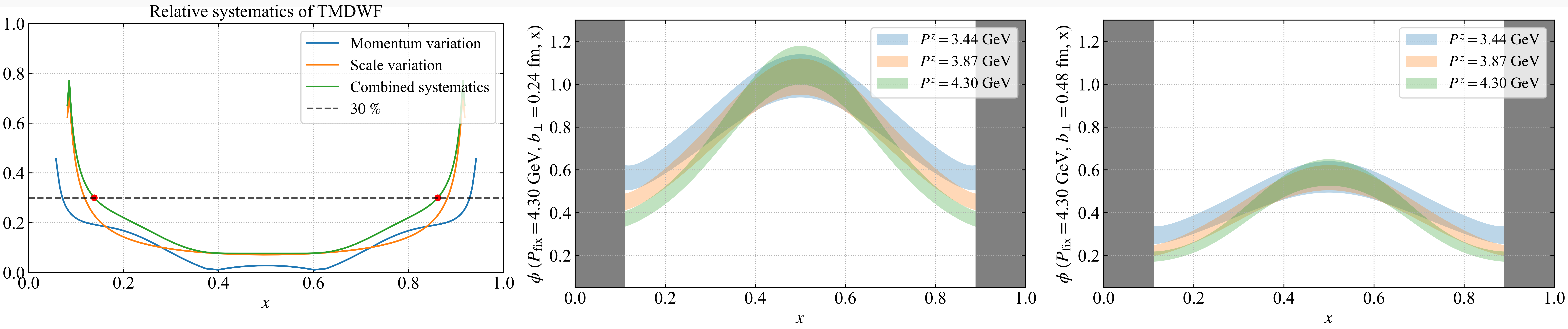
- **Our lattice results are consistent with the perturbation theory in the small $b_\perp$ regime;**
- **Different momenta of quasi-TMDWF give consistent results;**
- **Thanks to the absence of linear divergence, our final results of the intrinsic soft function can go beyond $b_\perp \sim 1$ fm.**

Pion quasi-TMD Wave Function



- We did a discretized Fourier transform because of the good convergence in the λ-space;
- The CG matrix elements decay to zero with the error bars remain almost constant, making the FT easy to be under control.

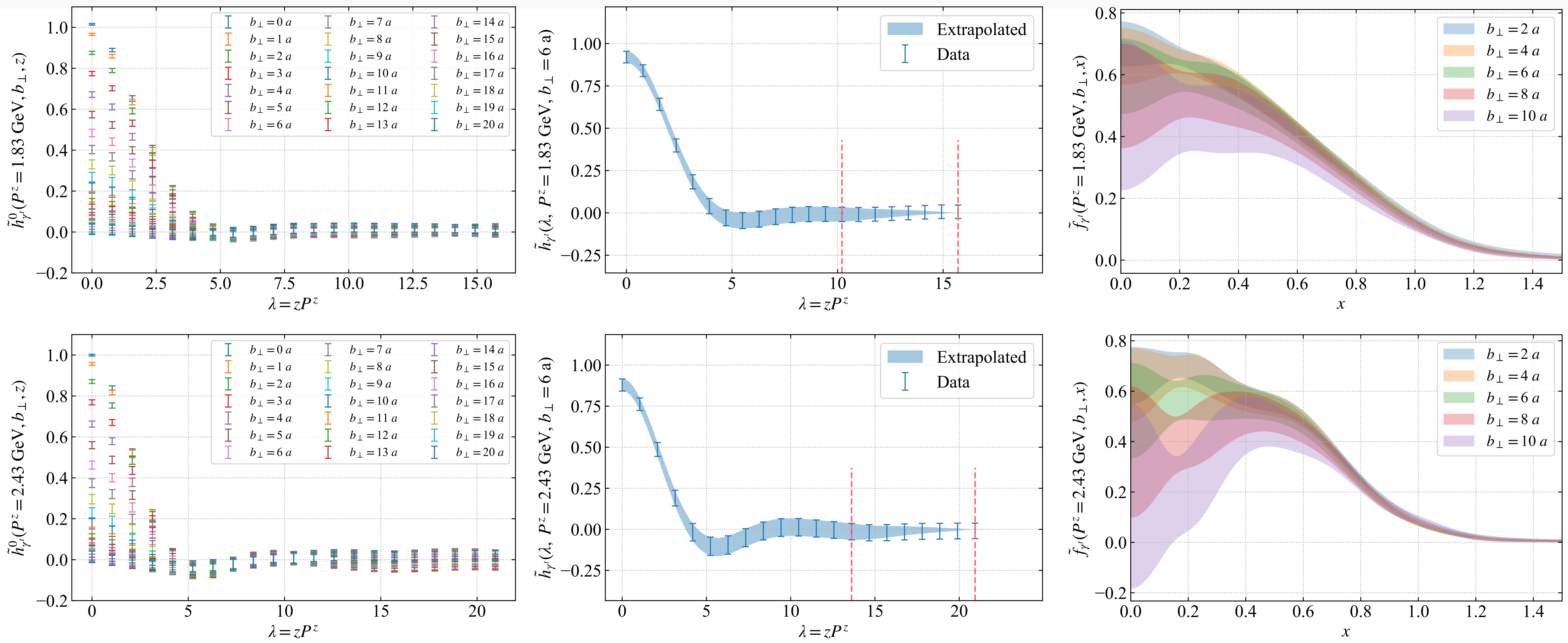
Pion TMD Wave Function



$$\sqrt{S_I\left(b_\perp;\mu\right)} \cdot \tilde{\phi}_\Gamma\left(x, b_\perp, P^z;\mu\right) = \phi(x, b_\perp;\mu,\zeta,\bar{\zeta}) H_\phi\left(x,\bar{x},P^z;\mu\right) \exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\rm MS}}\left(b_\perp;\mu\right)\right] + \text{Power corrections}$$

- **The variation between different momenta remains mild in the moderate x region, demonstrating the validity of power expansion in large P^z;**
- **The combined systematics are estimated from two sources:**
  - **Momentum variation: spread of central values between three momenta / mean of central values of three momenta**
  - **Vary the initial scale in the RG resummation of matching kernel by a factor of $\sqrt{2}$;**
- **The 30 % combined systematics are used to quantify the moderate x region that LaMET can make reliable predictions;**
- **The convergence between three momenta near the endpoint regions can be improved with larger Lorentz boost factor.**

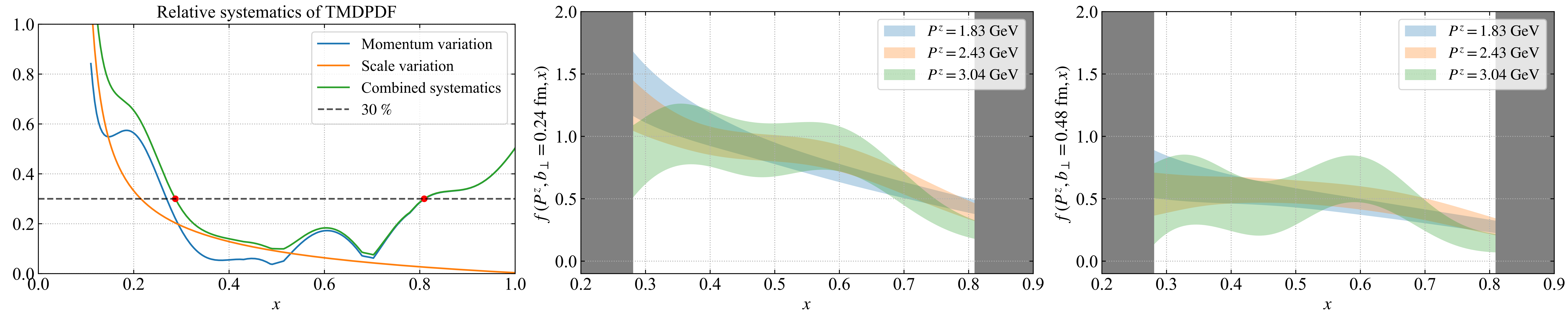
Pion quasi-TMD Beam Function



- **The CG matrix elements decay to zero with the error bars remain almost constant, making the FT easy to be under control.**
- **To remove the non-physical oscillation, we apply the extrapolation to make error bars converge to zero smoothly.**
- **Since quasi-TMD (in moderate x) is insensitive to the extrapolation strategies, the non-fit extrapolation is adopted here:**

$\tilde{h}^{\text{ext}} = w \cdot \tilde{h} + (1 - w) \cdot 0$, **where the weight $w(z)$ linearly decays from 1 to 0 within two red dashed lines below.**

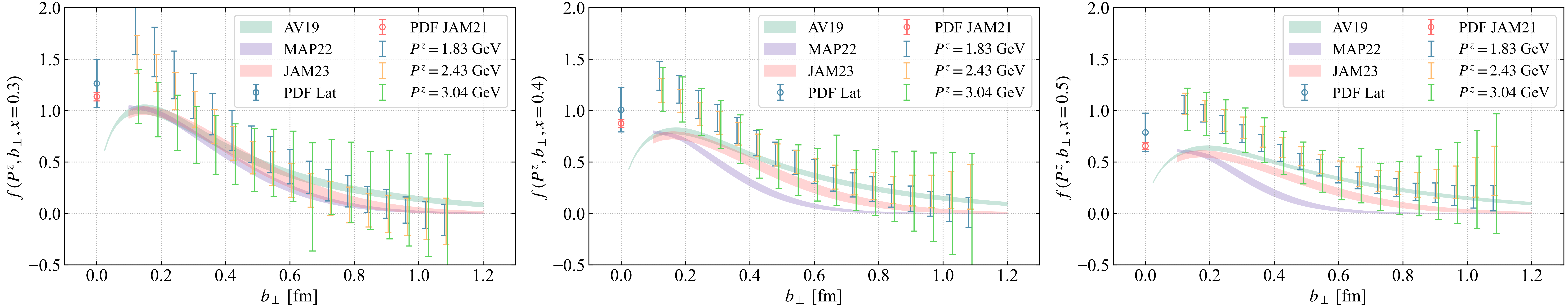
Pion TMDPDF in the x Space



$$\sqrt{S_I(b_\perp;\mu)} \cdot \tilde{f}_\Gamma(x, b_\perp, P^z;\mu) = f(x, b_\perp;\mu,\zeta)\, H_f(x, P^z;\mu) \exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}(b_\perp;\mu)\right] + \text{Power corrections}$$

- **The variation between different momenta remains mild in the moderate x region, demonstrating the validity of power expansion in large P^z;**
- **The combined systematics are estimated from two sources:**
  - **Momentum variation: spread of central values between three momenta / mean of central values of three momenta**
  - **Vary the initial scale in the RG resummation of matching kernel by a factor of $\sqrt{2}$;**
- **The $30\,\%$ combined systematics are used to quantify the moderate x region that LaMET can make reliable predictions;**

Pion TMDPDF in the $b_\perp$ Space

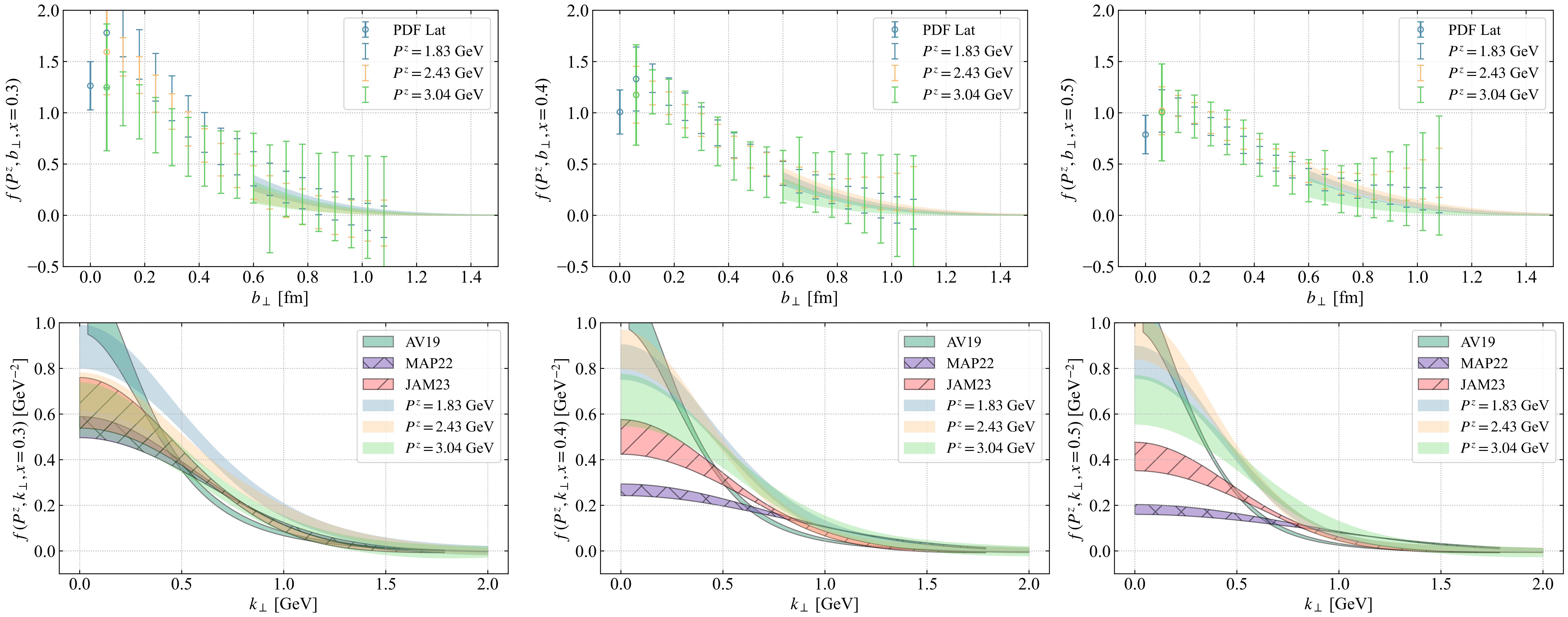


A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), Phys.Rev.D 107 (2023); P. C. Barry, et al. (JAM) Phys.Rev.D 108 (2023)

- **Thanks to the absence of linear divergence, we can calculate pion TMDPDF up to $b_\perp > 1$ fm;**
- **When x gets larger, the amplitude of TMDPDF is decreasing, while the the transverse correlation length stays roughly the same;**
- **When x gets closer to $x = 0.5$, we can find that the variance across different momenta becomes smaller, indicating the suppression of power correction;**
- **While when x gets closer to $x = 0.5$, the deviation from global analysis becomes larger, which may cause by the fact that the experimental data gives better constraints to the small x region;**

Pion TMDPDF in the $k_\perp$ Space

- **We can give the $k_\perp$-dependence thanks to the good SNR in CG;**
- **Extrapolate the large $b_\perp$ using a simple Gaussian form:** $f(b_\perp) = Ae^{-mb_\perp^2}$ **;**
- **Fourier transform to the $k_\perp$ space:** $\tilde{f}\left(k_\perp\right) = \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{\vec{b_\perp}\cdot\vec{k}_\perp} f\left(b_\perp\right) = \int \frac{d\left|b_\perp\right|}{2\pi} \left|b_\perp\right| \cdot J_0\left(\left|b_\perp\right| \cdot \left|k_\perp\right|\right) \cdot f\left(b_\perp\right).$



A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), Phys.Rev.D 107 (2023); P. C. Barry, et al. (JAM) Phys.Rev.D 108 (2023)

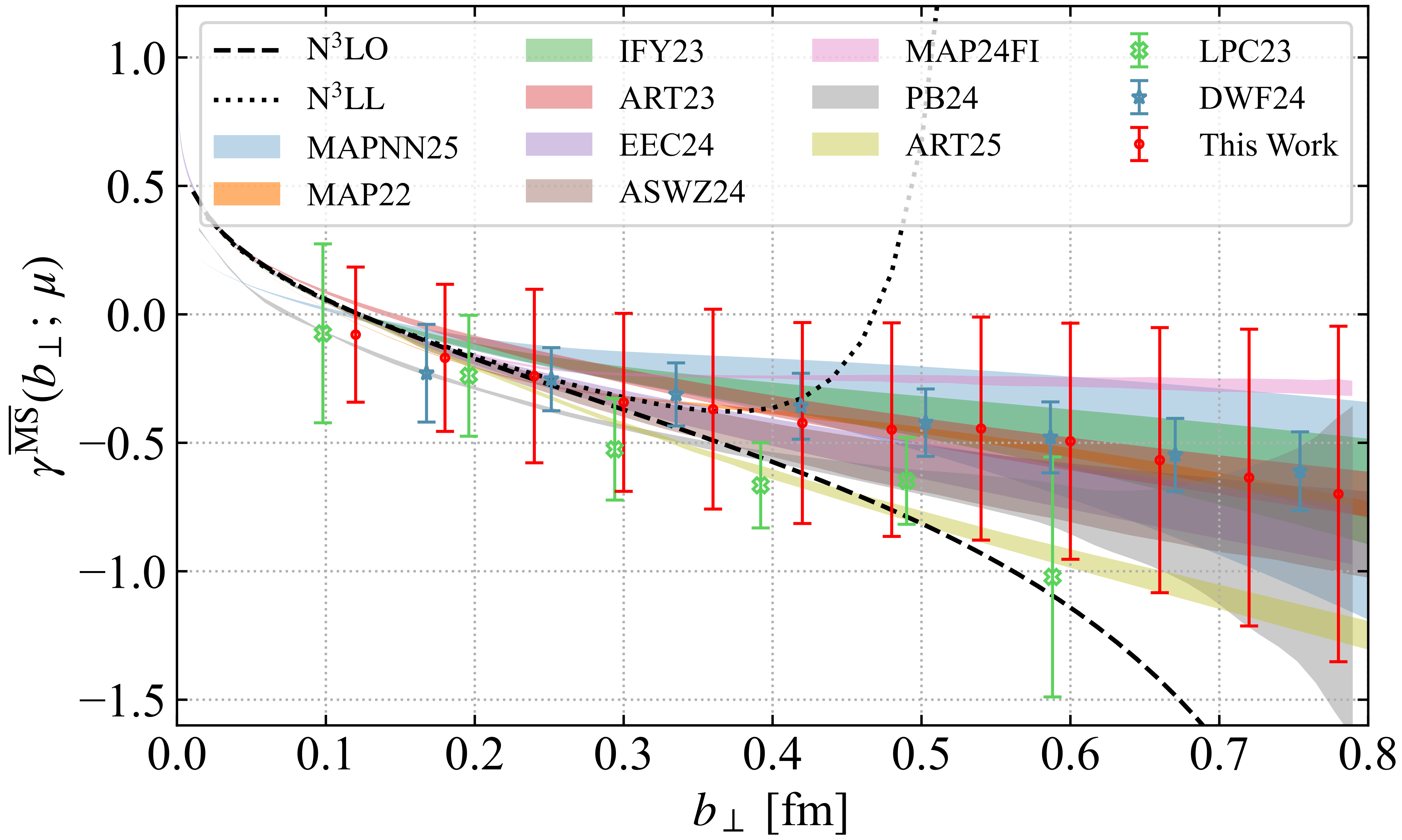
Summary

Summary

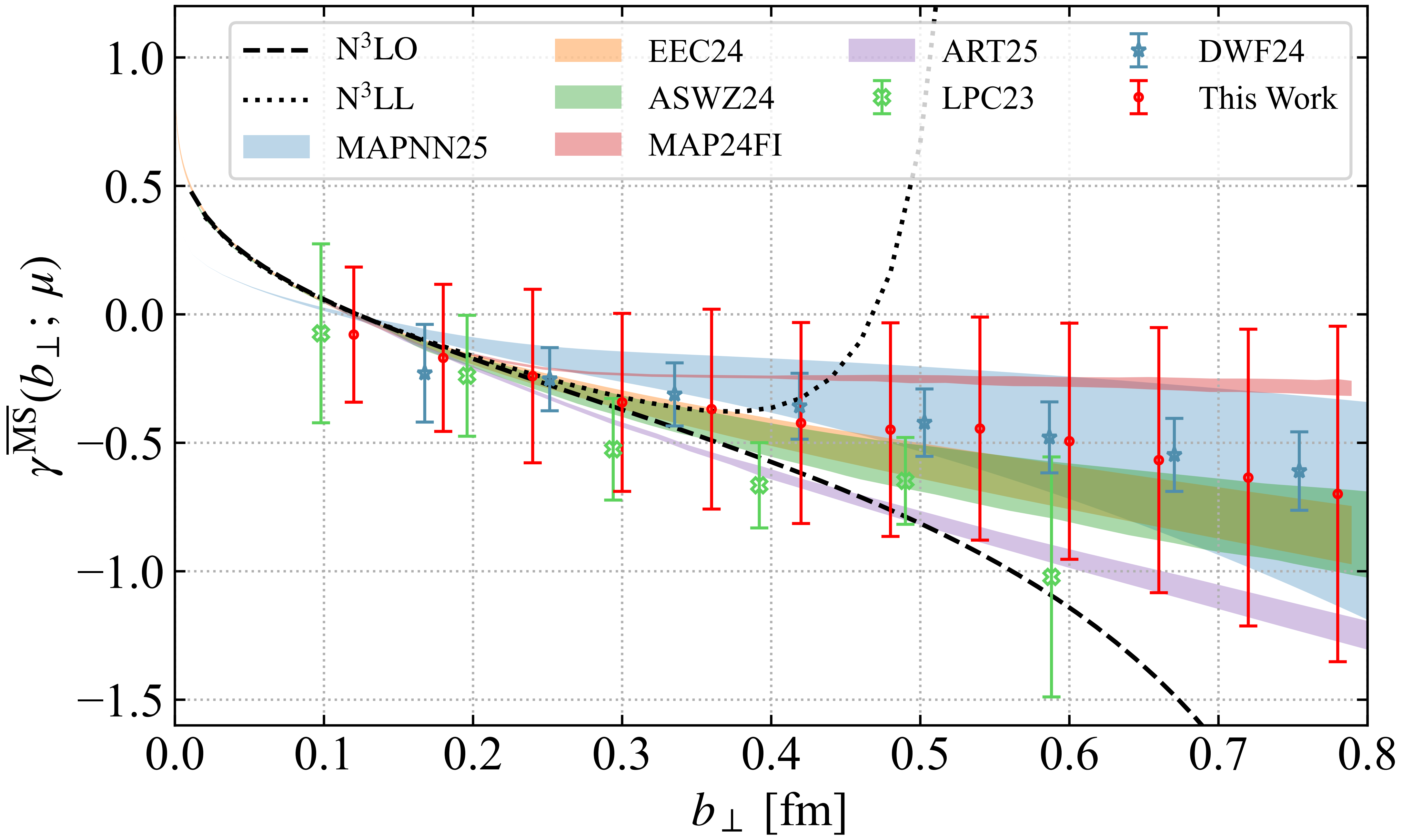
- This is the first lattice calculation of the pion unpolarized TMDPDF within LaMET framework;
- The novel CG method is employed to remove the linear divergence, so that to have a good SNR up to $b_\perp > 1$ fm;
- The soft function is extracted at NLL factorization using RG resummation, the results show consistency with perturbation theory;
- The TMDs, including CS kernel, intrinsic function, TMDWF and TMDPDF are calculated using the same lattice ensemble, and the results show consistency with existing studies, including phenomenology and lattice calculations;
- The outcome of this study highlights the efficacy of the CG quasi-TMD approach in probing the transverse momentum structure of hadrons;
- In the future work, we will apply the CG quasi-TMD approach on nucleon, and the lattice systematics like discretization effects and non-physical pion mass will be investigated in detail.

Backup

CS Kernel



N^{3}LO
N^{3}LL
MAPNN25
MAP22
IFY23
ART23
EEC24
ASWZ24
MAP24FI
PB24
ART25
LPC23
DWF24
This Work
$\gamma^{\overline{\mathrm{MS}}}(b_\perp;\mu)$
$b_\perp$ [fm]



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$b_\perp$ [fm]

Gauge Fixing in Lattice QCD

Continuous Theory

$$F_{\mathrm{CG}}[A,\Omega] \equiv \frac{1}{2}\sum_{\mu=1}^{3}\int d^4x A^a_{\Omega\mu}(x)A^{\mu a}_{\Omega}(x)$$

$$\begin{aligned}\delta F_{\mathrm{CG}}[A,\Omega] &= -\sum_{\mu=1}^{3}\int d^4x(D^{\Omega}_{\mu ab}\theta_b)A^{\mu a}_{\Omega}\\ &= -\sum_{\mu=1}^{3}\int d^4x(\partial_\mu\theta_a - g f^{cab}A^c_{\Omega\mu}\theta_b)A^{\mu a}_{\Omega}\\ &= \sum_{\mu=1}^{3}\int d^4x\theta_a(\partial_\mu A^{\mu a}_{\Omega})\end{aligned}$$

$* \; A_{\Omega\mu}(x) \equiv \Omega^\dagger(x)A_\mu(x)\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$

Lattice Theory

$$F_{\mathrm{CG}}[U,\Omega] \equiv -\Re\left[\mathrm{Tr}\sum_{x}\sum_{\mu=1}^{3}\Omega^\dagger(x+\hat{\mu})U_\mu(x)\Omega(x)\right]$$

Find stationary points of the functional value.

Gauge fixing criterion in this work: variation of functional satisfies $\delta F/F < 10^{-8}$.

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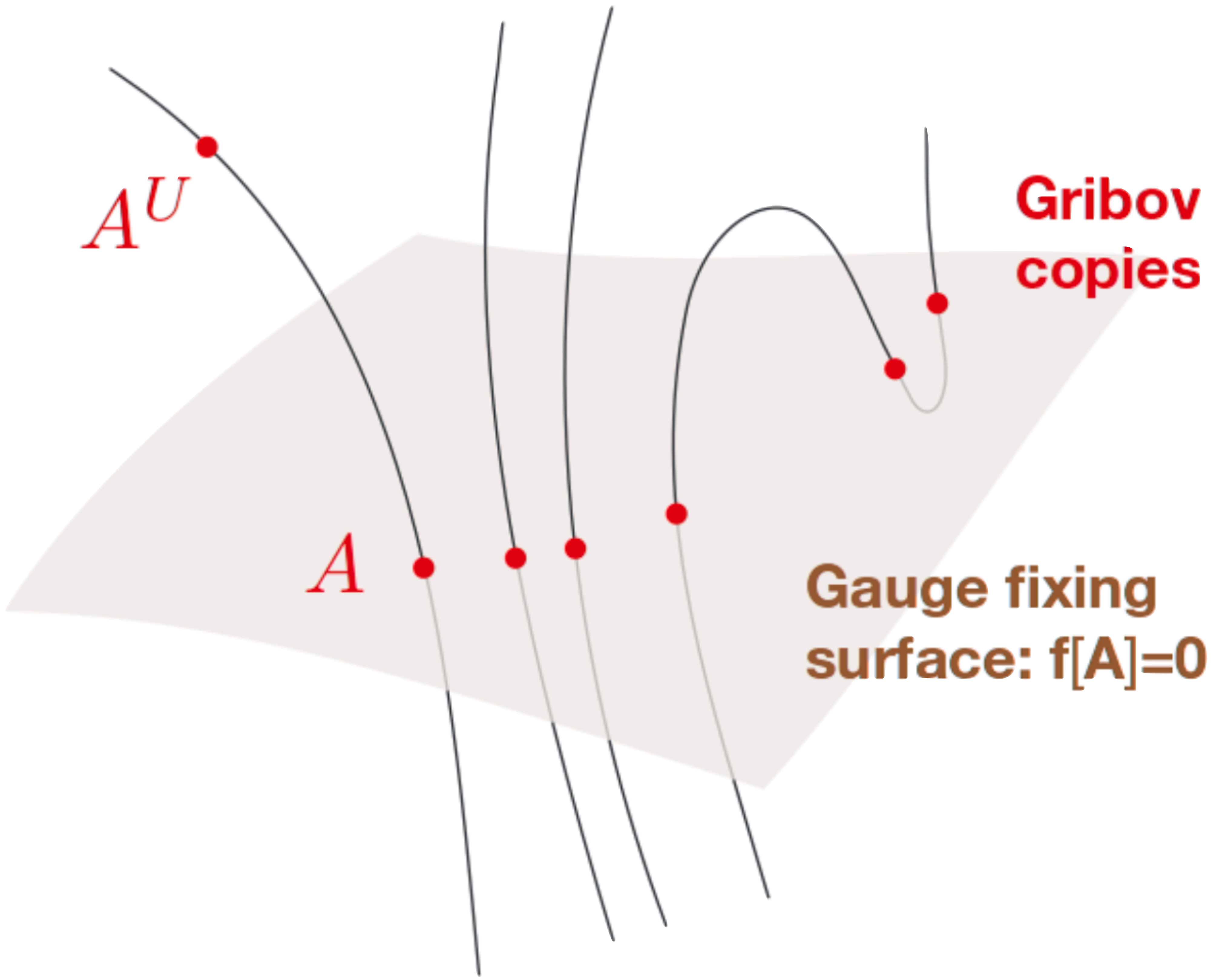
The gauge fixing condition may have many solutions in Lattice QCD.



Ph. D. Thesis of Diego Fiorentini

Criteria of Gauge Fixing

- **Variation of the functional**

$$\delta F/F < 10^{-8}$$

- **Residual gradient of the functional**

$$\theta^G \equiv \frac{1}{V}\sum_x \theta^G(x) \equiv \frac{1}{V}\sum_x \mathrm{Tr}\left[\Delta^G(x)\left(\Delta^G\right)^\dagger(x)\right],\ \Delta^G(x) \equiv \sum_\mu \left(A_\mu^G(x) - A_\mu^G(x-\hat{\mu})\right)$$