612 Final review

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Heat capacity

$$n = g\int \frac{dk}{2\pi^2}k^2 \cdot n_f\left(\frac{\epsilon-\mu}{T}\right)$$

With steep descent,

$$\frac{u}{n} = \frac{u_0+\Delta u}{n_0+\Delta n}$$

$$\Delta n = g\sum_{n=0} \frac{f^{(n)}(\epsilon)}{n!}\bigg|_{\epsilon=\mu} \cdot T^{n+1}C_n$$

$$\Delta u = g\sum_{n=0} \frac{(\epsilon\cdot f)^{(n)}(\epsilon)}{n!}\bigg|_{\epsilon=\mu} \cdot T^{n+1}C_n$$

$$C_n = \int dy\, y^n \cdot \Delta(y)$$

$$f(\epsilon) = \frac{(2m)^{3/2}}{4\pi^2}\epsilon^{1/2}$$

we got

$$\frac{u}{n} = \frac{U}{N} = \frac{3}{5}\epsilon_f\left(1+\frac{5}{12}\pi^2\left(\frac{T}{\epsilon_f}\right)^2+\dots\right)$$

$$C_V = \frac{dU}{dT}\bigg|_{N,V} = \frac{N\pi^2}{2\epsilon_f}T + O\left(T^2\right)$$

Lattice

$$\frac{1}{2}\log\left(\frac{1+x}{1-x}\right) = \tanh^{-1}(x)$$

<u>Critial exponent</u>

Specific heat C, order parameter Ψ, susceptibility χ and correlation length ξ.

With $t = \dfrac{T-T_c}{T}$, for $t > t_0$

$$C\sim t^{-\alpha},\ \Psi\sim 0,\ \chi\sim t^{-\gamma},\ \xi\sim t^{-\nu}$$

for $t < t_0$

$$C\sim(-t)^{-\alpha'},\ \Psi\sim(-t)^{-\beta},\ \chi\sim(-t)^{-\gamma'},\ \xi\sim(-t)^{-\nu'}$$

where $\alpha,\ \alpha',\ \beta,\ \gamma,\ \gamma' > 0$.

In ising model we have $\Psi = \langle \sigma \rangle$ and $\chi = \frac{\partial \sigma}{\partial h}\Big|_{h=0}$.

Two laws,

$$\alpha + 2\beta + \gamma = 2$$

$$\nu = \frac{2-\alpha}{d}, \ where\ d\ is\ dimension\ of\ space$$

Bubble

If we have a bubble of phase II with free energy density f_2 in the sea of phase I with free energy density f_1, and $f_2 < f_1$.

Then the free energy difference is

$$\Delta F = V(f_2 - f_1) + \sigma A = \frac{4\pi}{3} R^3 (f_2 - f_1) + 4\pi R^2 \sigma$$

$$\frac{\partial(\Delta F)}{\partial R} = 8\pi R\sigma - 4\pi R^2 (f_1 - f_2)$$

If $\frac{\partial(\Delta F)}{\partial R} < 0$, then $R > \frac{2\sigma}{f_1 - f_2} = R_c$. That means when bubble is big, it will grow bigger to get lower free energy, when bubble is small, it will shrink to get lower free energy.